Version 2
28/01/09

Authors: Dr C.A. Hall & Dr L. Xu

IIB Paper 4A3 Turbomachinery I Solutions 2009

1. a)
The specific speed can be used to determine the most suitable type of turbomachine for an application. For a known specific speed only one type of machine will have optimum performance.

Radial turbomachines are suited to low flow rates, or low ϕ. They also have large pressure ratios due to the additional pressure changes caused by changing flow radius and therefore tend to have high ψ. Thus, $N_s = \phi^{1/2}\psi^{-3/4}$ is very low for radial machines.

[20%]

b) (i)

$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{(\gamma-1)/\gamma\eta_p} \qquad \Rightarrow T_{02} = 300\times(4.5)^{0.4/(1.4\times0.9)} = \underline{483.6K}$$

[10%]

(ii)

$$\psi = \frac{\Delta h_0}{U^2} = \frac{c_p(T_{02}-T_{01})}{U^2} = \frac{1005\times(483.6-300)}{500^2} = 0.738$$

$$V_{\theta2,ideal} = (U + V_{r2}\tan\chi_2),\ V_{\theta2} = \sigma V_{\theta2,ideal}$$

Thus, $\Delta h_0 = UV_{\theta2} = \sigma U(U + V_{r2}\tan\chi_2) \Rightarrow \psi = \sigma\left(1 + \frac{V_{r2}}{U}\tan\chi_2\right)$

The above can be rearranged to give the ratio of radial velocity to blade speed,

$$\phi_2 = \frac{V_{r2}}{U} = \left(\frac{\psi}{\sigma}-1\right)\Big/\tan(\chi_2) = (0.738/0.85-1)/\tan(-30^0) = \underline{0.228}$$

Thus, the specific speed, $N_s = \phi^{1/2}\psi^{-3/4} = 0.228^{0.5}\times0.738^{-0.75} = \underline{0.6}$

[15%]

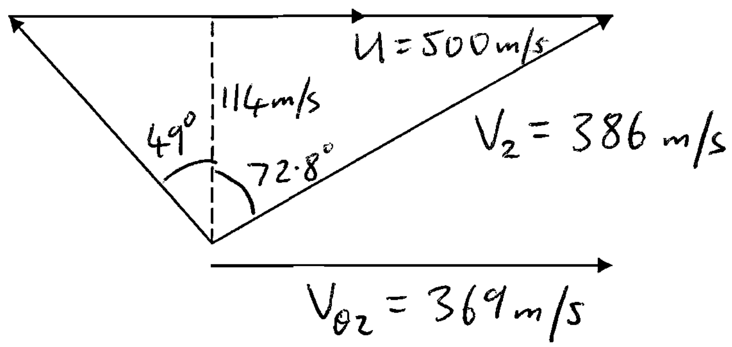
(iii)

$$V_{\theta2} = U\psi = 500\times0.738 = 369\,m\,s^{-1}$$

$$V_{r2} = U\phi_2 = 500\times0.228 = 114\,m\,s^{-1}$$

$$\alpha_2 = \tan^{-1}(V_{\theta2}/V_{r2}) = \tan^{-1}(\psi/\phi_2) = \tan^{-1}(0.738/0.228) = \underline{72.83^0}$$

$$\beta_2 = \tan^{-1}([V_{\theta2}-U]/V_{r2}) = \tan^{-1}([\psi-1]/\phi_2) = \tan^{-1}(-0.262/0.228) = \underline{-48.97^0}$$



[15%]

(iv)

$$V_2 = \sqrt{V_{\theta 2}^2 + V_{r2}^2} = \sqrt{369^2 + 114^2} = 386.2\ m\,s^{-1}$$

$$T_2 = T_{02} - V_2^2/(2c_p) = 483.6 - 386.2^2/2010 = 409.4\ K$$

Thus the absolute Mach number at impeller exit can be found:

$$M_2 = V_2/\sqrt{\gamma R T_2} = 386.2/\sqrt{1.4 \times 287.15 \times 409.4} = \underline{0.952}$$

Also, $p_{02}/p_2 = 1.7915 \quad \Rightarrow p_2 = 4.5 \times 100/1.7915 = 251.19\ kPa$

The impeller exit radius, $r_2 = 500/(50000 \times \pi/30) = 95.5\ mm$

From continuity, the exit area from the impeller normal to the radial direction is

$$A_{r2} = \frac{\dot{m}}{\rho_2 V_{r2}} = 2\pi\, r_2 h_2$$

$$\Rightarrow h_2 = \frac{\dot{m}}{2\pi\, r_2 \rho_2 V_{r2}} = \frac{\dot{m} R T_2}{2\pi\, r_2 p_2 V_{r2}} = \frac{1.2 \times 287.15 \times 409.4}{2\pi\, 0.0955 \times 251190 \times 114} = \underline{8.21\ mm}$$

