2009 PART IIB 4A13 COMBUSTION AND IC ENGINES
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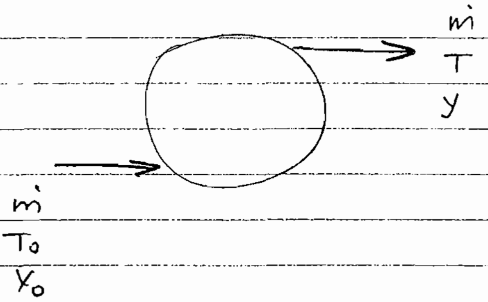
1 PSR

(a)



MASS BALANCE: $\dot m (Y_0 - Y) = \dot\omega V$ (1)

ENERGY BALANCE: $\dot m c_p (T - T_0) = \dot\omega Q V$ (2)

FROM (1) and (2):

$$Y_0 - Y = \frac{c_p (T - T_0)}{Q} \rightarrow Y = Y_0 - \frac{c_p (T - T_0)}{Q}$$

SUBSTITUTING INTO (2)

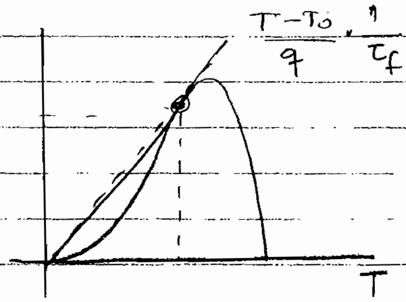
$$\dot m c_p (T - T_0) = A \rho Y \exp(-\theta_a/T) \, QV$$

$$= A \frac{\rho}{\rho_0} \rho_0 \left(Y_0 - \frac{c_p (T - T_0)}{Q} \right) \exp\left(-\frac{\theta_a}{T}\right) QV$$

$$\underbrace{\frac{\dot m}{\rho_0 V}}_{1/\tau_f} \cdot \frac{c_p (T - T_0)}{Q} = \underbrace{A}_{1/\tau_c} \left(\frac{T_0}{T}\right) \left(Y_0 - c_p \frac{(T - T_0)}{Q} \right) \exp\left(-\frac{\theta_a}{T}\right)$$

$$\boxed{\frac{\tau_c}{\tau_f} \frac{T - T_0}{q} = \left(\frac{T_0}{T}\right)\left(Y_0 - \frac{T - T_0}{q}\right) \exp\left(-\frac{\theta_a}{T}\right)}$$

(b) BLOW OFF TAKES PLACE WHEN THE slope OF THE CONVECTIVE TERM EQUALS THAT OF THE CHEMICAL TERM:



LET US DEFINE THE LHS + RHS FUNCTIONS

$$L(T) = \frac{\tau_c}{\tau_f}\,\frac{T-T_0}{q}$$

$$R(T) = \left(\frac{T_0}{T}\right)\left(Y_0 - \frac{T-T_0}{q}\right)\exp\left(-\frac{\theta_a}{T}\right)$$

$$\frac{dL}{dT} = \frac{\tau_c}{\tau_f}\,\frac{1}{q}$$

$$\frac{dR}{dT} = \left(-\frac{T_0}{T^2}\right)\left(Y_0 - \frac{T-T_0}{q}\right)\exp\left(-\frac{\theta_a}{T}\right) + \left(\frac{T_0}{T}\right)\left(-\frac{1}{q}\right)\exp\left(-\frac{\theta_a}{T}\right)$$

$$+\left(\frac{T_0}{T}\right)\left(Y_0 - \frac{T-T_0}{q}\right)\exp\left(-\frac{\theta_a}{T}\right)\left(+\frac{\theta_a}{T^2}\right)$$

