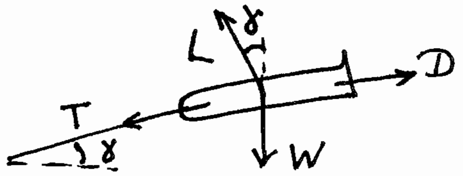
Solutions 2009

Qu 1

a)



Resolving parallel to the glide slope $\underline{\underline{D = T + W\sin\gamma}}$ [10%]

Resolving normal to the glide slope $\underline{\underline{L = W\cos\gamma}}$

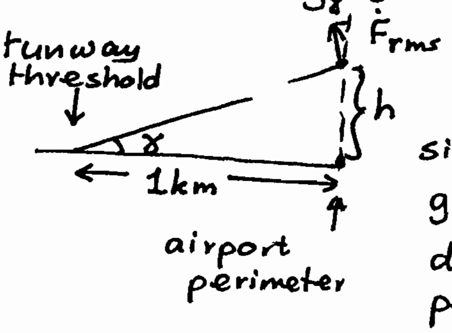
b) i) Additional drag $\Delta D = W\sin 6^\circ - W\sin 3^\circ$ where W = mass × g $= 140\times10^3\times9.81\,N$

$= \underline{\underline{71.7\ kN}}$ [10%]

ii) $\Delta D = \frac{1}{2}\rho_{air} U^2 C_D \times \text{area}$

Hence area of air brake $= \dfrac{71.68\times10^3}{\frac{1}{2}\,1.2\times60^2\times1} = \underline{\underline{33.2\ m^2}}$ [10%]

iii) This is a dipole source with axis in direction of unsteady force.



$$p_{rms} = \frac{\dot F_{rms}\cos\theta}{4\pi|\underline{x}|c_0}$$

since γ is small, maximum SPL on the ground outside airfield perimeter will be directly below as the aircraft crosses perimeter. Then $\cos\theta = 1$, $|\underline{x}| = h = 10^3\sin\gamma$ m.

Maximum $p_{rms} = \dfrac{\dot F_{rms}}{4\pi\sin6^\circ\times10^3\times343}$ N/m^2

where $\dot F_{rms} = \omega F_{rms} = 2\pi\dfrac{2\times U}{\sqrt{A}}\ \frac{1}{2}\rho_{air}U^2\,0.01\times A$

$= 9.38\ kN/s$

Hence maximum $p_{rms} = 0.21\ N/m^2$

$SPL = 20\log_{10}\left(\dfrac{0.21}{2\times10^{-5}}\right) = \underline{\underline{80\,dB}}$ [20%]

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Qu 1 cont.)

c) (i) a decrease in engine flight-idle thrust is always beneficial for noise. It reduces the engine noise and also the required drag which can be a major component of airframe noise. However, it is always important to be able to complete a go-around procedure safety and so how much the flight idle thrust can be reduced depends on how quickly the engines can spool-up. Smaller diameter fans with reduced moments of inertia are an advantage.

(ii) an increase in glide-slope angle increases the height of the aircraft above the ground and so by the inverse-square law leads to some reduction in the noise due to a fixed source. However, as illustrated by the example in part b) of this question, there is then a need for additional drag which could be noisy. While the drag can be produced quietly (SAX40 uses induced drag) or shielded from listeners on the ground, low-noise drag and/or other mitigation like shielding have to be considered as part of the design, if the additional noise due to the drag is not going to swamp the modest reduction due to the increased height. There are also operational issues associated with a mixed fleet with different types of aircraft approaching the same airport on different glide slopes which would reduce the capacity of the airport as well as requiring a doubling up, the infrastructure. The increase in vertical descent rate may make a go-around more difficult.