[20%]

c)

The total-to-static isentropic efficiency is defined as:

$$\eta_{TS} = \frac{h_{3s} - h_{01}}{h_{03} - h_{01}} = \frac{(p_3/p_{01})^{(\gamma-1)/\gamma} - 1}{T_{03}/T_{01} - 1}$$

$$p_3/p_2 = C_p(p_{02}/p_2 - 1) + 1 = 0.6 \times (1.7915 - 1) + 1 = 1.475$$

$$p_3/p_{01} = p_3/p_2 \times p_2/p_{01} = 1.475 \times 251.19/100 = 3.705$$

$$\Rightarrow \eta_{TS} = \frac{(3.705)^{(\gamma-1)/\gamma} - 1}{483.6/300 - 1} = \underline{0.741}$$

This efficiency is typical of a centrifugal compressor. It is low relative to the impeller polytropic efficiency because the diffusion process is so inefficient - only 60% of the dynamic head at exit from the impeller is recovered in the diffuser (in addition, isentropic efficiencies of compressors are lower than polytropic efficiencies).

[20%]

2. a)

As the blade pitch (or spacing) increases, the loading on each blade increases because each blade has to turn more flow. For low values of pitch-to-chord ratio, the large numbers of blades leads to a high surface area and therefore high loss. For high values of pitch-to-chord ratio, the loading is increased leading to high velocities on the suction surfaces of the blades and thus high losses. Therefore, there is an optimum pitch-to-chord ratio in the middle of this range that gives minimum loss.

[20%]

b)

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{A_x \cos\alpha_1 \, p_{01}} = Q(M_1) = \frac{\dot{m}\sqrt{c_p T_{02}}}{A_x \cos\alpha_2 \, p_{02}} \times \frac{\cos\alpha_2}{\cos\alpha_1} \times \frac{p_{02}}{p_{01}}, \quad T_{01}=T_{02} \text{ (cascade)}$$

$$\frac{p_{02}}{p_{01}} = \frac{Q(M_1)}{Q(M_2)} \times \frac{\cos\alpha_1}{\cos\alpha_2} = \frac{0.6295}{1.2756} \times \frac{\cos(22^0)}{\cos(-61.4^0)} = 0.9559$$

The ratio of inlet stagnation to exit pressure is then:

$$\frac{p_{01}}{p_2} = \frac{p_{02}}{p_2} \times \frac{p_{01}}{p_{02}} = \frac{1}{0.5721 \times 0.9559} = \underline{1.829}$$

The cascade loss coefficient can then be determined:

$$Y_P = \frac{p_{01}-p_{02}}{p_{01}-p_2} = \frac{1-p_{02}/p_{01}}{1-p_2/p_{01}} = \frac{1-0.9559}{1-1.829^{-1}} = \underline{0.0973}$$

Alternatively,

$$Y_P = \frac{p_{01}-p_{02}}{p_{02}-p_2} = \frac{1-p_{02}/p_{01}}{p_{02}/p_{01}-p_2/p_{01}} = \frac{1-0.9559}{0.9559-1.829^{-1}} = 0.1078$$

[20%]

c)

$$Z = \frac{\dot{m}\left|V_{\theta 2}-V_{\theta 1}\right|}{(p_{01}-p_2)c_x h} = \frac{\dot{m}\sqrt{c_p T_{01}}}{hs\cos\alpha_1 \, p_{01}} \times \frac{\left|V_1 \sin\alpha_1/\sqrt{c_p T_{01}} - V_2 \sin\alpha_2/\sqrt{c_p T_{01}}\right| \times hs\cos\alpha_1}{(1-p_2/p_{01})c_x h}$$

Rearranging to find the pitch to axial chord ratio,

$$\frac{s}{c_x} = \frac{(1-p_2/p_{01})Z}{Q(M_1) \times \left|V_{\theta 1}/\sqrt{c_p T_{01}} - V_{\theta 2}/\sqrt{c_p T_{01}}\right| \times \cos\alpha}$$

Note that the modulus ensures that the loading coefficient is positive regardless of the sign convention used for the flow angles.

