ENGINEERING TRIPOS PART IIB

Wednesday 23 April 2008 2.30 to 4

Module 4F1

CONTROL SYSTEM DESIGN

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Formulae sheet (3 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed
Supplementary pages: Two extra copies of Fig. 2 (Question 3).

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Briefly describe the advantages and disadvantages of using feedback to control a system. [25%]

(b) Suppose that $G(s)K(s)$ is stable, minimum phase and has at least second order roll-off at high frequencies and let $S(s)$ denote the sensitivity function.

(i) Show that [25%]

$$\int_0^\infty \ln|S(j\omega)|d\omega = 0.$$

(ii) Suppose the following specifications are required:

A: $|S(j\omega)| < \varepsilon$ for $0 \leq \omega \leq 1$,

B: $|S(j\omega)| < 1.2$ for $1 \leq \omega \leq 5$,

C: $|G(j\omega)K(j\omega)| < \frac{1}{\omega^2}$ for all $\omega \geq 5$.

Use C to obtain an upper bound on [25%]

$$\int_5^\infty \ln|S(j\omega)|d\omega.$$

[Hint: you may assume that

$$\int \ln(1-\omega^{-2})d\omega = \omega\ln(1-\omega^{-2}) + \ln\left(\frac{\omega+1}{\omega-1}\right).]$$

(iii) Hence find a postive number ε_0 such that the specifications are infeasible for $\varepsilon < \varepsilon_0$. [25%]

2 (a) Fig. 1 shows the block diagram of a two-degree-of-freedom control system where the plant to be controlled has a rational transfer function $G(s)$. A return-ratio transfer function $L(s) = C(s)G(s)$ has already been designed with a compensator $C(s)$ which internally stabilizes the plant. It is desired to choose $F(s)$, $K(s)$ and $H(s)$ to achieve a desired transfer function $T_{\bar{r}\to\bar{y}}$ relating $\bar{y}$ to $\bar{r}$ with $C(s) = F(s)K(s)$.

(i) State the minimum conditions that need to be imposed on $T_{\bar{r}\to\bar{y}}$. [10%]

(ii) Prove that the conditions of Part (a)(i) are necessary for internal stability. [15%]

(iii) Assuming only that $T_{\bar{r}\to\bar{y}}$ satisfies the conditions of Part (a)(i), describe how to construct $F(s)$, $K(s)$ and $H(s)$. [15%]

(b) A control system is to be designed for the plant $G(s) = 1/(s-2)$. A sensor has been selected which does not measure $\bar{y}(s)$ directly, but instead measures $\bar{w}(s) = F(s)\bar{y}(s)$ where $F(s) = \dfrac{s-1}{s+1}$.

(i) Find a $K(s)$ and $H(s)$ in Fig. 1 to achieve an internally stable closed-loop system and transfer function $T_{\bar{r}\to\bar{y}} = 1/(s+1)$. [Hint: a $K(s)$ with only one zero at -1 and only one pole is sufficient.] [40%]

(ii) If $K(s)$ is replaced by $kK(s)$ in your design where k is a constant, what is the range of k for which the closed-loop system is stable? [10%]

(iii) What advice should a control engineer give to his employer who requires a control system to be designed for this plant? [10%]

