ENGINEERING TRIPOS PART IIB

Monday 21 April 2008 2.30 to 4

Module 4F8

IMAGE PROCESSING AND IMAGE CODING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain what is meant by a *zero-phase* 2D filter and why this is a desirable property for an image processing filter. [10%]

(b) Consider the zero-phase ideal frequency response $H(\omega_1, \omega_2)$ shown in Fig. 1. H takes the value 1 in the shaded region and zero outside this region. The original image is sampled with spacings Δ_1 and Δ_2 in the u_1 and u_2 directions respectively.

By using standard results for lowpass/bandpass filters or otherwise, find the ideal impulse response $h(u_1, u_2)$ corresponding to $H(\omega_1, \omega_2)$. [50%]

(c) When an image $g(u_1, u_2)$ is sampled on a rectangular grid (spacings Δ_1 and Δ_2 as in part (b)), the sampled image $g_s(u_1, u_2)$ may be written as

$$g_s(u_1, u_2) = s(u_1, u_2) g(u_1, u_2)$$

where $s(u_1, u_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} \delta(u_1 - n_1\Delta_1, u_2 - n_2\Delta_2)$

By writing s as a Fourier series, find $G_s(\omega_1, \omega_2)$, the Fourier transform of g_s, in terms of $G(\omega_1, \omega_2)$, the Fourier transform of g. Hence explain the phenomenon of *aliasing*. [40%]

(cont.

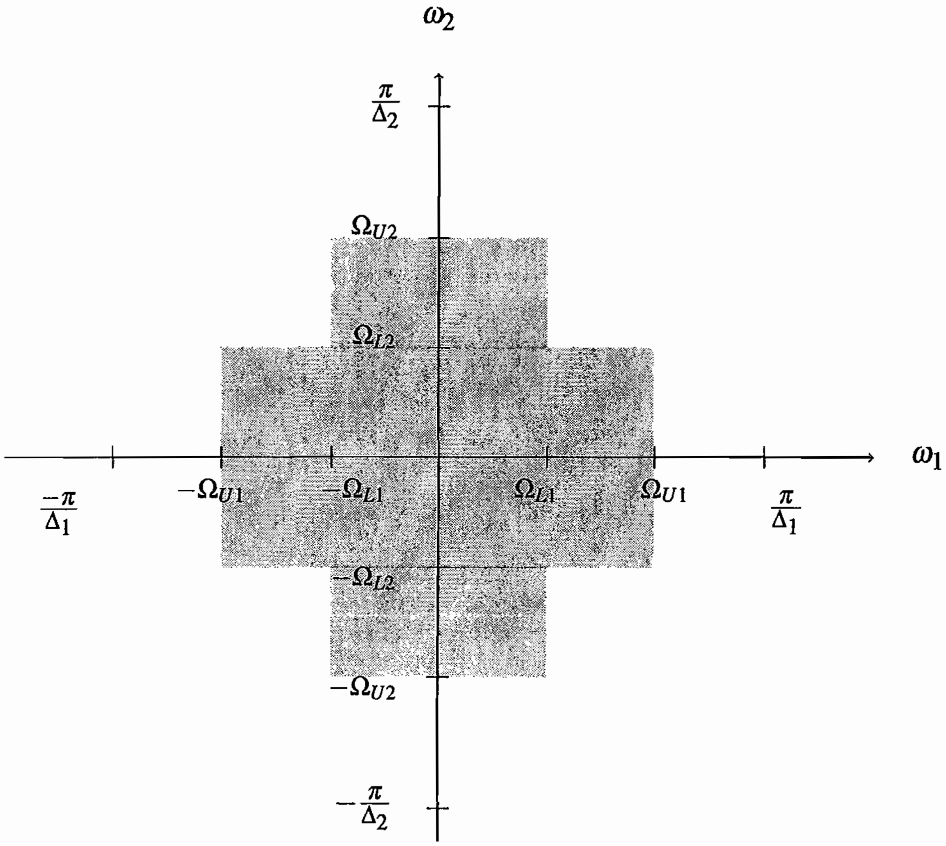

Fig. 1

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2 (a) For *spatially stationary processes*, $x(\mathbf{n})$, $y(\mathbf{n})$, where $\mathbf{n} = (n_1, n_2)$, define the *cross correlation function* $R_{xy}(\mathbf{n})$ and the *cross power spectrum* $P_{xy}(\boldsymbol{\omega})$, where $\boldsymbol{\omega} = (\omega_1, \omega_2)$. [10%]

The Wiener filter gives an estimate $\hat{x}(\mathbf{n})$ of the image $x(\mathbf{n})$ from the noisy measurements $y(\mathbf{n})$ via

$$\hat{x}(\mathbf{n}) = \sum_{\mathbf{q} \in \mathbf{Z}^2} g(\mathbf{q}) y(\mathbf{n} - \mathbf{q})$$

where $\mathbf{Z}$ denotes the set of integers. The frequency response of this optimal filter is given by $G(\boldsymbol{\omega}) = \frac{P_{xy}(\boldsymbol{\omega})}{P_{yy}(\boldsymbol{\omega})}$, where $P_{yy}(\boldsymbol{\omega})$ is the power spectrum of the noisy measurements $y(\mathbf{n})$.

