ENGINEERING TRIPOS PART IIA
ENGINEERING TRIPOS PART IIB

Friday 2 May 2008 2.30 to 4

Module 4D8

PRESTRESSED CONCRETE

*Answer **one** question from Section A and **two** questions from Section B. Questions from Section A carry **twice** as many marks as questions from Section B.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: None

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 The beam shown in Fig. 1 has a prestressing cable of cross sectional area 500 mm^2 placed 160 mm above the bottom fibre, and reinforcing steel of area 800 mm^2 placed 80 mm above the bottom. Before the concrete was cast, the cable was pre-tensioned to 800 N/mm^2. Young's modulus for both types of steel is 200 kN/mm^2. Young's modulus for short term loads on concrete is 25 kN/mm^2 and for long term loads is 10 kN/mm^2. Over a long period of time, the shrinkage strain of concrete is 300×10^{-6}.

(a) By considering compatibility of strain, derive relationships which will allow the determination of the stresses in the concrete at different stages. Use the effective modulus method, and assume that no significant forces other than the prestress act on the beam. [30%]

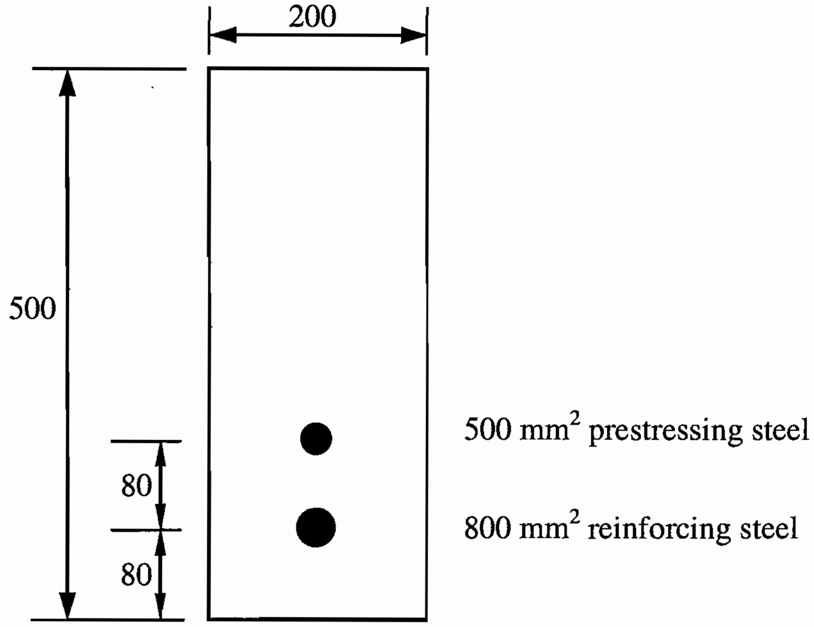
(b) Determine the mean stress in the concrete, the maximum stress in the concrete and the stress in the prestressing tendon after all creep and shrinkage have taken place. [30%]

(c) Immediately after prestressing the maximum compressive stress in the concrete was 7.0 N/mm^2 and the tensile stress in the prestressing steel was 770 N/mm^2. [These values were determined using the relationship specified in (a).]

Find the ratio between the final and initial stresses for both:
(i) concrete; and
(ii) prestressing steel. [30%]

Why does the concrete appear to have lost a different proportion of its prestress? Explain which ratio is more important to designers of prestressed concrete. [10%]

(cont.



(all dimensions in mm)

Fig. 1

2 Figure 2 shows the stress-strain curves for prestressing steel (as a solid line) and aramid fibres (as a dashed line). Assume that the prestressing steel is prestressed to 800 MPa, and that it is always designed so that at failure the steel is on the top, perfectly plastic, portion of its stress-strain curve. The initial elastic modulus of steel is 200 GPa. Aramid fibres are linear elastic (with a Young's Modulus of 120 GPa) and are brittle, so they must be designed so that they never reach the top of the stress-strain curve; they will be prestressed to 1000 MPa. The concrete may be assumed to have a cube strength f_{cu} and to fail when the compressive strain reaches 0.0035; the stress throughout the compression zone may be taken as $0.6 f_{cu}$. The strains in the concrete due to the prestress alone can be assumed to be negligible.