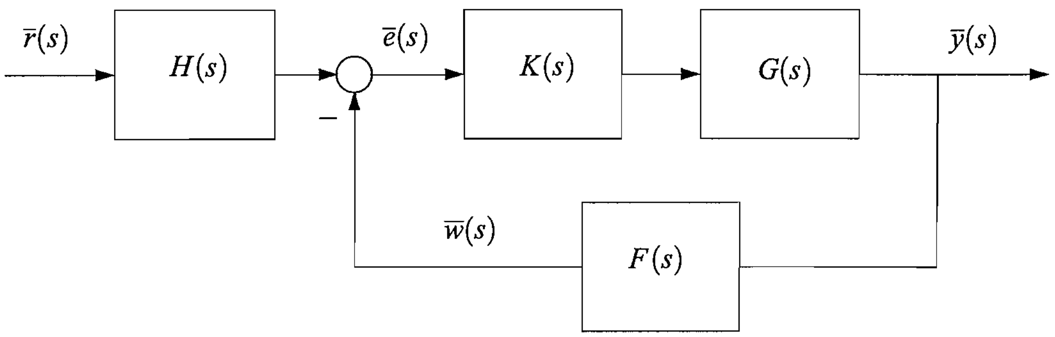


Fig. 1

(TURN OVER

3 Fig. 2 is the Bode diagram of a system $G(s)$ for which a feedback compensator $K(s)$ in the standard negative feedback configuration is to be designed. It may be assumed that $G(s)$ is a real-rational transfer function, and that all poles and zeros have moduli which lie within the range of frequencies shown on the diagram.

(a) (i) Sketch on a copy of Fig. 2 the expected phase of $G(j\omega)$ if $G(s)$ were stable and minimum phase. [10%]

(ii) Comment on whether $G(s)$ has any poles or zeros at the origin. [10%]

(iii) Determine whether $G(s)$ has any right half plane poles or zeros (it doesn't have both), and estimate their location (if there are any). [10%]

(iv) Comment on any limitations that this might impose on the achievable crossover frequency. [10%]

(b) Suppose a constant controller $K(s) = k$ is employed.

(i) Use a sketch of the Nyquist diagram to determine the number of right half plane poles of the closed-loop system for each k, as k varies over positive and negative values. [15%]

(ii) Find the value of k which gives a stable closed-loop system with the largest phase margin. [10%]

(c) A feedback compensator $K(s)$ is sought to provide internal stability of the closed-loop and satisfy $S(0) = 0$ where S is the sensitivity function. Explain why this is not achievable. [10%]

(d) Find a feedback compensator $K(s)$ which provides internal stability of the closed-loop and satisfies the following specifications:

A: $|G(j\omega)K(j\omega)| = 1$ at $\omega = 10$ rad/sec,

B: a phase margin of at least 45°.

Show on another copy of Fig. 2 the effect of this compensator on the return-ratio transfer function. [25%]

Two copies of Fig. 2 are provided on separate sheets. These should be handed in with your answers.

(cont.