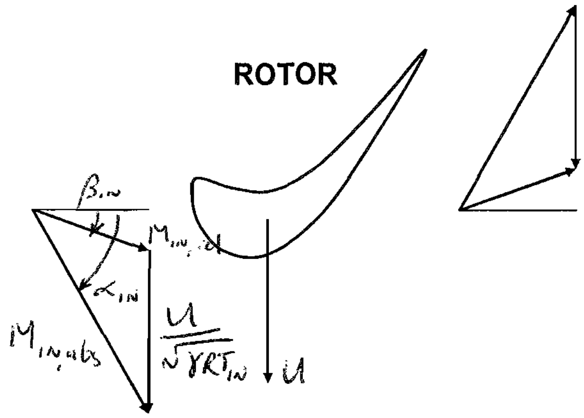
Putting in the values and using the compressible flow tables where needed,

$$\frac{s}{c_x} = \frac{(1-1.829^{-1}) \times 0.6}{0.6295 \times \left[0.1881.\sin(22^0) + 0.5431.\sin(61.4^0)\right] \times \cos(22^0)} = \underline{0.851}$$

[30%]

d)

Mach number triangles for the rotor:



$M_{in,rel} = 0.3, \beta_{in} = 22^0$

$M_{in,abs} = (M_{in,rel} \times \cos\beta_{in}) / \cos\alpha_{in} = (0.3 \times \cos 22^0) / \cos 70^0 = 0.8133$

$T_{in} = T_{0in} \times \left(1 + \frac{\gamma - 1}{2} M_{in,abs}^2\right)^{-1} = 550 \times (1 + 0.2 \times 0.8133^2)^{-1} = 485.7\,K$

$M_{blade} = (M_{in,abs} \times \sin\alpha_{in}) - (M_{in,rel} \times \sin\beta_{in}) = 0.8133 \times \sin 70^0 - 0.3 \times \sin 22^0 = 0.6519$

Thus the blade speed,

$U_{blade} = M_{blade} \times \sqrt{\gamma R T_{in}} = 0.6519 \times \sqrt{1.4 \times 287.15 \times 485.7} = \underline{288.1\,m\,s^{-1}}$

$r_{blade} = U_{blade} / \Omega_{blade} = 288.1 / (6000 \times \pi / 30) = 0.4584\,m$

$N_{blade} = \frac{2\pi . r_{blade}}{s} = \frac{2\pi . r_{blade}}{c_x . s / c_x} = \frac{2\pi \times 0.4584}{0.036 \times 0.851} = 94.0$

Therefore, 94 blades are required in the rotor

The flow coefficient, $\phi = \frac{V_x}{U} = \frac{M_{in,rel} \cos\beta_{in}}{M_{blade}} = \frac{0.3 \times \cos 22^0}{0.6519} = \underline{0.4267}$

[30%]

3. a.

As both the turbine nozzle and the exhaust nozzle are choked, the turbine pressure ratio and temperature ratio are fixed by the area ratio A_3/A_5 by continuity: $\dot m_4 = \dot m_5 \Rightarrow \sqrt{T_{04}}/P_{04}\cdot A_5 = \sqrt{T_{03}}/P_{03}\cdot A_3$

$$\left(\frac{P_{04}}{P_{03}}\right)^2 = \left(\frac{A_3}{A_5}\right)^2 \cdot \left(\frac{T_{04}}{T_{03}}\right)$$

. for constant $\eta_{p.t}$, γ, and A_3/A_5

$$\frac{P_{04}}{P_{03}} = \text{const}, \qquad \frac{T_{04}}{T_{03}} = \text{const}.$$

Along the operating line, work balance between the compressor and the turbine gives:

$$C_{p.c}(T_{02}-T_{01}) = C_{p.t}(T_{03}-T_{04}) = C_{p.t}T_{03}\left(1-\frac{T_{04}}{T_{03}}\right) = C_{p.t}\cdot k\, T_{03}$$

$$\therefore \quad \pi_c = \frac{P_{02}}{P_{01}} = \left(1+\frac{T_{02}-T_{01}}{T_{01}}\right)^{\frac{\eta_{p.c}\gamma}{\gamma-1}} = \left(1+k\frac{C_{p.t}}{C_{p.c}}\cdot\frac{T_{03}}{T_{01}}\right)^{\frac{\eta_{p.c}\cdot\gamma}{\gamma-1}} = fn(\eta_{p.c}, T_{03}) \quad (1)$$

as T_{01} is held constant/known on ground test bed.