(iii) a reduction in approach speed is extremely beneficial for noise since the airframe noise sources scale on the fifth or sixth-power of the velocity. However, as seen in part a) the lift force must equal $W\cos\gamma$ and so for the same glide-slope angle the lift must be retained even though the lift coefficient is reduced. Some high-lift devices like slats are noisy and for a low-noise aircraft should be avoided. SAX40 uces a drooped leading edge which does not generate as

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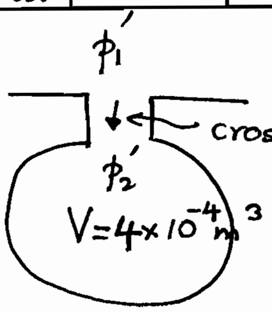
Qu1 c) iii) cont)

much additional lift as slats but was sufficient. A low speed approach can compromise the flight fuel burn if the high C_L on approach leads to an oversized wing. A lower approach speed also means less braking distance is required and so there is scope to use a displaced threshold and land further down the runway. This has the effect of keeping the aircraft higher above the ground outside the airfield leading to a further reduction in noise, but at the expense of more complicated operations and perhaps a duplication of infrastructure. [50%]
Slow approach speeds increase the time each aircraft spends on approach and so can reduce the number of planes landing / unit time, reducing the airport capacity.

Qu2.



$$p_1' - p_2' = \rho_0 l \frac{du}{dt} + \alpha u \qquad (1)$$

The pressure rise in the cavity is related to the stored air.

$$p_2' = c_0^2 \rho_2' \qquad \text{and} \qquad \rho_0 A u_2 = V \frac{\partial \rho_2}{\partial t}$$

Hence $\quad p_2' = \dfrac{c_0^2 A}{i\omega V} u_2 \rho_0$

Substituting into equation (1) gives $\quad p_1' = u\left(\rho_0 l i\omega + \dfrac{c_0^2 A \rho_0}{V i\omega} + \alpha\right)$

a) The rate of sound absorption $\propto \overline{p_1' u}$ and so we would like u to be as large as possible. It is therefore a good idea to choose A so that the Helmholtz resonator resonates at 250Hz

$$\rho_0 l i\omega_0 + \frac{c_0^2 A \rho_0}{V i\omega_0} = 0 \qquad \text{when } \omega_0 = 2\pi\, 250$$

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Qu 2 cont.)

$$\ell = 0.6d, \qquad A = \frac{\pi d^2}{4}$$

Hence $$0.6d\,\omega_0^2 = \frac{c_0^2 \pi d^2}{4V}$$

$$\Rightarrow d = \frac{4 \times 0.6\, \omega_0^2 V}{\pi c_0^2}$$

At $p = 1$ bar, $T = 600$k, $\rho_0 = 0.581$ kg/m^3, $c_0 = 491$ ms^{-1}.

$\underline{\underline{d = 3.13\text{mm.}}}$ [20%]

b) Rate of sound energy absorbed / hole $= \overline{p_1' u'}\, A$

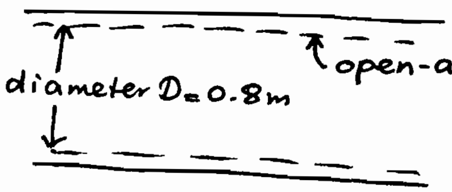
At 250 Hz $\quad u = \frac{p_1'}{\alpha}$, hence

rate of sound energy absorbed / hole $= \overline{p_1'^2}\,\frac{A}{\alpha} = \overline{p_1'^2}\,\frac{A}{\rho_0 c_0 \times 0.1}$

$$= (p_{1rms})^2 \frac{\pi d^2}{4\rho_0 c_0 \times 0.1} = \underline{\underline{2.7 \times 10^{-7} (p_{1rms})^2 \text{ m}^4\text{s/kg}}}$$

[40%]

c)



There is an open-area ratio of 10% and each hole has cross-sectional area $\frac{\pi d^2}{4}$. Hence there are N holes per unit area of liner

where $$N \frac{\pi d^2}{4} = 0.1 \quad \Rightarrow \quad N = \frac{0.4}{\pi d^2} = 1.3 \times 10^4 \text{ holes/m}^2$$

From part b) rate of sound energy / unit area of liner

$$= N\, p_{1rms}^2\, 2.7 \times 10^{-7}$$

Rate of sound energy absorbed in length dx of liner.

$$= N\, p_{1rms}^2\, 2.7 \times 10^{-7}\, \underbrace{\pi D\, dx}_{\text{liner surface area}}$$

= change in acoustic energy flow along duct in length dx.