For each material:

(a) Derive an expression for the limits on the cross-sectional area of the prestressing tendons that can be permitted for a rectangular section of effective depth d and breadth b. Indicate clearly whether this is an upper or lower limit. [40%]

(b) If the limiting area of tendon is provided, determine the corresponding ultimate applied moment and curvature. [40%]

(c) Describe qualitatively the mode of failure that will occur in pure bending in the following cases. Illustrate your answers with sketches of the strain distribution expected at failure, and indicate whether the failure load will be higher or lower than that determined in (b) above.

(i) The limit on the tendon area is just satisfied. [5%]

(ii) 20% less tendon material is provided than specified. [5%]

(iii) 20% more tendon material is provided than specified. [5%]

(iv) The limiting amount of tendon material is provided, but it is prestressed to only 80% of the values specified above. [5%]

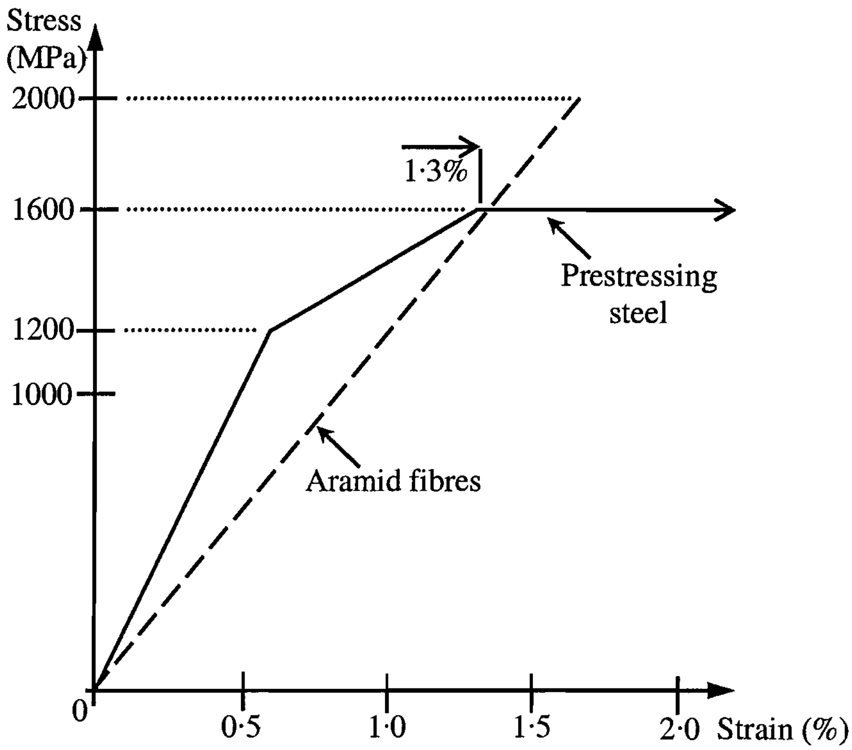

Stress
(MPa)
2000
1600
1200
1000
1·3%
Prestressing
steel
Aramid fibres
0
0·5
1·0
1·5
2·0
Strain (%)

Fig. 2

SECTION B

3 Show that the secondary moment, M_2 , over the internal support of a beam continuous over three supports can be found from

$$M_2 = \frac{\int_{length} \frac{\beta P e}{EI} dx}{\int_{length} \frac{\beta^2}{EI} dx}$$

where P is the force in the tendon, EI is the flexural stiffness of the beam, x is a position in the beam measured from one end and e is the tendon eccentricity. What is the variable β and how is it defined? [50%]

Hence show that the tendon defined by the profile shown in Fig. 3 is concordant. The tendon force and the beam stiffness may be assumed not to vary along the beam. [50%]

