Q1
(a)
Exposure is minimized by minimizing the <u>time</u> of exposure, maximizing <u>distance</u> from the source and the use of <u>shielding</u> where appropriate.

αs are highly charged and heavy and are absorbed by the outer layers of skin (dead tissue), so they are only dangerous when ingested or inhaled. To work with αs no shielding is required but work should be carried out in glove boxes to ensure the material cannot enter the body.

βs do require some shielding and precautions must be taken to ensure that they do not enter the body. It must be remembered that when βs are attenuated by shielding they give rise to secondary radiation (bremsstrahlung, a form of EM radiation) that may require secondary shielding.

γs are much more difficult to attenuate and cause serious damage to tissue even if the source does not enter the body. Significant shielding using materials such as lead is required.      [30%]

(b)
Assume a point source.

Co-60 emits a β and two γs (1.17 and 1.33 MeV).

Assume the β does not penetrate 1 m of air.

Assume air does not attenuate γs significantly.

Energy flux $\phi_e = \dfrac{AE}{4\pi r^2}$

So at 1 m $\phi_e = \dfrac{18.5 \times 10^9 \times (1.17 + 1.33) \times 1.602 \times 10^{-13}}{4\pi(1)^2} = 5.896 \times 10^{-4} \, \mathrm{J\,m^{-2}s^{-1}}$

Flux absorbed in distance $\Delta x \approx \phi_e \Sigma_e \Delta x$

∴ Energy absorbed by body of area $S$, thickness $\Delta x \approx \phi_e \Sigma_e S \Delta x$

∴ Absorbed dose rate $D \approx \dfrac{\phi_e \Sigma_e S \Delta x}{\rho S \Delta x} = \phi_e \dfrac{\Sigma_e}{\rho}$

∴ Equivalent absorbed dose rate $E \approx W_R \phi_e \dfrac{\Sigma_e}{\rho}$

For γs $W_R = 1$, so assuming the body to be 100% water

∴ $E \approx W_R \phi_e \dfrac{\Sigma_e}{\rho} = 1 \times 5.896 \times 10^{-4} \times \dfrac{3.0}{1000} = 1.769 \, \mu\mathrm{Sv\,s^{-1}}$      [30%]

(c)
Assume the casual worker will stay more than 1 m away from the source.

Required dose rate limit $3 \, \mu\mathrm{Sv\,hr^{-1}} \equiv 0.833 \, \mathrm{nSv\,s^{-1}}$

∴ Reduction required from shield $R = \dfrac{1.769 \times 10^{-6}}{0.833 \times 10^{-9}} = 2124$

Using linear build-up model

∴ $(1 + \mu t)\exp(-\mu t) = R^{-1}$

Taking natural logs

∴ $\ln(1 + \mu t) - \mu t = -\ln R$

∴ $\mu t = \ln R + \ln(1 + \mu t)$

This can be used as a basis for iterative solution: $\mu t_{i+1} = \ln R + \ln(1 + \mu t_i)$

For $R = 2124 \Rightarrow \mu t = 10.065$

For lead $\mu = 45.2 \text{ m}^{-1} \Rightarrow t = \dfrac{10.065}{45.2} = 0.223 \text{ m}$ — quite thick!

For concrete $\mu = 9.42 \text{ m}^{-1} \Rightarrow t = \dfrac{10.065}{9.42} = 1.068 \text{ m}$ — would ensure the worker stayed 1 m away!

Given the cost of lead and the amount required it would probably be better to opt for a concrete shield. The source could, for example, to be stored in a purpose-built pit sunk into the floor of the laboratory, with a long handle being used to access the source when it needs to be taken out. [40%]

Q2

(a)

The solution to the one-group diffusion equation in Cartesian geometry with cubic symmetry is

$$\phi = \phi_0 \cos\left(\frac{\pi x}{2L}\right)\cos\left(\frac{\pi y}{2L}\right)\cos\left(\frac{\pi z}{2L}\right)$$

The power density will be proportional to the flux as the reactor is assumed to be homogeneous in one-group diffusion theory.

The channel power is thus proportional to $\int_{-L}^{L} \phi\, dx$ assuming the channels run in the $x$ direction.