Qu 2 c cont.)

Energy flow rate along the duct $= \frac{\pi D^2}{4} I$

where $I =$ intensity $= \frac{p'^2}{\rho_0 c_0}$ in a plane wave

Hence over a length δx

$$\frac{\pi D^2}{4} \delta I = - N p_{1rms}^2 \, 2.7 \times 10^{-7} \pi D \, \delta x$$

$$\frac{\pi D^2}{4} \delta I = - N I \rho_0 c_0 \pi D \, 2.7 \times 10^{-7} \delta x$$

i.e $\quad \delta I = - I \times 5.01 \, \delta x$

or $\quad \frac{dI}{dx} + IK = 0 \quad$ where the constant $K = 5.01 \, m^{-1}$

i.e $\quad I(x) = I_0 e^{-5.01 x}$

Attenuation / unit length in dB $= 10 \log_{10}\left(\frac{I(1)}{I_0}\right)$

$$= 10 \log_{10}\left(e^{-5.01}\right)$$

$$= \underline{-21.8 \, dB}$$

[40%]

3 (a) $$p'(\underline{x},t) = \frac{\hat{x}_i \hat{x}_j}{4\pi c_0^4 |\underline{x}|} \frac{d^2 S_{ij}}{dt^2}\left(t - \frac{|\underline{x}|}{c_0}\right)$$

Assume turbulence lengthscale l_0 and velocity time scale u_0, leading to time scale l_0/u_0

$$S_{ij} = \int T_{ij}\, dV \;; \qquad T_{ij} \approx \rho u_i u_j$$

$$\therefore \quad T_{ij} \sim O(\rho_0 u_0^2) \quad \text{and} \quad S_{ij} \sim O(\rho_0 u_0^2 . l_0^3)$$

source volume

$$\frac{d^2}{dt^2} \sim O\left(\left(\frac{l_0}{u_0}\right)^{-2}\right), \text{ie } (\text{timescale})^{-2}$$

$$\therefore \quad p' \sim \frac{1}{c_0^4 |\underline{x}|} \frac{u_0^2}{l_0^2} . \rho_0 u_0^2 l_0^3$$

$$p' \sim \rho_0 \frac{l_0}{|\underline{x}|} \left(\frac{u_0}{c_0}\right)^4 .$$

Acoustic intensity $= p'u'$, $p' = c_0^2 \rho'$

and $u' = \dfrac{p'}{\rho_0 c_0}$, plane wave impedance

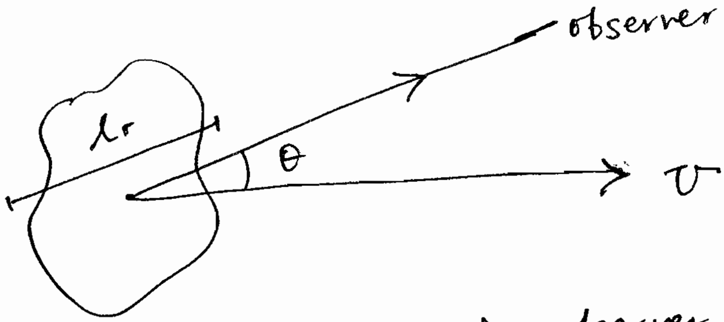
$$\therefore \text{ intensity } \propto (\rho')^2 \sim \underline{\underline{\left(\frac{u_0}{c_0}\right)^8}} .$$ 5/20

timescale ω^{-1}, $\omega = u_0/l_0$ replaced by

$\dfrac{\omega}{D}$ where $D = 1 - M\cos w$.

Qu3 cont.)

