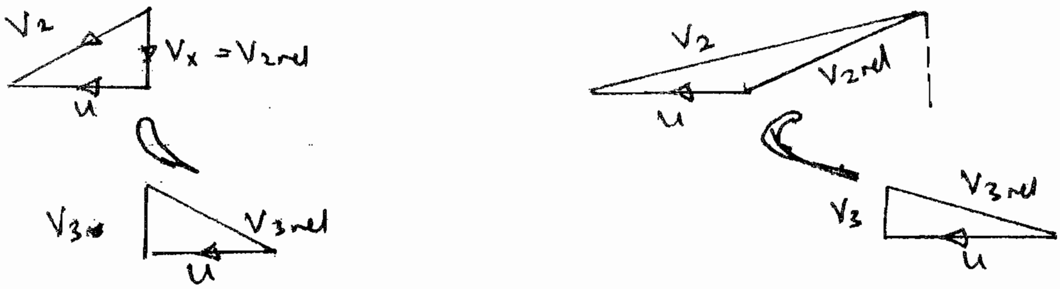
C

1 (a) (i) Reaction effects the symmetry of velocity triangles eg:



50% reaction Turbine 0% reaction turbine

Most turbines have ~ 50% reaction as this evens out the pressure changes. Some turbines are low reaction because this helps reduce the over-shroud leakage losses. Compressors tend to be 50% reaction or slightly above.

(ii). SRE: $\frac{\partial p}{\partial r} = \rho \frac{V_\theta^2}{r}$

$$T\,ds = dh - \frac{dp}{\rho} \Rightarrow T\frac{\partial s}{\partial r} = \frac{\partial h}{\partial r} - \frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$\therefore \frac{\partial h}{\partial r} = T\frac{\partial s}{\partial r} + \frac{V_\theta^2}{r}$$

(b) $T_{01} = 1400\,K$

$\alpha_1(r) = 0 \qquad \alpha_3(r) = 0$

$r_h / r_t = 0.6 = \text{const}$

1 cont (b)(i) @ r_m, $M_{2m} = 0.7$ $\alpha_{2m} = 74.6°$ $T_{03m} = 1170K$

Isentropic ∴ $ds = 0$ & $\partial s/\partial r = 0$

$$\text{Reaction} = \frac{h_2 - h_3}{h_1 - h_3} = \frac{T_2 - T_3}{T_1 - T_3} \quad (c_p = \text{const})$$

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2 \Rightarrow T_2 = \frac{1400}{1.0980} = 1275K$$

To find T_1, need M_1 etc ∴ use conservation of mass

$$\frac{\dot m \sqrt{c_p T_{01}}}{s\, p_{01}} = \frac{\dot m \sqrt{c_p T_{02}}}{s \cos\alpha_2 \, p_{02}} \cdot \sqrt{\frac{T_{01}}{T_{02}}} \cdot \frac{p_{01}}{p_{02}} \cdot \frac{s\cos\alpha_2}{s}$$

$$= F(M_{2m}) \cdot 1 \cdot 1 \cdot \cos 74.6$$

$$= 1.1705 \cos 74.6 = 0.3108$$

Interpolating in the tables gives $M_1 = 0.1421$

$$\& \; T_1 = \frac{T_{01}}{1 + \frac{\gamma-1}{2} M_1^2} = \frac{T_{01}}{1.004} = 1394.4K.$$

To find T_3, similar process:

$$\frac{\dot m \sqrt{c_p T_{03}}}{s\, p_{03}} = \frac{\dot m \sqrt{c_p T_{02}}}{s \cos\alpha_2 \, p_{02}} \frac{p_{02}}{p_{03}} \sqrt{\frac{T_{03}}{T_{02}}} \frac{s \cos\alpha_2}{s}$$

Now Turbine is isentropic ∴ $\frac{p_{02}}{p_{03}} = \left(\frac{T_{02}}{T_{03}}\right)^{\gamma/\gamma-1}$

$$\therefore \frac{\dot m \sqrt{c_p T_{03}}}{s\, p_{03}} = 1.1705 \cdot \left(\frac{1400}{1170}\right)^{3.5-1/2} \cdot \cos 74.6 = 0.5325$$

$\Rightarrow M_3 = 0.2497$; $T_3 = T_{03}/1.0125 = 1155.6$

$$\therefore \Lambda = \frac{1275 - 1156}{1394 - 1156} = \underline{\underline{0.500}}$$

Q1 cont b(ii) Free vortex is common because when meridional curvature is small and radial variation of loss is small, meridional velocity is constant. This helps the hub reaction stay high in low hub-tip ratio machines. It also helps in multistage machines as work variation with radius is small so radial variations of Po etc remain small and do not escalate.