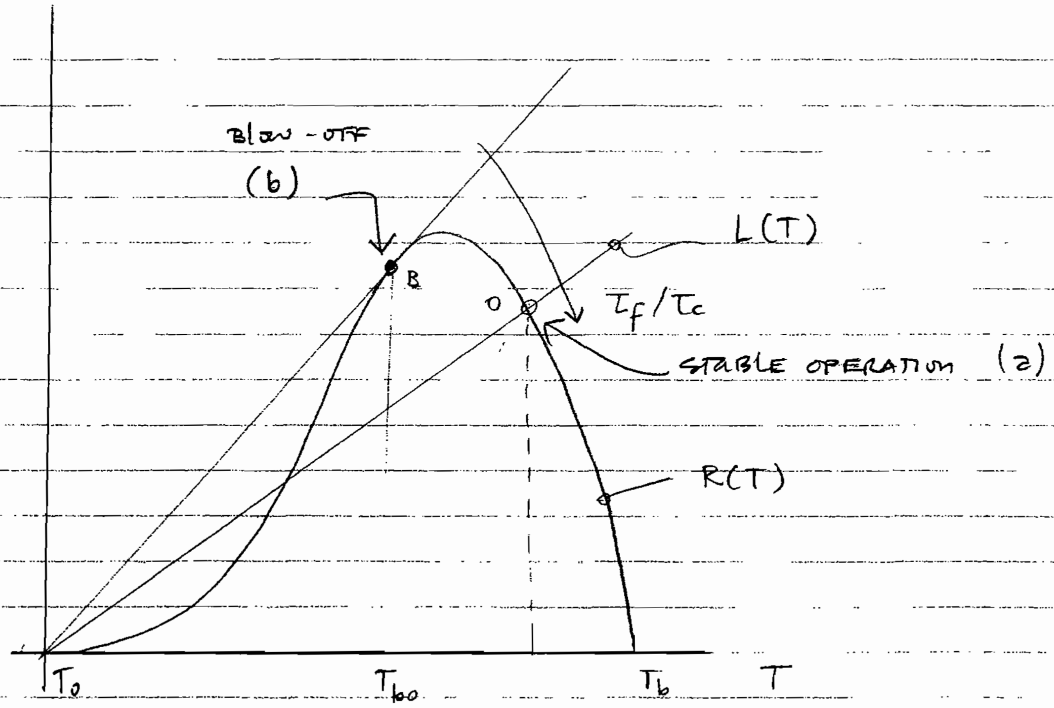
$$\frac{dL}{dT} = \frac{dR}{dT} \Longrightarrow$$

$$\frac{\tau_c}{\tau_f}\,\frac{1}{q} = \left(\frac{T_0}{T}\right)\left[-\frac{1}{T}\left(Y_0 - \left(\frac{T-T_0}{q}\right)\right) - \frac{1}{q} + \left(Y_0 - \frac{T-T_0}{q}\right)\frac{\theta_a}{T^2}\right]\exp\left(-\frac{\theta_a}{T}\right)$$

$$= \frac{1}{T}\,\frac{T_0}{T}\left[\left(Y_0 - \left(\frac{T-T_0}{q}\right)\right)\left[\frac{\theta_a}{T} - 1\right] - \frac{T}{q}\right]\exp\left(-\frac{\theta_a}{T}\right)$$

$$\boxed{\frac{\tau_c}{\tau_f} = \left(\frac{q}{T}\right)\left(\frac{T_0}{T}\right)\left[\left(Y_0 - \left(\frac{T-T_0}{q}\right)\right)\left(\frac{\theta_a}{T} - 1\right) - \frac{T}{q}\right]\exp\left(-\frac{\theta_a}{T}\right)}$$

(d)



(c)

THE STEADY STATE OPERATING POINT IS DETERMINED BY THE INTERSECTION OF THE CONVECTIVE ENERGY TRANSPORT L(T), AND THE REACTIVE ENERGY RELEASE R(T). AT T_b, ALL REACTANT IS CONSUMED AND NO REACTION IS POSSIBLE. AS THE FLOW TIME T_f DECREASES (OR AS THE CHEMICAL TIME T_c INCREASES), LESS REACTION HAPPENS, UNTIL AT THE POINT OF BLOW-OFF (B) THE REACTION CANNOT RELEASE ENOUGH ENERGY TO MAINTAIN THE REACTION TEMPERATURE.