Source volume is l_0^3 for stationary source, but in motion volume is $l_r l_0^2$



effective lengthscale in observer direction is elongated, $l_r = \frac{l_0}{D}$

$\therefore$ in above $(p')^2 \to p_0^2 \left(\frac{l_0}{|\underset{\sim}{x}|}\right)^2 \left(\frac{u_0}{c_0}\right)^8 \cdot \frac{1}{D^4} \cdot \frac{1}{D^2}$

$\uparrow$ frequency $\uparrow$ volume

$$\therefore (p')^2 \sim \frac{(u_0/c_0)^8}{(1-M\cos\theta)^6}$$

5/20

(b)

seek solution of form

$\phi = f(y)\exp[ikV(t - x/V)]$

$\phi_{xx} + \phi_{yy} = \frac{\phi_{tt}}{c_0^2} \to -k^2 f + f'' = -k^2 M^2 f$ $\quad M = V/c_0$

$f'' + (M^2-1)k^2 f = 0$

$M>1$ $\quad f = A\exp[i\sqrt{M^2-1}\,ky] + B\exp[-i\sqrt{M^2-1}\,ky]$

want outgoing wave at infinity $\therefore$ choose $-$ root.

Also $p = -\rho_0 \frac{\partial\phi}{\partial t} = -\rho_0 ikV\phi$

$p(0) = -\rho_0 ikV f(0)\exp[ikV(t - x/V)]$

$= P\exp[ikV(t - x/V)]$

Qu3 cont.)

$$\therefore \quad P_0 = -ikV\rho_0 f(0), \quad f(0) = \frac{iP_0}{kV\rho_0}$$

$$\therefore \quad p(x,y,t) = P_0 \exp\left[ikV(t - x/V) - i\sqrt{M^2-1}\,ky\right]$$

$$\underset{\sim}{v}(x,y,t) = \nabla\phi = \left(-ikf(y), \; f'(y)\right)\exp\left(ikV(t - x/V)\right)$$

$$= \left(\frac{P_0}{V\rho_0}, \; \frac{P_0\sqrt{M^2-1}}{V\rho_0}\right)\exp\left[ikV\left(t-\frac{x}{V}\right) - i\sqrt{M^2-1}\,ky\right]$$

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when $V < c_0$, $f(y)$ obeys eq^n of form

$$f'' - (1-M^2)k^2 f = 0,$$

which has sol^{n}s $\exp\left[\pm\sqrt{1-M^2}\,ky\right]$.

Reject sol^n $+$, as growing at infinity

$\therefore$ left with acoustic field which decays like $\exp\left[-\sqrt{1-M^2}\,ky\right]$ in the wall-normal direction. Evanescent.

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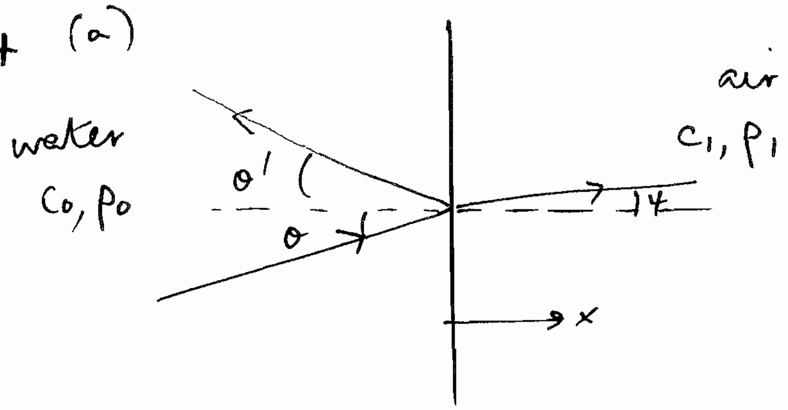
4 (a)



$c_1 < c_0$

in $x<0$ $\quad p(x,y,t) = I \exp(i\omega t - ik_0\cos\theta\, x - ik_0 \sin\theta\, y)$

$\qquad + R\exp(i\omega t + ik_0 \cos\theta' x - ik_0 \sin\theta' y)$

in $x>0$ $\quad p(x,y,t) = T\exp(i\omega t - ik_1\cos\psi\, x - ik_1\sin\psi\, y)$

$k_0 = \omega/c_0, \quad k_1 = \omega/c_1$

Continuity of pressure on $x=0$:

$I\exp(-ik_0\sin\theta\, y) + R\exp(-ik_0\sin\theta' y) = T\exp(-ik_1\sin\psi\, y)$

holds for all $y \Rightarrow$ exponentials cancel

$\Rightarrow \underline{\theta' = \theta}$ and $\underline{\dfrac{\sin\theta}{C_0} = \dfrac{\sin\psi}{c_1}}$ 2/20

Also $\quad I + R = T$ ——— (1).