Now: $\frac{\partial h}{\partial r} = \frac{V_\theta^2}{r}$ & $rV_{\theta_2} = \text{const} = k$

$\Rightarrow V_{\theta_2}^2 = k^2/r^2$

$\therefore$

$$\frac{\partial h_2}{\partial r} = \frac{k^2}{r^3}$$

$$\Rightarrow \quad h_2 = -\frac{1}{2}\frac{k^2}{r^2} + \text{const}$$

$$= -\frac{1}{2}\frac{(r_m V_{\theta_{2m}})^2}{r^2} + \text{const}$$

Now $h_1 = \text{const}$ & $h_3 = \text{const}$ $\therefore$

$$\Lambda(r) = \frac{h_2(r) - h_3}{h_1 - h_3} = \frac{-\frac{1}{2}\left(\frac{r_m}{r}\right)^2 V_{\theta_{2m}}^2 + \text{const} - h_3}{h_1 - h_3}$$

When $r = r_m$, $\Lambda_m = 0.5$

$$\therefore \quad 0.5 = \frac{-\frac{1}{2}V_{\theta_{2m}}^2 + \text{const} - h_3}{h_1 - h_3}$$

$$\therefore \quad \text{const} = \tfrac{1}{2}(h_1 + h_3) + \tfrac{1}{2}V_{\theta_{2m}}^2$$

Q1 cont. $\therefore$ $$\Lambda = \frac{\frac{1}{2}\left(h_1 + h_3 + V_{\theta 2m}^2\right) - \frac{1}{2}V_{\theta 2m}^2\left(\frac{r_m}{r}\right)^2 - h_3}{h_1 - h_3}$$

$$= \frac{1}{2}\,\frac{\left(h_1 - h_3 + V_{\theta 2m}^2\left(1 - (r_m/r)^2\right)\right)}{h_1 - h_3}$$

But $V_{\theta 2m}^2 = \qquad \alpha_{2m} = 2(h_{02} - h_{2m})\sin^2\alpha_{2m}$

$$\therefore \quad \Lambda = \frac{1}{2}\left[1 + \frac{2(h_{02} - h_2)\sin^2\alpha_{2m}\left(1 - (r_m/r)^2\right)}{h_1 - h_3}\right]$$

$$= \frac{1}{2}\left[1 + \frac{2(T_{02} - T_{2m})}{(T_1 - T_3)}\sin^2\alpha_{2m}\left(1 - (r_m/r)^2\right)\right]$$

as required

$r_m = 0.8\, r_t$

$r_h = 0.6 r_t = \frac{0.6}{0.8} r_m = 0.75 r_m$

$$\therefore \quad \Lambda = \frac{1}{2}\left[2\frac{(1400 - 1275)}{(1394 - 1156)}\sin^2 74.6\left(1 - \left(\frac{r_m}{r}\right)^2\right)\right]$$

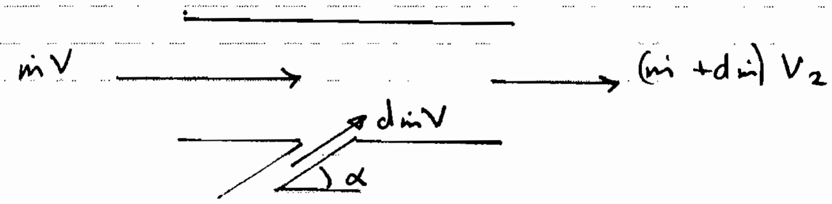
$$\Rightarrow \quad \Lambda = 0.5252 \sin 74.6 \left(1 - \left(\frac{r_m}{r}\right)^2\right)$$

$\Lambda_h = \underline{\underline{0.12}}$

$\Lambda_t = \underline{\underline{0.675}}$

(iii) The low hub reaction means that there is little overall acceleration. Combined with the high turning, the thick blades and low pitch chord ratio, this means that the peak velocities will be high and there deceleration after the peak on the suction surface will be large.