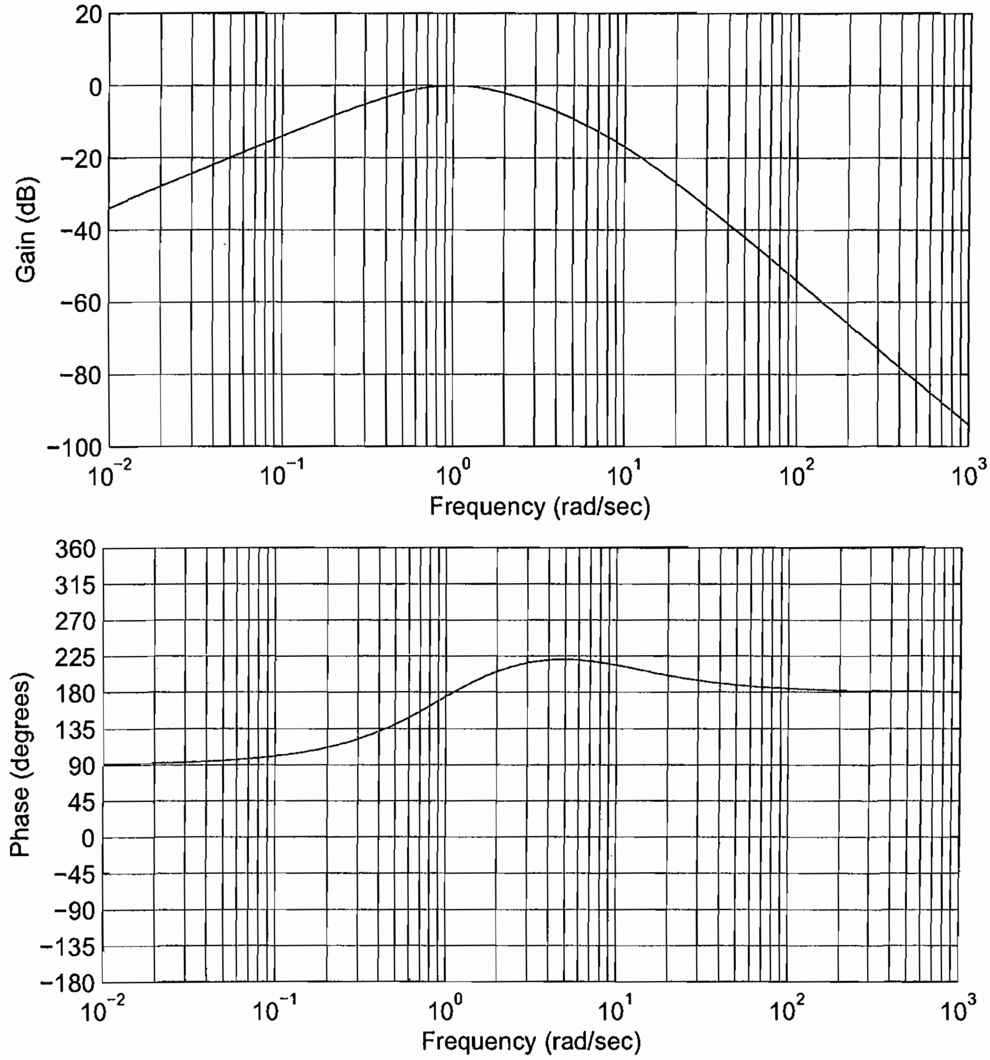


Fig. 2

END OF PAPER

Formulae sheet for Module 4F1: Control System Design

To be available during the examination.

1 Terms

For the standard feedback system shown below, the **Return-Ratio Transfer Function** $L(s)$ is given by

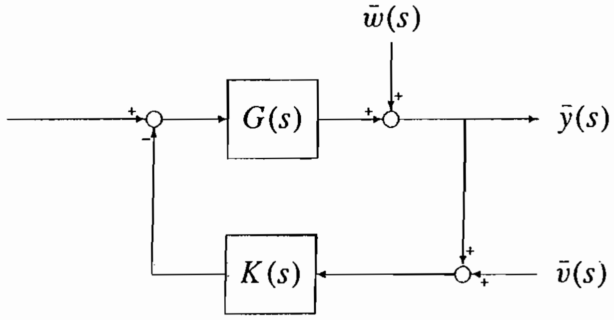
$$L(s) = G(s)K(s),$$

the **Sensitivity Function** $S(s)$ is given by

$$S(s) = \frac{1}{1 + G(s)K(s)}$$

and the **Complementary Sensitivity Function** $T(s)$ is given by

$$T(s) = \frac{G(s)K(s)}{1 + G(s)K(s)}$$



The closed-loop system is called **Internally Stable** if each of the *four* closed-loop transfer functions

$$\frac{1}{1 + G(s)K(s)}, \quad \frac{G(s)K(s)}{1 + G(s)K(s)}, \quad \frac{K(s)}{1 + G(s)K(s)}, \quad \frac{G(s)}{1 + G(s)K(s)}$$

are stable (which is equivalent to $S(s)$ being stable and there being no right half plane pole/zero cancellations between $G(s)$ and $K(s)$).
A transfer function is called **real-rational** if it can be written as the ratio of two polynomials in s, the coefficients of each of which are purely real.

2 Phase-lead compensators

The phase-lead compensator

$$K(s) = \alpha \frac{s + \omega_c/\alpha}{s + \omega_c \alpha}, \quad \alpha > 1$$

achieves its maximum phase advance at $\omega = \omega_c$, and satisfies:

$$|K(j\omega_c)| = 1, \quad \text{and} \quad \angle K(j\omega_c) = 2\arctan\alpha - 90°.$$

3 The Bode Gain/Phase Relationship

If

1. $L(s)$ is a real-rational function of s,
2. $L(s)$ has no poles or zeros in the *open* RHP ($\mathrm{Re}(s) > 0$) and
3. satisfies the normalization condition $L(0) > 0$.

then

$$\boxed{\angle L(j\omega_0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{d}{dv}\log|L(j\omega_0 e^v)| \log\coth\frac{|v|}{2}\,dv}$$