<u>NOTES</u>: STRAIGHTFORWARD QUESTION, LITTLE ALGEBRA.

COMMON PROBLEM: WRITING BLOW-OFF CONDITION AS $\frac{d}{dT} = 0$!

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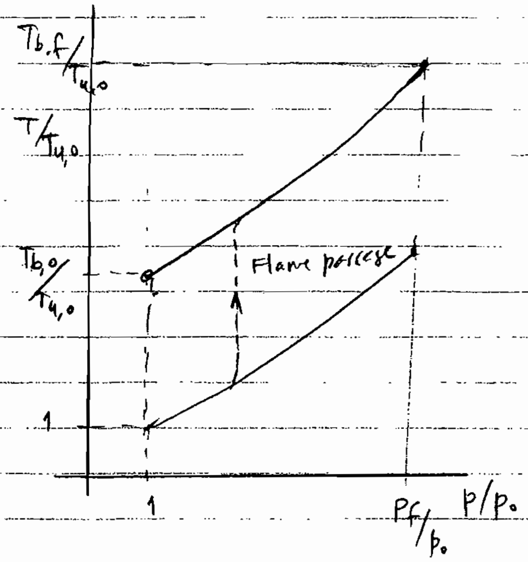
Q [2] NOTE: NOT POPULAR QUESTION, PERHAPS BECAUSE IT LOOKS LONG.

(a) Final temperature of reactants:

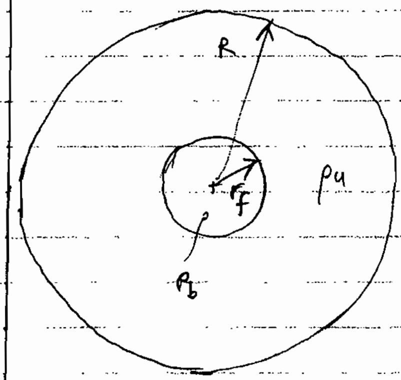


$$m C_v (T_{b,f} - T_{u,o}) = m Y_o Q$$

$$T_{b,f} = T_{u,o} + \frac{Y_o Q}{C_v}$$

$$\frac{T_{b,f}}{T_{u,o}} = 1 + \frac{Y_o Q}{C_v T_{u,o}} \qquad (1)$$

(b) From mass conservation:



$$\frac{4\pi}{3}\left[(R^3 - r_f^3)\rho_u + r_f^3 \rho_b\right] = \frac{4\pi}{3} R^3 \rho_{u,o} \quad \cancel{\tfrac{4\pi}{3} R^3 \rho_{u,o}}$$

$$\left(1 - \left(\frac{r_f}{R}\right)^3\right)\frac{\rho_u}{\rho_{u,o}} + \left(\frac{r_f}{R}\right)^3 \underbrace{\frac{\rho_b}{\rho_{u,o}}}_{\frac{\rho_b}{\rho_{b,o}}\frac{\rho_{b,o}}{\rho_{u,o}}} = 1 \qquad (2)$$

From assumption of isentropic compression:

$$\rho_u = \rho_{u,o}\,(p/p_o)^{1/\gamma} \qquad \rho_b = \rho_{b,o}\,(p/p_o)^{1/\gamma} \qquad (3)$$

Substituting (3) into (2) and using $\hat{r} = r_f/R$ and $\hat{p} = p/p_o$, we have:

$$\left(1 - \hat{r}^3\right)\hat{p}^{1/\gamma} + \hat{r}^3 \frac{\rho_{b,o}}{\rho_{u,o}} \hat{p}^{1/\gamma} = 1$$

$$\left[1 - \left(1 - \frac{\rho_{b,o}}{\rho_{u,o}}\right)\hat{r}^3\right]\hat{p}^{1/\gamma} = 1 \qquad (4)$$

(b) Across the thin, constant pressure flame we apply:

MASS CONSERVATION:

$$\rho_{b,0} u_{b,0} = \rho_{u,0} u_{u,0}$$

$$\boxed{u_{u,0} = \frac{u_{b,0}\,\rho_b}{\rho_u}} \qquad (5)$$

ENERGY CONSERVATION @ CONSTANT PRESSURE

$$(\rho u)\, c_p (T_{b,0} - T_{u,0}) = (\rho u)\, Y_0 Q$$

$$\boxed{\frac{T_{b,0}}{T_{u,0}} = 1 + \frac{Y_0 Q}{c_p T_{u,0}}}$$

$$\boxed{\frac{\rho_{u,0}}{\rho_{b,0}} = \frac{T_{b,0}}{T_{u,0}} = 1 + \frac{Y_0 Q}{c_p T_{u,0}}} \qquad (6)$$

(d) FROM (6) WE HAVE:

$$\frac{\rho_{b,0}}{\rho_{u,0}} = \frac{T_{u,0}}{T_{b,0}} = \left(1 + \frac{Y_0 Q}{c_p T_{u,0}}\right)^{-1} = (1+q)^{-1}$$

$$\left(1 - \frac{\rho_{b,0}}{\rho_{u,0}}\right) = 1 - \frac{1}{1+q} = \frac{q}{1+q} \xrightarrow{(4)} \left(1 - \frac{q}{1+q}\hat{r}^3\right) = \hat{p}^{-1/\gamma} \qquad (7)$$

DIFFERENTIATING (4) W.R.T. TIME, WE HAVE

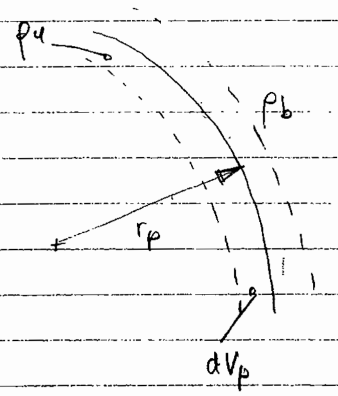
$$-\frac{q}{1+q} \cdot 3\hat{r}^2 \frac{d\hat{r}}{dt} = -\frac{1}{\gamma}\,\hat{p}^{-1/\gamma - 1} \frac{d\hat{p}}{dt}$$

SUBSTITUTING (7): $\hat{r}^3 = \left(1 - \hat{p}^{-1/\gamma}\right)(1+q)/q$

$$\frac{q}{1+q}\, 3 \left[\left(1 - \hat{p}^{-1/\gamma}\right)\left(\frac{1+q}{q}\right)\right]^{2/3} \frac{d\hat{r}}{dt} = \frac{1}{\gamma}\,\hat{p}^{-(1/\gamma + 1)} \frac{d\hat{p}}{dt}$$

$$\boxed{\frac{d\hat{r}}{dt} = \frac{1+q}{3q\gamma}\,\hat{p}^{-\left(\frac{\gamma+1}{\gamma}\right)} \left[\left(1 - \hat{p}^{-1/\gamma}\right)\frac{q+1}{q}\right]^{-2/3} \frac{d\hat{p}}{dt}} \qquad (8)$$

(e) Assuming that the reactant mixture is stationary ahead of the flame, the laminar flame speed is given by:



$$dV_p = +4\pi r_f^2\, dr_f$$

$$dV_R = \underbrace{-4\pi r_f^2\, u_L\, dt}_{\text{prop, only}} - \underbrace{\frac{1}{\gamma}\frac{V_R}{p}\,dp}_{\text{compression}}$$

$$p V_R^{\gamma} = \text{const}$$

$$V_R^{\gamma}\, dp + \gamma p V_R^{\gamma-1}\, dV_R = 0$$

$$dV_R = \frac{1}{\gamma} V_R \frac{dp}{p}$$

$$+\cancel{4\pi r_f^2}\frac{dr_f}{dt} - \cancel{4\pi r_f^2} u_L - \frac{1}{\gamma}\frac{4\pi(R^3 - r_f^3)}{p}\frac{dp}{dt} = 0$$