Also with continuity between compressor and turbine,

$$\frac{\dot m\sqrt{C_{p.c}T_{01}}}{T_{01}\, A_1} = \frac{\dot m\sqrt{C_{p.t}T_{03}}}{P_{03}\cdot A_3}\cdot\sqrt{\frac{T_{01}}{T_{03}}}\cdot\frac{A_3}{A_1}\cdot\frac{P_{02}}{P_{01}} \quad (a)$$

$$= K_1\cdot\sqrt{\frac{T_{01}}{T_{03}}}\cdot\frac{A_3}{A_1}\left(1+k\frac{C_{p.t}}{C_{p.c}}\cdot\frac{T_{03}}{T_{01}}\right)^{\frac{\eta_{p.c}\cdot\gamma}{\gamma-1}} = fn(\eta_{p.c}, T_{03}) \quad (2)$$

equations (1) and (2) give a relationship between the turbine entry temperature T_{03} and compressor operating point as fixed by π_c and $\frac{\dot m\sqrt{T_{01}}}{P_{01}}$.

Mass flow balance between the compressor and the turbine as expressed in equation (a) indicates a linear relationship between π_c and $\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}$ for a given T_{03}, as shown in fig 4. As the efficiency reduces at lower engine speed, π_c and $\frac{\dot{m}\sqrt{T_{01}}}{P_{01}}$ will be lower for the same T_{03} and the operating line is to move towards the surge line, which makes the match of the compressor stage more difficult at part speeds.