The ratio of the power in the central channel to the mean power per channel is then the same as the radial form factor:

$$F_R = \frac{\phi_0}{\phi_{ave}} \quad \text{where} \quad \phi_{ave} = \frac{1}{4L^2}\int_{-L}^{L}\int_{-L}^{L} \phi_0 \cos\left(\frac{\pi y}{2L}\right)\cos\left(\frac{\pi z}{2L}\right)dz\,dy$$

$$\therefore \quad \phi_{ave} = \frac{\phi_0}{4L^2}\int_{-L}^{L}\frac{2L}{\pi}\cos\left(\frac{\pi y}{2L}\right)\left[\sin\left(\frac{\pi z}{2L}\right)\right]_{-L}^{L}dy = \frac{\phi_0}{4L^2}\int_{-L}^{L}\frac{4L}{\pi}\cos\left(\frac{\pi y}{2L}\right)dy$$

$$\therefore \quad \phi_{ave} = \frac{\phi_0}{\pi L}\int_{-L}^{L}\cos\left(\frac{\pi y}{2L}\right)dy = \frac{\phi_0}{\pi L}\frac{2L}{\pi}\left[\sin\left(\frac{\pi y}{2L}\right)\right]_{-L}^{L} = \frac{4\phi_0}{\pi^2}$$

Thus $F_R = \dfrac{\phi_0}{\phi_{ave}} = \dfrac{\pi^2}{4}$. [20%]

(b)

From the steady flow energy equation applied to a channel, the channel power $P$ is related to the coolant temperature rise by

$$P = \dot{m}c_p\left(T_{co} - T_{ci}\right)$$

The coolant inlet temperature $T_{ci}$ will be the same for all channels. Thus, if $T_{co}$ is also the same and $c_p$ is constant, then $P \propto \dot{m}$.

From part (a) the mean channel power $P_{ave} = \dfrac{4}{\pi^2}P_{cen}$ where $P_{cen}$ is the maximum channel power

(the power of the central channel). Hence $\dot{m}_{ave} = \dfrac{4}{\pi^2}\dot{m}_{cen} = 0.405\dot{m}_{cen}$, so a reduction of 59.5% in coolant flow rate is required. [15%]

(c)

The temperature distributions can be determined using Ginn's equation. In this case, as the extrapolation lengths are negligible:

$$\theta = 2\left(\frac{T - T_{ci}}{T_{co} - T_{ci}}\right) - 1 = \sin\frac{\pi x}{2L} + Q\cos\frac{\pi x}{2L}$$

where $Q = \dfrac{\pi \dot{m}c_p}{UA}$.

If the coolant inlet and outlet temperatures are the same, the maximum temperature within a channel depends on the value of $Q$. If heat transfer coefficients and specific heat capacities are unchanged, then $Q \propto \dot{m}$. Thus, as $\dot{m}$ is lower for the mean channel, the maximum fuel temperature will therefore be lower and the hot-spot limitation will not apply. [15%]

(d)
If the maximum temperature is the same in both channels then

$$\theta^2_{\max} = \left[2\left(\frac{T_{\max} - T_{ci}}{T_{co} - T_{ci}}\right)_{\text{cen}} - 1\right]^2 = 1 + Q^2_{\text{cen}}$$

$$\therefore \quad 2(T_{\max} - T_{ci})_{\text{cen}} = (T_{co} - T_{ci})_{\text{cen}}\left(1 + \sqrt{1 + Q^2_{\text{cen}}}\right)$$

and

$$\theta^2_{\max} = \left[2\left(\frac{T_{\max} - T_{ci}}{T_{co} - T_{ci}}\right)_{\text{ave}} - 1\right]^2 = 1 + Q^2_{\text{ave}}$$

$$\therefore \quad 2(T_{\max} - T_{ci})_{\text{ave}} = (T_{co} - T_{ci})_{\text{ave}}\left(1 + \sqrt{1 + Q^2_{\text{ave}}}\right)$$

where subscript "cen" indicates the central channel and subscript "ave" the mean channel.
The coolant inlet temperatures are the same so $(T_{\max} - T_{ci})_{\text{ave}} = (T_{\max} - T_{ci})_{\text{cen}}$.

$$\therefore (T_{co} - T_{ci})_{\text{cen}}\left(1 + \sqrt{1 + Q^2_{\text{cen}}}\right) = (T_{co} - T_{ci})_{\text{ave}}\left(1 + \sqrt{1 + Q^2_{\text{ave}}}\right)$$