$$u_L = \frac{dr_f}{dt} - \frac{1}{\gamma}\left(\frac{R^3}{r_f^2} - \frac{r_f^3}{r_f}\right)\frac{1}{p}\frac{dp}{dt}$$

$$\frac{u_L}{R} = \hat{r}\left[\frac{d\hat{r}}{dt} + \frac{1}{\gamma}\left(\frac{1}{\hat{r}} - 1\right)\frac{1}{\hat{p}}\frac{d\hat{p}}{dt}\right]$$

But $\frac{d\hat{r}}{dt}$ is a function of $\frac{d\hat{p}}{dt}$, so we can substitute and solve.

Acceptable answers are also that locally,

$\rho_u u_L = \rho_b u_b = \rho_b \frac{dr_f}{dt}$, and a description of

how one might use $p(t)$ to calculate ρ_b and ρ_u, which are

different from $\rho_{u,0}$ and $\rho_{b,0}$

$u_L = \frac{\rho_b}{\rho_{b,0}}\frac{\rho_{u,0}}{\rho_u}\frac{dr_f}{dt}$ ≈ The problem with this analysis

is that ρ_b is not uniform — each element burns at different P, T —
even though the bulk use of $\frac{\rho_b}{\rho_{b,0}}$ is correct in (2).

Q3, Solution

(a) Isentropic compression, $pV^{\gamma} = const$, so we have $pV^{\gamma} = p_1V_1^{\gamma} = p_2V_2^{\gamma} = k$

The work done is given by $W_{12} = \int_1^2 p\,dV = \int_1^2 \frac{k}{V^{\gamma}}dV = \frac{k}{1-\gamma}\left[V_2^{(-\gamma+1)} - V_1^{(-\gamma+1)}\right]$

So

$$W_{12} = \frac{1}{1-\gamma}\left(p_2V_2^{\gamma}V_2^{1-\gamma} - p_1V_1^{\gamma}V_1^{1-\gamma}\right) = \frac{p_1V_1}{\gamma-1}\left(1-\frac{p_2V_2}{p_1V_1}\right) = \frac{p_1V_1}{\gamma-1}\left(1-\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right) = \frac{p_1V_1}{\gamma-1}\left(1-\left(\frac{V_1}{V_2}\right)^{\gamma-1}\right)$$

(b) $W_{12} = \frac{p_iV_m}{\gamma-1}\left(1-\left(\frac{V_m}{V_c}\right)^{\gamma-1}\right)$,

$$W_{34} = \frac{p_3V_c}{\gamma-1}\left(1-\left(\frac{V_c}{V_m}\right)^{\gamma-1}\right)$$

(c) For the constant volume combustion, $\frac{p_2}{T_2} = \frac{p_3}{T_2+\Delta T_c}$, and

$p_2 = p_i\left(\frac{V_m}{V_c}\right)^{\gamma}$ $T_2 = T_i\left(\frac{V_m}{V_c}\right)^{\gamma-1}$ therefore

$$p_2 = 0.5(9)^{1.4} = 10.84\,bar, T_2 = 288*9^{.4} = 693.6\,K \therefore p_3 = \frac{(693.6+1400)*10.84}{693.4} = 32.73\,bar$$

$$W_{12} = \frac{0.5V_m}{0.4}\left(1-(9)^{0.4}\right) = -1.76V_m$$

$$W_{34} = \frac{32.73V_m}{0.4*9}\left(1-\left(\frac{1}{9}\right)^{0.4}\right) = 5.32V_m$$