Note that

$$\log\coth\frac{|v|}{2} = \log\left|\frac{\omega+\omega_0}{\omega-\omega_0}\right|, \text{ where } \omega = \omega_0 e^v.$$

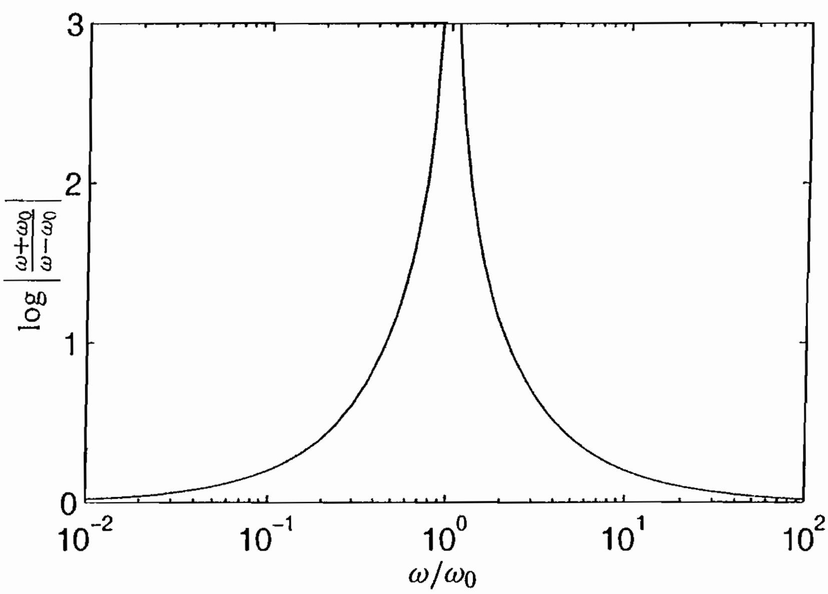


Figure 1:

If the slope of $L(j\omega)$ is approximately constant for a sufficiently wide range of frequencies around $\omega = \omega_0$ we get the *approximate form of the Bode Gain/Phase Relationship*

$$\angle L(j\omega_0) \approx \frac{\pi}{2}\left.\frac{d\log|L(j\omega_0 e^v|)}{dv}\right|_{\omega=\omega_0}.$$

4 The Poisson Integral

If $H(s)$ is a real-rational function of s which has no poles or zeros in $\mathrm{Re}(s) > 0$, then if $s_0 = \sigma_0 + j\omega_0$ with $\sigma_0 > 0$

$$\boxed{\log H(s_0) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sigma_0}{\sigma_0^2 + (\omega - \omega_0)^2} \log H(j\omega)\, d\omega}$$

and

$$\boxed{\log |H(s_0)| = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\cosh v \cos\theta}{\sinh^2 v + \cos^2\theta} \log |H(j|s_0|e^v)|\, dv}$$

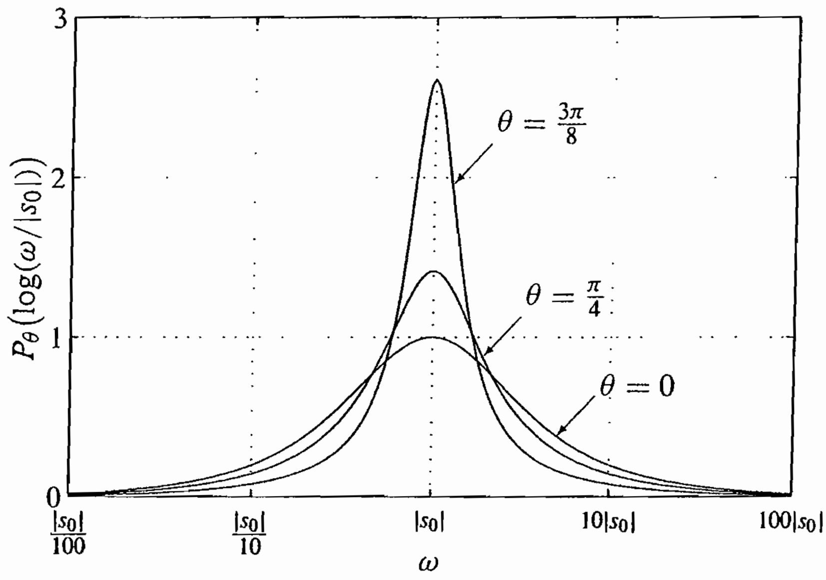
where $v = \log\left(\frac{\omega}{|s_0|}\right)$ and $\theta = \angle(s_0)$. Note that, if s_0 is real, so $\angle s_0 = 0$, then