The coolant outlet temperatures are now not the same but are related to the channel power by
$P = \dot{m}c_p(T_{co} - T_{ci})$, so

$$P_{\text{ave}} = \dot{m}_{\text{ave}}c_p(T_{co} - T_{ci})_{\text{ave}}$$

and

$$P_{\text{cen}} = \dot{m}_{\text{cen}}c_p(T_{co} - T_{ci})_{\text{cen}}$$

$$\therefore \quad \frac{P_{\text{ave}}}{P_{\text{cen}}} = \frac{4}{\pi^2} = \frac{\dot{m}_{\text{ave}}(T_{co} - T_{ci})_{\text{ave}}}{\dot{m}_{\text{cen}}(T_{co} - T_{ci})_{\text{cen}}} = R\frac{(T_{co} - T_{ci})_{\text{ave}}}{(T_{co} - T_{ci})_{\text{cen}}}$$

where $R = \dot{m}_{\text{ave}}/\dot{m}_{\text{cen}}$.

$$\therefore \quad \frac{\pi^2 R}{4}\left(1 + \sqrt{1 + Q^2_{\text{cen}}}\right) = \left(1 + \sqrt{1 + Q^2_{\text{ave}}}\right)$$

Now $Q \propto \dot{m}$

$$\therefore \quad \frac{Q_{\text{ave}}}{Q_{\text{cen}}} = \frac{\dot{m}_{\text{ave}}}{\dot{m}_{\text{cen}}} = R$$

$$\therefore \quad \frac{\pi^2 R}{4}\left(1 + \sqrt{1 + Q^2_{\text{cen}}}\right) = \left(1 + \sqrt{1 + R^2 Q^2_{\text{cen}}}\right)$$

Given a value for $Q_{\text{cen}}$, $R$ can be found from this equation.

If $Q_{\text{cen}} = \sqrt{8}$

$$\therefore \quad \pi^2 R = \left(1 + \sqrt{1 + 8R^2}\right)$$

$$\therefore \quad \left(\pi^2 R - 1\right)^2 = 1 + 8R^2$$

$$\therefore \quad \pi^4 R^2 - 2\pi^2 R + 1 = 1 + 8R^2$$

$$\therefore \quad \left(\pi^4 - 8\right)R^2 = 2\pi^2 R$$

$$\therefore \quad R = \frac{2\pi^2}{\pi^4 - 8} \text{ or } R = 0 \text{ (unphysical)} \qquad [50\%]$$

Q3
(a)

Xenon-135, which is produced directly in small quantities as a fission product and by the decay of the more commonly produced iodine-135, has a very high capture cross-section for thermal neutrons and therefore captures these extensively. This affects the neutron balance for a reactor and, if not compensated for by the withdrawal of control rods, for example, can take the reactor subcritical. To maintain steady-state operation the unavoidable equilibrium xenon-135 population must be allowed for.

When a reactor is shut down xenon-135 will continue to be produced by the decay of iodine-135, but will no longer be eliminated by neutron capture. The xenon-135 population will therefore rise initially. Xenon-135 is radioactive and decays to caesium-135 with a half-life of about 9 hours, so the post-shutdown xenon-135 population does eventually decrease, but there may be of several hours during which there is insufficient excess reactivity available to overcome the effect of the xenon poisoning and restart the reactor.                    [15%]

(b)

In steady state $\quad \dfrac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I = 0 \;\Rightarrow\; I_0 = \dfrac{\gamma_i \Sigma_f \phi}{\lambda_i}$

$$\dfrac{dX}{dt} = \lambda_i I - \lambda_x X - \phi\sigma X = 0 \;\Rightarrow\; X_0 = \dfrac{\lambda_i I_0}{\lambda_x + \phi\sigma} = \dfrac{\gamma_i \Sigma_f \phi}{\lambda_x + \phi\sigma}$$

For a high power reactor the flux $\phi$ is high, so $\phi\sigma \gg \lambda_x$ and $X_0 \to \dfrac{\gamma_i \Sigma_f}{\sigma}$

The reactivity loss (poisoning) $\rho = -\dfrac{\sigma X}{v \Sigma_f}$, so $\rho_0 = -\dfrac{\gamma_i \Sigma_f}{v \Sigma_f} = -\gamma_i / v$                    [20%]

(c)

After shutdown $\phi = 0$, so $\quad \dfrac{dI}{dt} = -\lambda_i I \qquad\qquad (1)$

$$\dfrac{dX}{dt} = \lambda_i I - \lambda_x X \;\Rightarrow\; \dfrac{dX}{dt} + \lambda_x X = \lambda_i I \qquad (2)$$

At $t = 0$, $I = I_0$ and $X = X_0$ if shut down from steady state.