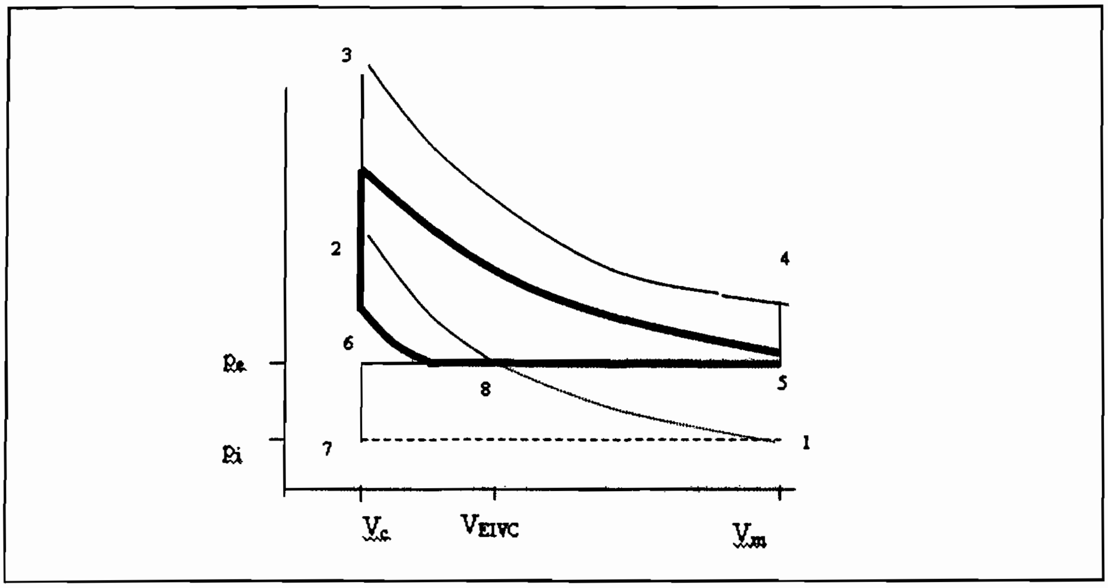
Thus the gross imep = $\frac{(W_{34}+W_{12})}{V_m - V_c} = \frac{(5.32-1.76)}{1-1/9} = 4.004\,bar$

(d) The pumping work is equal to $W_{pump} = (pi - pe)((Vm - Vc)$
And the pmep is $= (p_i - p_e) = 0.5\,bar$ (by convention a +ve quantity), so
$W_{pump} = (pi - pe)(Vm - Vc) = 0.5(1-1/9) = 0.444Vm$
The net imep is thus 4.004 – 0.5 = 3.504 bar

(e) The unthrottled cycle is shown by the heavy lines. The inlet valve is closed late, and as the area of the diagram needs to equal the net work from the throttled cycle, the compression begins at a volume less than V_8

Though not required in the question, an iterative solution to find the value of V_9 that gives a cycle that has the same net imep as the throttled cycle gives $V_9 = 0.395V_m$ (and then $p_4 = 1.07\ bar$) By comparison, the value of V_8 (for the throttled cycle is found

from $V_8 = V_m\left(\frac{p_i}{p_e}\right)^{1/\gamma} = V_m(0.5)^{1/1.4} = 0.61V_m$.