$$\frac{\cosh v \cos\theta}{\sinh^2 v + \cos^2\theta} = \frac{1}{\cosh v}.$$

We define

$$P_\theta(v) = \frac{\cosh v \cos\theta}{\sinh^2 v + \cos^2\theta}$$

and give graphs of P_θ below.



The indefinite integral is given by

$$\int P_\theta(v)\, dv = \arctan\left(\frac{\sinh v}{\cos\theta}\right)$$

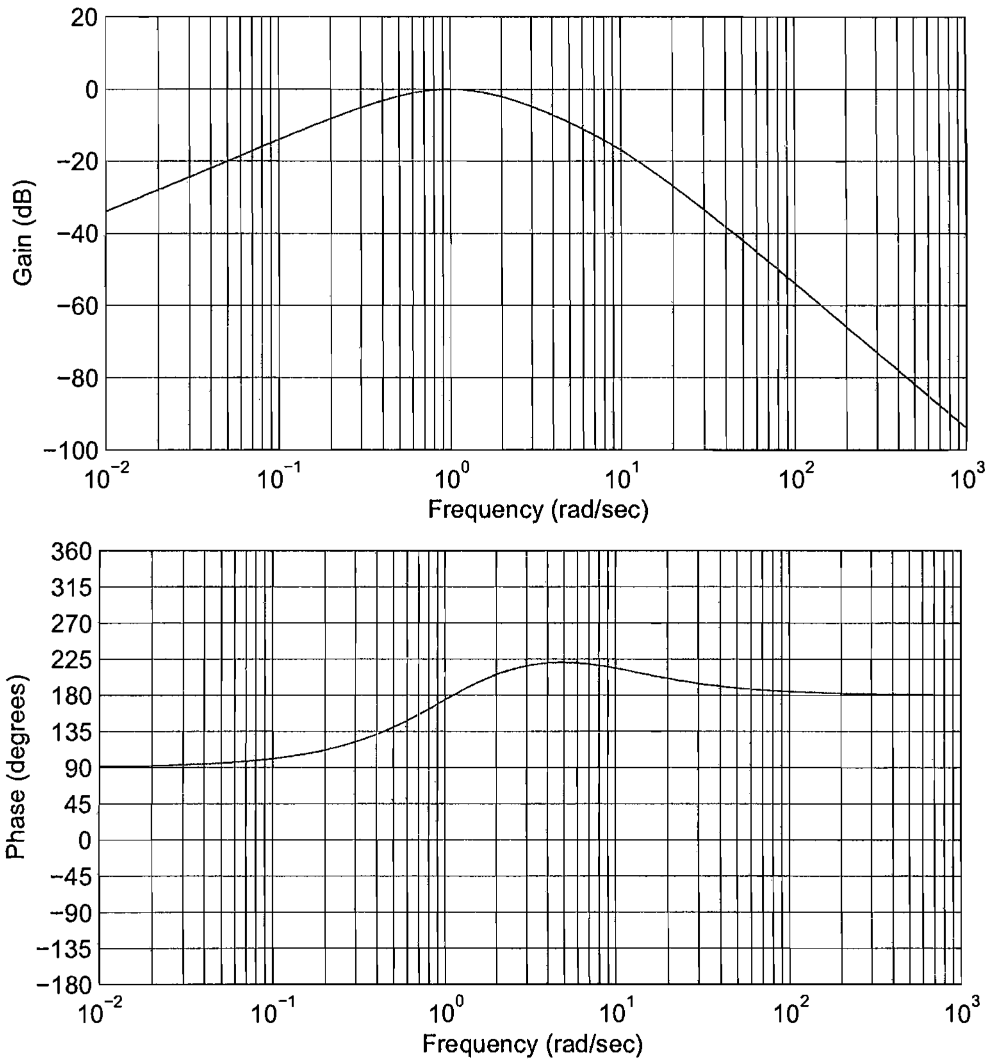
and

$$\frac{1}{\pi}\int_{-\infty}^{\infty} P_\theta(v)\, dv = 1 \quad \text{for all } \theta.$$

G. Vinnicombe
M.C. Smith
November 2002

ENGINEERING TRIPOS PART IIB

Wednesday 23 April 2008, Module 4F1, Question 3.