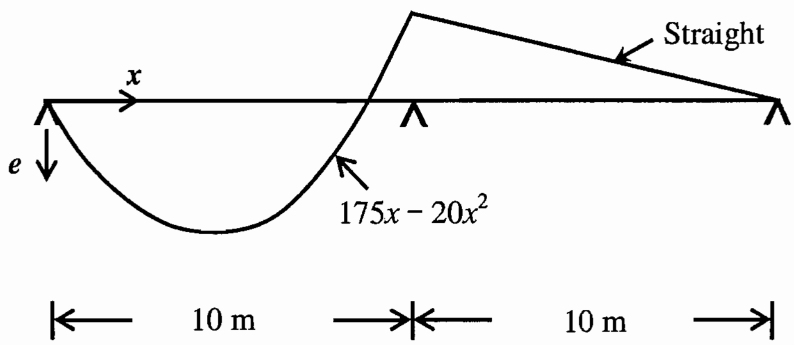


Fig. 3

4 A prestressing cable follows a path in three dimensions given in the table below. The effect of friction on the cable force is given by

$$P = P_0.e^{-(\mu\theta + kx)}$$

where P_0 is the force at the jack, μ is the coefficient of friction between the tendon and the duct, θ is the cumulative change in angle, k is the wobble factor and x is the distance from the jack. If the tendon area is A and its Young's modulus is E, take P_0/AE as 0.004. Take μ as 0.2 and k as 0.006 m^{-1}.

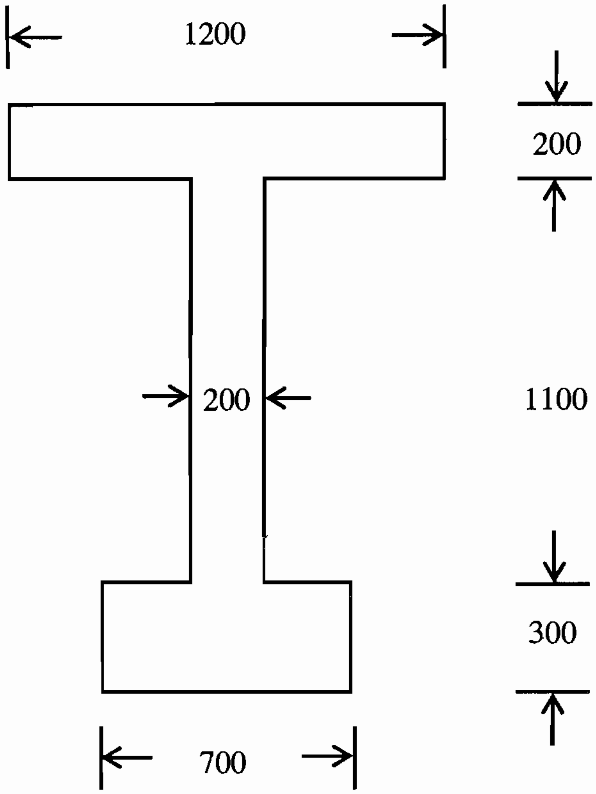
(a) Plot the variation in force in the tendon with distance from the jack while the jack is still attached. [50%]

(b) On the same plot, show the variation in prestressing force if the wedges "draw-in" by 3 mm at the end when that jack force is released. [50%]

Distance from jack x (m)	Vertical eccentricity e (m)	de/dx	Horizontal eccentricity s (m)	ds/dx
0	0	0.2	0	0
2	0.36	0.16	0	0
4	0.64	0.12	0	0
6	0.84	0.08	0	0
8	0.96	0.04	0	0
10	1	0	0	0
12	1	0	0.104	0.096
14	1	0	0.352	0.144
16	1	0	0.648	0.144
18	1	0	0.896	0.096
20	1	0	1	0
22	Then straight both vertically and horizontally			

5 A prestressed beam is designed to resist sagging bending moments ranging from 2000 kNm to 6000 kNm . The permissible stresses in concrete are 2 MPa in tension and 16 MPa in compression. A section has been chosen as shown in Fig. 4; the second moment of area of the section is $2.29 \times 10^{11}\ \text{mm}^4$ and the centroid is located 863 mm above the bottom fibre.

Draw a Magnel diagram and choose a suitable prestressing force and eccentricity. [100%]



(all dimensions in mm)

Fig. 4

END OF PAPER