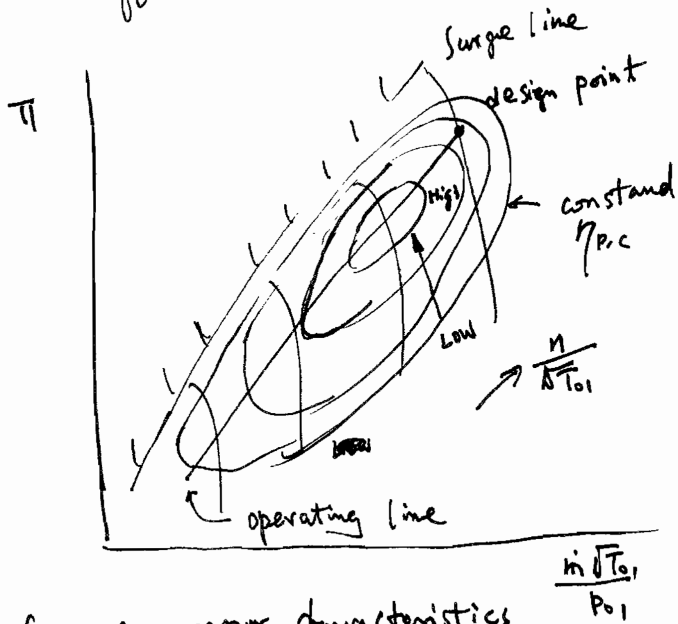


fig 1, compressor characteristics

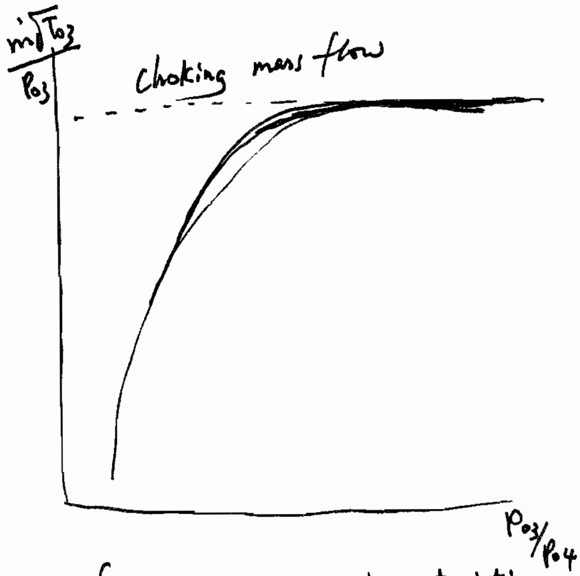


fig 2. turbine characteristics

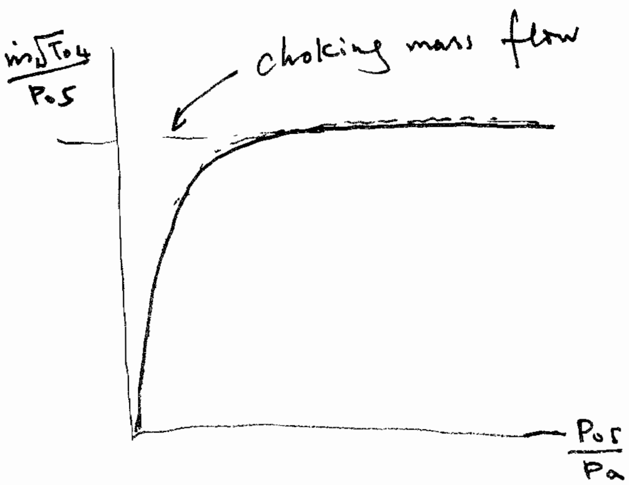
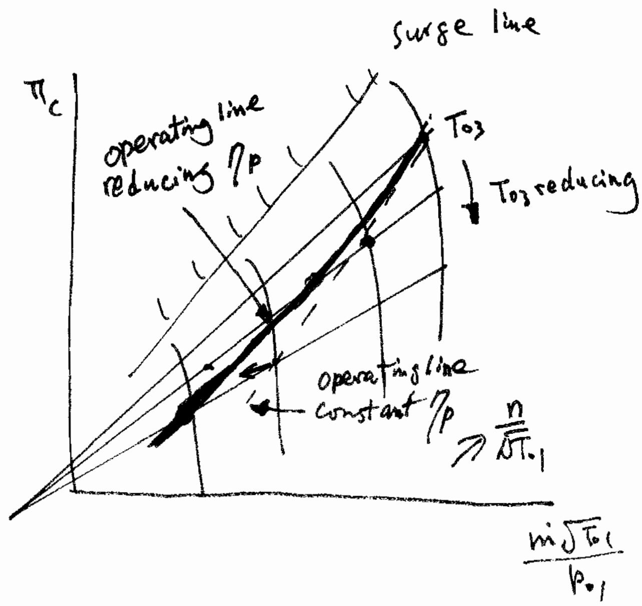


fig 3. nozzle characteristics



A high speed multistage compressor has a converging meridional flow path to accommodate the continuity requirement as the density of the air will become higher towards the rear stages. At part speed the pressure ratio is reduced therefore the area designed for high pressure ratio at high speed will become too small for the air flow with reduced density. As the result, the operating points for typical frontal, middle and rear stages will change from what are shown in fig 1 to fig. 2. This will cause the compressor to choke at rear stages and push the front stages into stall/surge. This could be partially relieved by designing the frontal stages at negative incidence and the rear stages at positive incidence. This will have small impact on the design speed efficiency of the compressor but greatly improve its part speed efficiency. However during the acceleration it is still very hard to avoid the rear stage choking to push the whole compressor into instability. Common engineering solutions for this problem are:

1). Bleeding off air from the middle of the compressor at the part peed to reduce the mass flow rate at rear stages. This is very easy to complement but result in low efficiency and will affect the rate of engine acceleration as the turbine power is proportional to the mass flow rate entering the turbine, which has be reduced whilst the mass flow rate in the front part of the compressor remains high.

2). Using variable stators in first half of the compressor to change the stator angle settings at the different speed to achieve the highest efficiency and stability margin. This is rather effective but is mechanically complicated (actuating and sealing) and adds weigh to the compressor. It also adds the cost of the compressor.

3). Split the compressor into multi spool which can rotate at different speeds. By setting the spools at different speeds the compressor can achieve the best matching. But this adds huge mechanical complexities to the engine and could be a very costly solution.

Fig 1. Stage characteristics at design speed

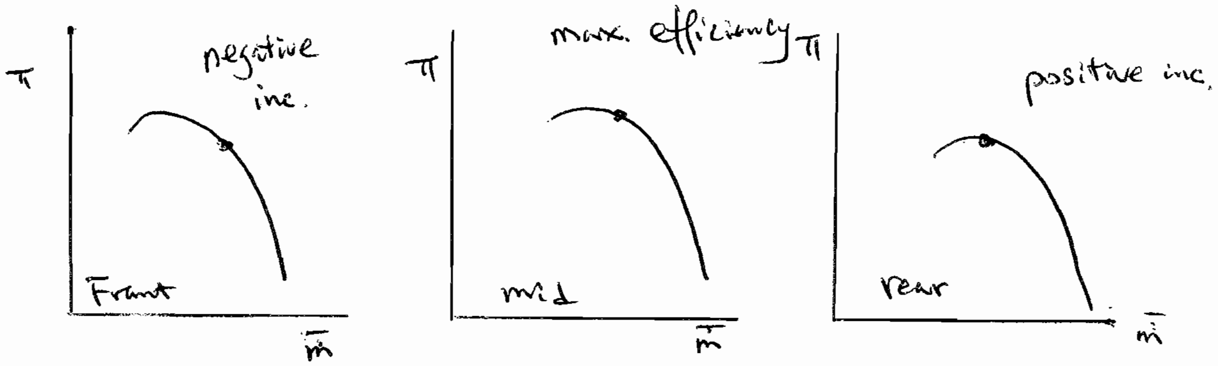


Fig 2 stage characteristics at part speeds