Extra copy of Fig. 2: Bode diagram of $G(s)$ for Question 3.

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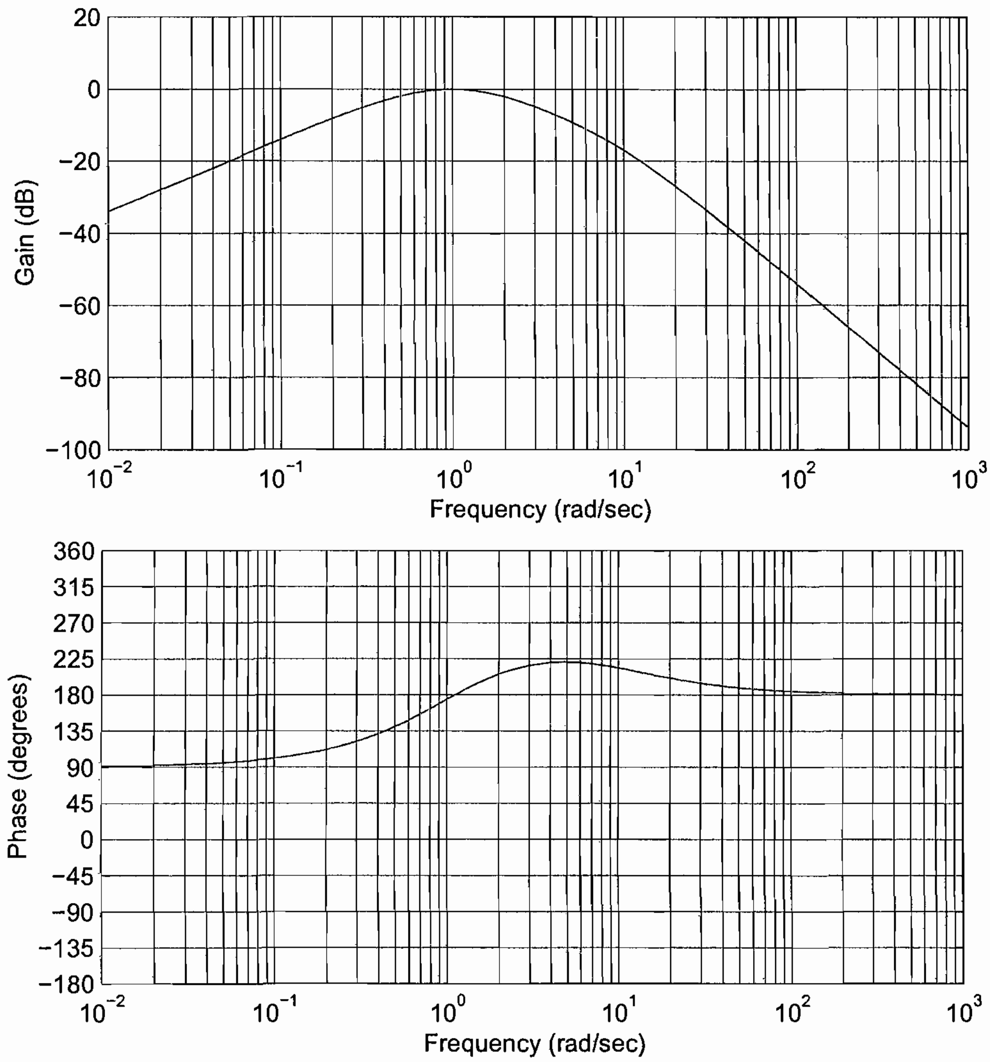
Wednesday 23 April 2008, Module 4F1, Question 3.



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