Solution of (1) is, by inspection:
$$I = I_0 \exp(-\lambda_i t)$$

For (2) complementary function is, by inspection:
$$X = A \exp(-\lambda_x t)$$

Particular integral is, by inspection, of the form:
$$X = B \exp(-\lambda_i t)$$

$\therefore \qquad -\lambda_i B \exp(-\lambda_i t) + \lambda_x B \exp(-\lambda_i t) = \lambda_i I_0 \exp(-\lambda_i t)$

$\therefore \qquad B = \dfrac{\lambda_i I_0}{\lambda_x - \lambda_i}$

Thus the general solution is

$$X = A \exp(-\lambda_x t) + \dfrac{\lambda_i I_0}{\lambda_x - \lambda_i} \exp(-\lambda_i t)$$

Boundary condition: $X = X_0$ at $t = 0$:

$\therefore \qquad A = X_0 - \dfrac{\lambda_i I_0}{\lambda_x - \lambda_i}$

$$\therefore \qquad X = \left[ X_0 - \frac{\lambda_i I_0}{\lambda_x - \lambda_i} \right] \exp(-\lambda_x t) + \frac{\lambda_i I_0}{\lambda_x - \lambda_i} \exp(-\lambda_i t)$$

From the datasheet $\gamma_i = 0.064$, $\sigma = 3.5$ Mbarns, $\nu = 2.43$

$$\lambda_i = \frac{\ln 2}{t_{1/2i}} = \frac{\ln 2}{6.7} = 0.10345 \text{ hr}^{-1} = 2.874 \times 10^{-5} \text{ s}^{-1}$$

$$\lambda_x = \frac{\ln 2}{t_{1/2x}} = \frac{\ln 2}{9.2} = 0.07534 \text{ hr}^{-1} = 2.093 \times 10^{-5} \text{ s}^{-1}$$

$$I_0 = \frac{\gamma_i \Sigma_f \phi}{\lambda_i} = \frac{0.064 \times 5 \times 10^{17}}{2.874 \times 10^{-5}} \Sigma_f = 1.113 \times 10^{21} \Sigma_f$$

$$X_0 = \frac{\gamma_i \Sigma_f \phi}{\lambda_x + \phi\sigma} = \frac{0.064 \times 5 \times 10^{17}}{2.093 \times 10^{-5} + 5 \times 10^{17} \times 3.5 \times 10^{-22}} \Sigma_f = 1.633 \times 10^{20} \Sigma_f$$

$$\frac{\lambda_i I_0}{\lambda_x - \lambda_i} = \frac{0.10345}{0.07534 - 0.10345} \times 1.113 \times 10^{21} \Sigma_f = -4.096 \times 10^{21} \Sigma_f$$

Thus after 3 hours

$$X = \left[ 1.633 \times 10^{20} \Sigma_f + 4.096 \times 10^{21} \Sigma_f \right] \exp(-0.07534 \times 3) - 4.096 \times 10^{21} \Sigma_f \exp(-0.10345 \times 3)$$

$$\therefore \ X = 3.945 \times 10^{20} \Sigma_f$$

Poisoning to be overcome $\rho = -\dfrac{\sigma X}{\nu \Sigma_f} = -\dfrac{3.5 \times 10^{-22} \times 3.945 \times 10^{20} \Sigma_f}{2.43 \Sigma_f} = -0.0568$

The steady-state poisoning $\rho_0 = -\dfrac{\sigma X_0}{\nu \Sigma_f} = -\dfrac{3.5 \times 10^{-22} \times 1.633 \times 10^{20} \Sigma_f}{2.43 \Sigma_f} = -0.0235$

So the *additional* reactivity required for restart (above that required to maintain steady-state operation) $= 0.0568 - 0.0235 = 0.0333$ [65%]

Q4

(a)

Reprocessing starts with the shearing of the fuel pins to release the spent fuel, which is then dissolved in hot concentrated nitric acid. Virtually all the spent fuel is dissolved in this process. The solution is then clarified by centrifugation and passed to the first extraction column. Here it is contacted with a solvent tri-butyl phosphate dissolved in odourless kerosene (TBP/OK) and the plutonium and uranium pass into the solvent phase leaving the fission products in the aqueous phase. The two phases separate and the fission products go to evaporation, vitrification and long-term storage. The organic phase containing the Pu and U is then treated with a reducing agent which reduces the valency of the Pu. This renders the Pu soluble in dilute nitric acid so the organic stream is now mixed with dilute nitric acid in a second contactor and the Pu is separated off in the aqueous stream leaving the U still in the organic stream. The U is finally recovered from the organic stream by backwashing with very slightly acidified water.

Both Pu and U are further purified using TBP/OK washing and backwashing, the Pu and U are usually finally recovered as oxides.