Q2 (a)



Loss of KE:

$$LKE = \tfrac{1}{2}\dot m V^2 + \tfrac{1}{2} d\dot m\, V^2 - \tfrac{1}{2}(\dot m + d\dot m)V_2^2 \quad (1)$$

p = const & ignoring streamline normal momentum, streamline momentum becomes

$$\dot m V + d\dot m\, V\cos\alpha = (\dot m + d\dot m)\, V_2 \quad (2)$$

$$\Rightarrow \quad \frac{V_2}{V} = \frac{\dot m + d\dot m\cos\alpha}{\dot m + d\dot m} = \frac{1 + d\dot m/\dot m\,\cos\alpha}{1 + d\dot m/\dot m} \quad (3)$$

$$(1) \Rightarrow \quad LKE = \tfrac{1}{2}V^2(\dot m + d\dot m)\left(1 - \left(\frac{V_2}{V}\right)^2\right)$$

$$(3) \Rightarrow \quad LKE = \tfrac{1}{2}V^2(\dot m + d\dot m)\left(1 - \left(\frac{\dot m + d\dot m\cos\alpha}{\dot m + d\dot m}\right)^2\right)$$

$$= \tfrac{1}{2}V^2\left(\dot m + d\dot m - \frac{\dot m^2}{\dot m + d\dot m} - \frac{2\dot m\, d\dot m\cos\alpha}{\dot m + d\dot m} + H.O.T\right)$$

$$\simeq \frac{1}{2}V^2\left(\dot m + d\dot m - \dot m\left(1 - \frac{d\dot m}{\dot m}\right) - \frac{2\dot m\, d\dot m\cos\alpha}{\dot m}\right)$$

Taylor Series Expansion

Ignored $(d\dot m)$ as $d\dot m \ll \dot m$

$$= \frac{1}{2}V^2\left(2d\dot m - 2d\dot m\cos\alpha\right)$$

$$= \underline{\underline{d\dot m\, V^2(1 - \cos\alpha)}}$$

Q2 cont (a) $Tds = dh - \frac{dp}{\rho}$

If p = const $Tds = dh$

If $h_{0_{rel}}$ = const (assumes constant radius of stream-lines)
$Tds = -d(KE_{REL})$

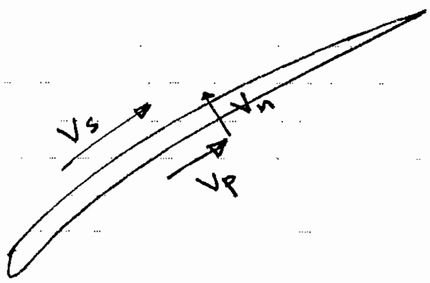
$\therefore$ loss of K.E. is the 'lost work'

As tip gaps ~ 1% of span, the leakage flow is small $\therefore$ above equation can be used to calculate tip leakage losses. However, it is assumed that all flow mixes out and this is not the case (a tip vortex extends downstream of the rotor). This also means that streamwise pressure changes will have a (small) effect.

(b) Low speed $\therefore$ incompressible $\therefore$

$$\frac{\Delta P_0}{\rho} = \text{loss of } KE$$

& $\xi = \dfrac{\Delta P_0}{\frac{1}{2}\rho V_1^2}$



Consider a small part of the chord:

$\therefore$ $d\dot{m}_{leak} = \rho V_n \, t \, dC$
$= \rho V_n \, t C d\bar{C}$

Because flow is isentropic up to the point of mixing,

$$V_p^2 + V_n^2 = V_s^2$$

$$\therefore V_n^2 = V_s^2 - V_p^2$$

$$V_n = \sqrt{V_s^2 - V_p^2}$$

2 cont

$\therefore$

$$d\dot{m}_{leak} = \rho\sqrt{V_s^2 - V_p^2}\ t.c.\,d\bar{c}$$

$\therefore$ for this small part of the chord, rate of loss of KE becomes

$$d(LKE) = \left(\sqrt{V_s^2 - V_p^2}\ t.c.\,d\bar{c}\right) V_s^2 (1 - \cos\alpha)$$

But $\cos\alpha = \dfrac{V_p}{V_s}$ $\therefore$

$$d(LKE) = V_s^3\sqrt{1 - \left(\frac{V_p}{V_s}\right)^2}\ t.c.\,d\bar{c}\left(1 - \frac{V_p}{V_s}\right)$$

