ENGINEERING TRIPOS PART IIB

Wednesday 7 May 2008 9 to 10.30

Module 4F6

SIGNAL DETECTION AND ESTIMATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 a) Prove the *Neyman-Fisher Factorisation* theorem and discuss why it is useful in *Estimation Theory.* [20%]

b) The *scalar exponential family* of probability density functions for a random variable x may be written as

$$p(x|\theta) = \exp(A(\theta)B(x) + C(x) + D(\theta))$$

If data $x(n)$ for $n = 0, 1, 2, \ldots, N-1$ are observed which are *iid* and whose probability density function belongs to this family, show that a *sufficient statistic*, $T(x)$, for the parameter θ is given by

$$T(x) = \sum_{n=0}^{N-1} B(x(n))$$

[40%]

c) Find the sufficient statistic for the estimation of a constant level signal in the presence of *White Gaussian noise.* [40%]

2 a) Outline a proof of the *Cramer-Rao Lower Bound* inequality and show that

$$E\left(\left(\frac{\partial \ln p(x|\theta)}{\partial \theta}\right)^2\right) = -E\left(\frac{\partial^2 \ln p(x|\theta)}{\partial \theta^2}\right)$$

using the standard notation. [40%]

b) Calculate the Cramer-Rao lower bounds for the estimation of the slope and intercept of a straight line model fitted to data, $d(n)$, given by

$$d(n) = A + Bn + w(n)$$

where $n = 0, 1, 2, 3, \ldots, N-1$ and $w(n)$ is white Gaussian noise of variance σ^2. [40%]

c) Show that for $N \geq 3$ it is easier to estimate B than it is to estimate A. [20%]

3 a) Describe the role of numerical Bayesian methods in statistical inference and give examples for the cases of parameter estimation and model selection. [20%]

b) Outline how one would obtain expressions for the marginal probability density functions for the intercept, c, and slope, m, of a straight line in the presence of white Gaussian noise, $w(n)$, which can be modelled as

$$d(n) = mx(n) + c + w(n)$$

[40%]

given data $d(n)$, for $n = 0, 1, 2, 3, ..., N-1$.

c) Show how the Gibbs sampler may be applied to this problem when the noise variance is known and show that, in this case, the problem reduces to that of being able to sample from simple distributions. Derive expressions for these distributions. [40%]

4 (a) Describe the MAP and Bayes criteria applied to detection theory and discuss the advantages and disadvantages of the Neyman-Pearson decision rule over the above two criteria. [25%]

(b) Describe how the threshold of the Neyman-Pearson decision rule may be obtained from the ROC curve. [15%]

(c) It is required to detect a line given by

$$s(n) = A + Bn$$

where $n = 0, 1, 2, ..., N-1$, in additive white Gaussian noise of variance σ^2, where A and B are known constants.

(i) Show that the data may be written in the form of a *general linear model.* [30%]

(ii) Determine the Neyman-Pearson detector for this problem. [30%]

END OF PAPER