ENGINEERING TRIPOS PART IIB

Monday 5 May 2008 2.30 to 4

Module 4F3

NONLINEAR AND PREDICTIVE CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain briefly the *describing function* method of predicting limit cycles in feedback systems. [25%]

(b) Figure 1 shows the input-output characteristic of a memoryless nonlinearity:

$$\begin{aligned} f(e) &= \begin{cases} 0, & \text{if } 0 \le e \le 1, \\ e-1, & \text{if } 1 < e < 2 \\ 1, & \text{if } e \ge 2 \end{cases} \\ f(-e) &= -f(e) \end{aligned}$$

Show that the describing function $N(E)$ of this nonlinearity satisfies [15%]

$$0 \le N(E) \le 0.5$$

(Note that it is not necessary to calculate the describing function, and that $\int_0^{\pi/2} \sin^2\theta d\theta = \pi/4$.)

(c) The nonlinearity shown in Fig.1 is placed in a negative feedback loop, as shown in Fig.2, with a linear system whose transfer function is

$$\frac{a}{s(s+1)^2}$$

Show that the describing function method does not predict any limit cycle if $a = 1$. [20%]

(d) Show, using the *circle criterion*, that the feedback loop shown in Fig.2 is globally asymptotically stable if $a = 1$. [20%]

(e) What do the two methods (describing function and circle criterion) predict if the gain of the linear system shown in Fig.2 is increased to $a = 3$? [20%]

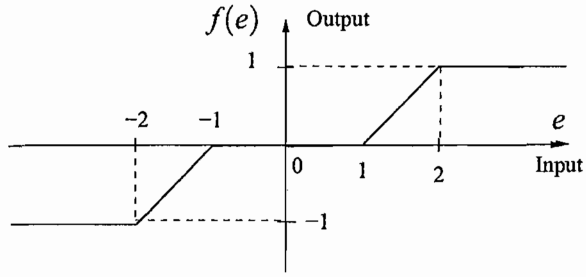

Fig. 1

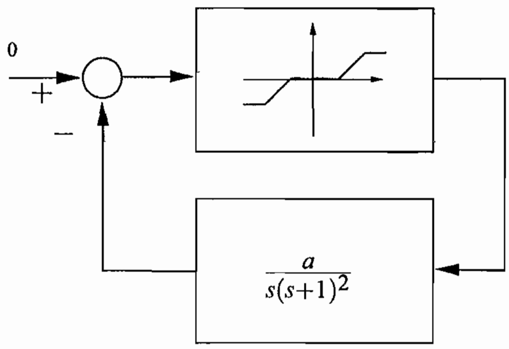

Fig. 2

(TURN OVER

2 (a) Describe how the *direct* and *indirect* methods of Lyapunov are used to investigate stability of equilibria of dynamic systems. [30%]

(b) A nonlinear system is defined by the equations

$$\begin{aligned} \dot{x}_1 &= x_2 - x_1 \\ \dot{x}_2 &= -x_1 + (ax_1 + bx_2)^2 x_2 \end{aligned}$$

where a and b are positive constants.

(i) This system has three equilibria. Find them. [20%]

(ii) Find the linearisation of this system in the neighbourhood of each equilibrium. [25%]

(iii) Show that one of the equilibria is stable, and that the other two are unstable if $a > 0$ and $b > 0$. [25%]

3 (a) Discuss the advantages and disadvantages of predictive control. Illustrate your discussion by reference to some application. [30%]

(b) Predictive control is to be applied to a system modelled as

$$x_{k+1} = Ax_k + Bu_k$$

The cost function to be used is

$$J(x_0, u_0, \ldots, u_{N-1}) = x_N^T P x_N + \sum_{k=0}^{N-1} \left(x_k^T Q x_k + u_k^T R u_k \right)$$

where x_0 is the latest measured state, and $x_1, \ldots, x_N$ are predictions of the state obtained by iterating the model. The predictive control law is obtained by finding the sequence $(u_0^*, \ldots, u_{N-1}^*)$ which minimises $J(x_0, u_0, \ldots, u_{N-1})$ and applying the first element u_0^* as the input to the plant. The conditions $P \geq 0$, $Q > 0$, $R > 0$ hold.

(i) What is meant by a *control Lyapunov function* in this context? [15%]

(ii) What is meant by a *terminal control law* in this context? [15%]

(iii) Assuming a linear terminal control law

$$u_k = Kx_k,$$

derive a condition relating A, B, K, P, Q, R which, if satisfied, ensures closed-loop stability of the origin. [40%]

4 (a) Write down the standard form of a *quadratic programming (QP)* optimisation problem. [10%]

(b) Predictive control is to be applied to a single-input system whose linear model is

$$x_{k+1} = Ax_k + Bu_k$$

with a cost function

$$\sum_{k=0}^{N-1} \left(x_{k+1}^T Q x_{k+1} + u_k^T R u_k \right)$$

and constraints

$$|u_k| \leq U$$

for some $U > 0$. Show how the predictive control problem can be written in the standard form of a quadratic programming problem if $N = 2$, assuming that the full state vector x_k can be measured at each step. [50%]

(c) Why is $N = 2$ likely to be too short a horizon in practice? What factors limit the length of horizon that can be used in practice? [20%]

(d) Outline briefly how the predictive control problem should be modified if offset-free tracking of piecewise-constant set-points is required. [20%]

END OF PAPER