Assume that the observed image can be modelled as the convolution of the true image and a point-spread function $h(\mathbf{n})$, plus additive noise $d(\mathbf{n})$, i.e.

$$y(\mathbf{n}) = \sum_{\mathbf{m} \in \mathbf{Z}^2} h(\mathbf{m}) x(\mathbf{n} - \mathbf{m}) + d(\mathbf{n})$$

Using this, evaluate $P_{xy}(\boldsymbol{\omega})$ and $P_{yy}(\boldsymbol{\omega})$ and hence derive the more usual form of the Wiener filter [40%]

$$G(\boldsymbol{\omega}) = \frac{H^*(\boldsymbol{\omega}) P_{xx}(\boldsymbol{\omega})}{|H(\boldsymbol{\omega})|^2 P_{xx}(\boldsymbol{\omega}) + P_{dd}(\boldsymbol{\omega})}$$

It can be assumed that the observation noise is uncorrelated with the image but you should detail any other assumptions made.

(b) The 8×8 image shown in Fig. 2 has greyscale levels of 1 through to 8. Sketch the histogram of this image and then perform histogram equalisation. Detail clearly the transformed values onto which the original greylevels are mapped and sketch the histogram of the equalised image using integer bins. Comment on how well the process has worked and suggest a way in which the process might be improved. [50%]

(cont.

1	2	3	4	5	6	7	8
2	2	3	4	5	6	7	7
3	3	4	5	4	5	6	6
4	4	5	4	4	5	5	5
5	5	5	4	4	5	4	4
6	6	5	4	5	4	3	3
7	7	6	5	4	3	2	2
8	7	6	5	4	3	2	1

Fig. 2

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3 (a) The elements of a Discrete Cosine Transform (DCT) matrix, $\mathbf{T}$ of size $n \times n$, are defined by:

$$t_{1i} = \sqrt{\frac{1}{n}} \quad \text{for } i = 1, \ldots, n$$

$$t_{ki} = \sqrt{\frac{2}{n}} \cos\left(\frac{\pi(2i-1)(k-1)}{2n}\right) \quad \text{for } i = 1, \ldots, n, \; k = 2, \ldots, n$$

Calculate the DCT matrix for $n = 4$ and show that it is orthonormal. [20%]

(b) If $\mathbf{Y}$ is the 2-dimensional DCT of a 4×4 subimage $\mathbf{X}$, explain how $\mathbf{T}$ is used to calculate $\mathbf{Y}$ from $\mathbf{X}$, and how $\mathbf{T}$ may also be used to calculate $\mathbf{X}$ from $\mathbf{Y}$. Why is orthonormality of $\mathbf{T}$ a useful property here? [25%]

(c) Explain in words what are the 16 basis functions of this 2-dimensional transform. [10%]

(d) Derive an expression, in terms of the rows of $\mathbf{T}$, to show how a given component y_{ij} of $\mathbf{Y}$ contributes to the 16 pixels of the output subimage $\mathbf{X}$, when performing the inverse 2-dimensional DCT. [20%]

(e) Hence explain how a 4×4-pixel region of an image, with pixel intensities that are uniform over the region, can be represented using just one component of $\mathbf{Y}$. What is the significance of this result for the coding of typical images? [25%]

4 (a) Sketch a block diagram of a typical image coder and decoder, showing the 3 main stages of each. Why is it desirable for the *Transform* and *Inverse Transform* blocks in the encoder and decoder to have the property of *Perfect Reconstruction*? In which block does the main source of coding distortion occur? [25%]

(b) The analysis and reconstruction parts of a two-band filter bank, which forms part of a Wavelet Transform, are shown in Fig. 3(a) and Fig. 3(b). Show that

$$\hat{Y}_0(z) = \tfrac{1}{2}[Y_0(z) + Y_0(-z)] \quad \text{and} \quad \hat{Y}_1(z) = \tfrac{1}{2}[Y_1(z) + Y_1(-z)]$$

and hence derive an expression for $\hat{X}(z)$ in terms of $X(z)$, $X(-z)$ and the four filter z-transfer functions. [20%]

(c) If $G_0(z) = zH_1(-z)$, find an expression for $G_1(z)$ in order to eliminate any aliased components from $\hat{X}(z)$. [10%]

(d) Letting $P(z) = H_0(z)G_0(z)$, obtain an expression for $P(z) + P(-z)$ in order to ensure Perfect Reconstruction from the filter bank. What constraints does this place on the coefficients of $P(z)$, assuming $P(z)$ also needs to be zero phase? [20%]

(e) Describe briefly, with sketches, how this filter bank would be used to form a 2-dimensional Wavelet Transform for images, and discuss the desirable properties that would be needed for the filters H_0 and G_0. [25%]

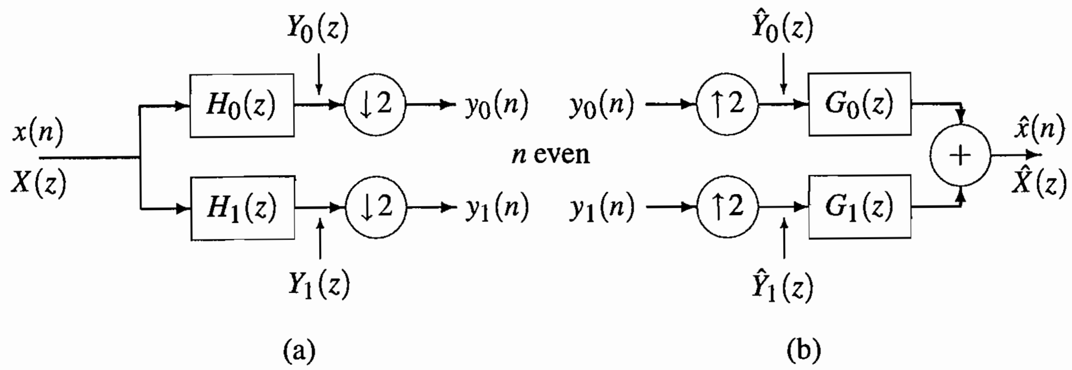

Fig. 3

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