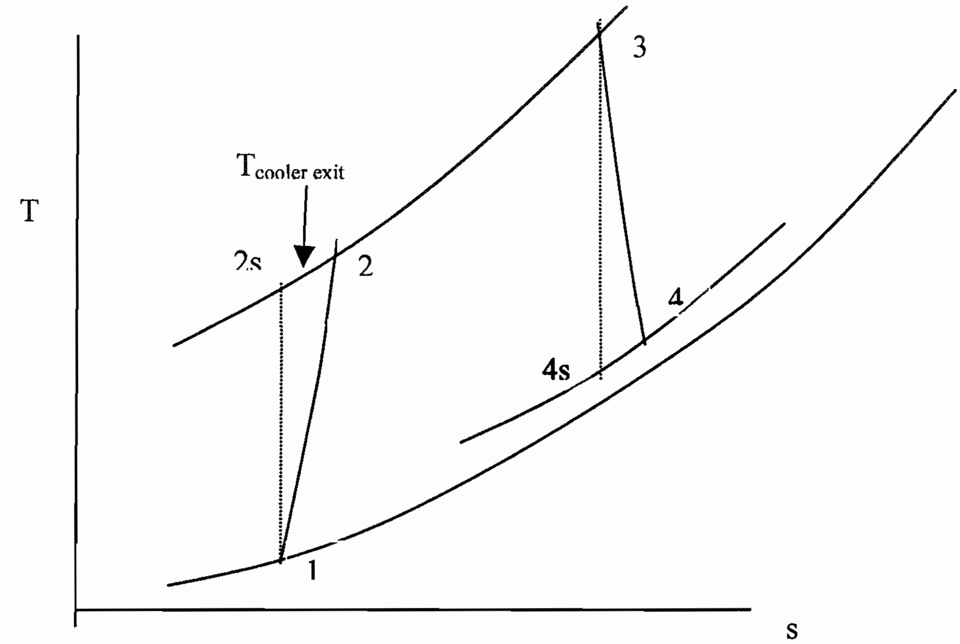
Clearly the efficiencies are much higher than are actually achieved in practice. The assumption of constant volume combustion and adiabatic conditions both lead to the implausible efficiencies. Also the assumption of negligible losses through the valves is untenable. The latter assumption is especially important here, where we are looking at the benefit from late inlet valve closing – shutting the inlet valve at "8" is during a period of very high gas flow – and the inlet valve closing period is necessarily extended, since it is cam operated – therefore there will be significant throttling losses during the closure, and this has a significant effect on the efficiency gain.

In engines where the valve lift is adjustable, the valves may be used as throttles for light load operation instead of the conventional throttle – for example by late inlet valve opening. While at first sight this would seem to offer no benefit, in fact, since the turbulence generated by the valve increases the mixing processes in the cylinder and mass burning rate during combustion, there can be real benefits, especially at low engine speed.

Note: this is a straightforward question. The main difficulty is understanding the shape of the corresponding unthrottled cycle.

Q4 Soln

(a)



(b) For isentropic compression 1 – 2s

$$\frac{T_{2s}}{T_1}=\left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}=\frac{T_{2s}}{288}=\left(\frac{2}{0.95}\right)^{1.4-1/1.4},\therefore T_{2s}=356.26K$$

Now from the definition of the compressor isentropic efficiency,

$$\eta_c=\frac{T_{2s}-T_1}{T_2-T_1}\quad\therefore\quad T_2=385.51K$$

and for the isentropic expansion 3 – 4s

$$\frac{T_{4s}}{T_3}=\left(\frac{p_3}{p_4}\right)^{\gamma-1/\gamma}=\frac{T_{4s}}{873}=\left(\frac{1.05}{1.8}\right)^{1.33-1/1.33},\therefore T_{4s}=763.72K$$

From the definition of the turbine isentropic efficiency

$$\eta_t=\frac{T_3-T_4}{T_3-T_{4s}}\quad\therefore\quad T_4=785.6K$$

(c) The compressor power is $W_c=\dot{m}_{air}\,c_{p,air}(T_2-T_1)=\dot{m}_{air}\,98.98kW$

From the air fuel ratio, $\dot{m}_{ex}=\dot{m}_{air}\left(1+\frac{1}{18}\right)$

So the turbine power may be written

$W_t=\dot{m}_{ex}\,c_{p,ex}(T_3-T_4)=\dot{m}_{air}\,106.82kW$

Therefore the mechanical efficiency is $\eta_{mech}=98.98/106.82=93\%$

(d) If an intercooler were fitted of effectiveness 0.5, then

$$\varepsilon = 0.5 = \frac{T_2 - T_{\text{cooler exit}}}{T_2 - T_1}, \therefore T_{cooler\ exit} = 337\,K$$

The power will increase approximately as the volumetric efficiency at constant AFR. The volumetric efficiency will increase in proportion to the density increase, i.e. as 1/T. Therefore the % increase will be 386/337 - 1 = 14.5%.

Notes: Straightforward question with turbocharger matching and intercooler.