Now

$$d\xi = \frac{d(LKE)}{\frac{1}{2}\dot{m}V_1^2}$$

where

$$\dot{m} = \rho V_1 \cos\alpha_{1,rel}\, h.p$$

$\therefore$

$$d\zeta = \frac{2ct}{hp\cos\alpha_{1,rel}}\left(\frac{V_s}{V_1}\right)^3\sqrt{1 - \left(\frac{V_p}{V_s}\right)^2}\left(1 - \frac{V_p}{V_s}\right)d\bar{c}$$

$\therefore$ loss coeffic $= \int d\xi$

$$\zeta = \frac{2ct}{hp\cos\alpha_{1,rel}}\int_0^1\left(\frac{V_s}{V_1}\right)^3\sqrt{1 - \left(\frac{V_p}{V_s}\right)^2}\left(1 - \frac{V_p}{V_s}\right)d\bar{c}$$

Equation shows that loss is proportional to ratio of leakage flow area (ct) to inlet-flow area ($hp\cos\alpha_{1,rel}$) $\therefore$ loss is proportional to ratio of $\dot{m}_{leak}/\dot{m}$ & increasing tip gap leads to a proportional increase in the loss

2 cont. (c) given: $t/h = 0.02$ $p/c = 0.8$ $\alpha_{1\,rel} = 30°$

$V_s/V_1 \leq 1.1$

De Haller $\Rightarrow$ $V_2 > 0.7 V_1$

Now $V_2 \simeq V_p$ & Let $V_s = 1.1 V_1$

$\therefore$ $V_p \simeq \frac{0.7}{1.1} V_s$

Substituting into formula gives (assuming values constant along the chord)

$$\xi = \frac{2 \quad 0.02}{0.8 \;.\; 0.866}\left[1.1^3\left(1 - \frac{0.7}{1.1}\right)\sqrt{1 - \left(\frac{0.7}{1.1}\right)^2}\right]$$

$$= \underline{\underline{0.0216}}$$

(d) $Tds = dh_0 - dp_0/\rho_0 \Rightarrow 0 = dh_{0\,min} - dp_0/\rho$

$$\therefore \eta = \frac{\Delta h_{0\,min}}{\Delta h_0} \simeq 1 - \frac{T\Delta s}{\Delta h_0} = 1 - \frac{\text{loss}}{\text{work}}.$$

$$\text{Loss} = \frac{\frac{1}{2}V_1^2\xi}{0.3} = \frac{1}{2}\left(\frac{V_x}{\cos\alpha_{1\,rel}}\right)^2 \xi \div 0.3$$

$$\text{Work} = \frac{\Delta H_0}{U^2} = \psi$$

$$\therefore \eta = 1 - \frac{1}{2}\frac{V_x^2}{U^2}\frac{\xi}{\cos\alpha_1}\frac{1}{\psi}\frac{1}{0.3} = 1 - \frac{1}{2}\phi^2\frac{\xi}{\cos\alpha_1}\frac{1}{\psi}\frac{1}{0.3}$$

$$= 1 - \frac{1}{2}\frac{0.65^2 \,.\, 0.0216}{0.866^2}\frac{1}{0.5}\frac{1}{0.3} = \underline{0.954}$$

3(a) Forced vortex $\Rightarrow V_\theta = kr$

$V_{\theta 1} = 0 \quad V_{x1} = \text{const.} \qquad h_{01} = \text{const}$

$\dot{Q} = 15\ m^3/s \qquad r_t = 0.5\ m \qquad r_h/r_t = 0.6$

$V_{\theta t} = 20\ m/s \qquad \therefore V_{\theta 2} = 20\dfrac{r}{r_t} = \underbrace{40}_{k} r$

(i) SRE with uniform losses across the span $(\partial s/\partial r = 0)$
& ρ = constant

$$T ds = dh - \frac{dp}{\rho}$$

$$\Rightarrow \quad \frac{\partial h}{\partial r} = \frac{\partial p}{\partial r} \quad \& \quad \frac{\partial p}{\partial r} = \rho \frac{V_\theta^2}{r}$$

$$\& \quad \frac{\partial h_0}{\partial r} = \frac{\partial h}{\partial r} + V_x \frac{\partial V_x}{\partial r} + V_\theta \frac{\partial V_\theta}{\partial r}$$

$$\therefore \quad \frac{\partial h_0}{\partial r} = V_x \frac{\partial V_x}{\partial r} + \frac{V_\theta}{r}\frac{\partial (r V_\theta)}{\partial r}$$