Waste arises at all stages of the process. The remains of the fuel pins are encapsulated in cement; the dissolver off gas is scrubbed to remove nitrous fumes, iodine and C14. Some krypton is discharged to the atmosphere. The fission product stream usually referred to as highly active liquor (HAL) is the most dangerous of all the waste arising but its volume is very small. It is evaporated to reduce volume then encapsulated in a glass matrix (vitrification) and placed in stainless steel containers for long-term storage.

Wastes also arise from the purification of the U and Pu products. These are referred to as medium active liquors (MAL). They are also evaporated and mixed with the HAL for vitrification. Low active waste streams arising from a number of sources are subject to chemical treatment before discharge to the environment. There are also a number of low level solid wastes arising which go to the controlled landfill at Drigg in Cumbria.                               [40%]

(b)

**Open cycle**

$$F = P + W \qquad (1)$$
$$x_F F = x_P P + x_W W \qquad (2)$$

$$\therefore \quad F = 500 + W$$
$$0.007F = 0.035P + 0.003W$$
$$0.003F = 0.003P + 0.003W \qquad 0.003 \times (1)$$

$$\therefore \quad 0.004F = 0.032P$$

$$\therefore \quad F = 4000, \quad W = 3500$$

Amount of natural uranium required is 4000 tonnes

$$SWU = P(-\ln x_P) + W(-\ln x_W) - F(-\ln x_F)$$

$$= 500(-\ln 0.035) + 3500(-\ln 0.003) - 4000(-\ln 0.007)$$
$$= 1676.2 + 20332.0 - 19847.4$$
$$= 2160.8 \text{ tonnes SWU} \qquad\qquad [20\%]$$

(c)
For open cycle

Feed cost $= 4000 \times 200{,}000 = \$8E8$
SWU cost $= 2160.8 \times 100{,}000 = \$2.1608E8$
Disposal cost $= 500 \times 400{,}000 = \$2E8$
Total cost $= \$1.21608E9$

**Closed cycle**

Recovered uranium $= 0.96 \times 500 = 480$ tonnes
Reprocessing loss 1% $= 4.8$ tonnes
Available uranium $= 475.2$ tonnes

Mass balance on recovered uranium:

$$475.2 = P_r + W_r \qquad (3)$$

$$0.008 \times 475.2 = 0.035P_r + 0.003W_r$$

$$\therefore \quad 3.8016 = 0.035P_r + 0.003W_r$$
$$1.4256 = 0.003P_r + 0.003W_r \qquad 0.003 \times (3)$$
$$\therefore \quad 2.376 = 0.032P_r$$

$\therefore$ Recovered product at 3.5% U235 $P_r = 74.25$ tonnes

Recovered tails $= 475.2 - 74.25 = 400.95$ tonnes

Product from fresh feed $= 500 - 74.25 = 425.75$ tonnes

$\therefore$ Amount of fresh feed required $= (4000/500) \times 425.75 = 3406$ tonnes

SWU for reprocessed feed is given by:

$$74.25(-\ln 0.035) + 400.95(-\ln 0.003) - 475.2(\ln -0.008)$$
$$= 248.9 + 2329.2 - 2294.4$$
$$= 283.7$$

SWU for fresh feed $= 2160.8 \times (425.75/500) = 1839.9$

Total SWU with reprocessing $= 283.7 + 1839.9 = 2123.6$

Reprocessing cost $= 500 \times 1{,}000{,}000 = \$5E8$
Fresh feed cost $= 3406 \times 200{,}000 = \$6.812E8$
Total SWU cost $= 2123.6 \times 100{,}000 = \$2.1236E8$
Total closed cycle cost $= \$1.39356E9$

Thus, the closed cycle costs more than the open cycle but it would not take a large increase in uranium costs to make reprocessing economically viable. [40%]

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Answers

Q1 (b) $1.769 \, \mu Svs^{-1}$

   (c) Suitable lead shield would be 0.223 m thick, concrete 1.068 m thick.

Q2 (b) 59.5% reduction

   (d) $\dfrac{\pi^2 R}{4}\left(1+\sqrt{1+Q_{cen}^2}\right)=\left(1+\sqrt{1+R^2 Q_{cen}^2}\right)$ where $R = \dot{m}_{ave}/\dot{m}_{cen}$

Q3 (c) 0.0333 additional reactivity

Q4 (b) 4000 tonnes, 2160.8 tonnes SWU

   (c) The total cost of the open cycle is $\$1.21608 \times 10^9$; the closed cycle costs $\$1.39356 \times 10^9$