$h_0(r) = \text{const}$ before rotor & $V_\theta = kr$

$$\therefore \quad -\frac{(\frac{1}{2}V_x^2)}{r} = k\frac{d}{dr}(kr^2)$$

$$-d(\tfrac{1}{2}V_x^2) = k\, d(kr^2)$$

$$-\tfrac{1}{2}V_x^2 = k^2r^2 + \text{Const.} = c$$

$$\therefore V_x = \sqrt{2}\,k\sqrt{c - r^2}$$

3 cont

$$\dot{Q} = \int_{r_h}^{r_t} V_x \, 2\pi r \, dr$$

$$\dot{Q} = \int_{r_h}^{r_t} \sqrt{2}\,k\,\sqrt{c-r^2}\;\pi\, r\,dr$$

$$\dot{Q} = \pi\sqrt{2}\,k \int_{r_h}^{r_t} \sqrt{c-r^2}\; d(r^2)$$

$$\dot{Q} = \pi\sqrt{2}\,k \left[\frac{2}{3}\left(c-r^2\right)^{3/2}\right]_{r_h}^{r_t}$$

Comparing the results : $\sqrt{2}k = \sqrt{3200}$; $c = 0.4502$

$$\therefore\ \dot{Q} = \pi \cdot \sqrt{3200}\ \frac{2}{3}\left[\left(0.4502 - 0.5^2\right)^{3/2} - \left(0.4502 - 0.3^2\right)^{3/2}\right]$$

$$= 15.0\ m^3/s \quad \underline{\text{qed}}$$

(ii) At stator exit, V_x = const. At inlet, V_x ≠ const
∴ there is a mass flow redistribution radially. This redistribution causes streamline curvature. The hub-strong profile would tend to cause the pressure to rise at the hub, which would help to relieve the large variation of V_x given by the S.R.E. equation.
Uniform Po usually implies uniform losses, but endwall losses tend to be higher than at midspan

3(b) Throughflow:

- Method: Flow is axisymmetric. Method is inviscid with correlations used to predict losses. Grid is formed by quasi-orthogonals (fixed) and streamlines (move during the calculation). In analysis mode, the user specifies the flow angles at the trailing edges. In design mode, the user specifies a work distribution and the code calculates flow angles. First march along streamlines setting stagnation enthalpy (from Euler's work equation) and entropy (from loss correlations). Then use the meridional streamline curvature equation to evaluate the variation of meridional velocity along each quasi-orthogonal (ensuring continuity is satisfied). Work out new streamline positions and iterate until converged.
- Strengths: Method is very quick (whole turbine in seconds)
- Weaknesses: Assumption of axisymmetric flow (cannot cater for streamsurface twist and distortion) and reliance on correlations for loss.
- Used early in design process (after mean-line analysis) to define layout of machine. Gives boundary conditions (flow angles, stagnation properties) for use in section design.

Finite-volume Navier-Stokes:

- Method: Grid is fixed (usually quadrilaterals or triangles in 2D). Conserve mass, momentum and energy for each cell. March in time using the unsteady control volume equations until flow properties no longer change (steady-state) and conservation is satisfied. Can calculate losses directly but turbulence needs to be modelled (grid not fine enough to resolve turbulence).
- Strengths: Do not need correlations for loss. Can calculate actual geometry (blades and all leakage paths, etc).
- Weaknesses: Much slower than throughflow (hours for a 3D calculation of one stage).
- Used: 2D versions are used to design blade sections. 3D versions calculate the whole blade at once and can be used either to design sections or as a check on the 3D blade shape produced by stacking 2D sections.

3(c)(i) In a fluid, information can be transmitted either by convection or by waves. It is common to consider the convected properties as being entropy and vorticity, and the waves are pressure (acoustic) waves. Convected properties move with the flow, pressure waves propagate at V+a downstream and V-a upstream (where V is the velocity normal to the boundary plane and a is the sonic velocity). If V>a, both waves propagate downstream. In 2D subsonic flow, we have 2 convected properties (entropy and vorticity) and 2 waves (one downstream and one upstream). So we must specify 3 BCs at inlet and 1 at outlet. We typically fix stagnation pressure, stagnation temperature and flow angle at inlet, and static pressure at outlet.

3(c)(ii)Although the solver appears not to require an exit BC, one must be being applied. The solver will set a pressure at exit and evaluate the massflow through the domain. If the massflow is too low, the exit pressure will be reduced, and vice-versa.

ANSWERS: MODULE 4A11: TURBOMACHINERY II

Q1	(b)(ii)	0.12	0.675
Q2	(c)	0.02	
	(d)	0.95	