Continuity of normal velocity on $x=0$:

$p = -\rho_0 \dfrac{\partial\phi}{\partial t} = -\rho_0 i\omega\phi \to \phi = \dfrac{ip}{\rho_0\omega}$

$\therefore$ on $x \le 0^-$ $\quad \dfrac{\partial\phi}{\partial x} = \dfrac{i}{\rho_0\omega}\left(-ik_0\cos\theta\, I + ik_0\cos\theta\, R\right)\exp(\dots)$

on $x = 0^+$ $\quad \dfrac{\partial\phi}{\partial x} = \dfrac{i}{\rho_1\omega} T\left(-ik_1\cos\psi\right)\exp(\dots)$

Equating $\Rightarrow \quad \dfrac{\cos\theta}{\rho_0 C_0}(I - R) = \dfrac{T\cos\psi}{\rho_1 c_1}$ ——— (2)

Add (1) and (2)

$$2I = T\left(1 + \frac{\rho_0 c_0 \cos\psi}{\rho_1 c_1 \cos\theta}\right)$$

$$\therefore T = \frac{2 I \rho_1 c_1 \cos\theta}{\rho_1 c_1 \cos\theta + \rho_0 c_0 \cos\psi}$$

subtract (1) and (2)

$$2R = T\left(1 - \frac{\rho_0 c_0 \cos\psi}{\rho_1 c_1 \cos\theta}\right)$$

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$$\therefore R = \frac{I\,(\rho_1 c_1 \cos\theta - \rho_0 c_0 \cos\psi)}{\rho_1 c_1 \cos\theta + \rho_0 c_0 \cos\psi}$$

All energy transmitted $\Rightarrow$ $R = 0$

$$\therefore \rho_1 c_1 \cos\theta = \rho_0 c_0 \cos\psi$$

$$\rho_1^2 c_1^2 \cos^2\theta = \rho_0^2 c_0^2 (1 - \sin^2\psi) = \rho_0^2 c_0^2 \left(1 - \frac{c_1^2 \sin^2\theta}{c_0^2}\right)$$

$$\rho_1^2 c_1^2 - \rho_0^2 c_0^2 = \sin^2\theta\,(\rho_1^2 c_1^2 - \rho_0^2 c_1^2)$$

$$\Rightarrow \sin^2\theta = \frac{\rho_1^2 c_1^2 - \rho_0^2 c_0^2}{c_1^2(\rho_1^2 - \rho_0^2)}$$

$$\sin\theta = \frac{\sqrt{\rho_0^2 c_0^2 - \rho_1^2 c_1^2}}{c_1 \sqrt{\rho_0^2 - \rho_1^2}}$$

ie special incidence angle. 2/20

(b) $\dfrac{\sin\theta}{c_0(x)} = \dfrac{\sin\theta_0}{c_0(0)}$, $\tan\theta = y'$

$\Rightarrow \sin\theta = \dfrac{y'}{\sqrt{1+y'^2}}$

$\therefore \dfrac{y'}{\sqrt{1+y'^2}} = \dfrac{e^{\alpha x} c_0(0)\sin\theta_0}{c_0(0)}$ $\quad y'^2 = e^{2\alpha x}(1+y'^2)\sin^2\theta_0$

$\rightarrow y'^2 = \dfrac{\sin^2\theta_0 e^{2\alpha x}}{1-\sin^2\theta_0 e^{2\alpha x}} \Rightarrow y' = \dfrac{\sin\theta_0 e^{\alpha x}}{\sqrt{1-\sin^2\theta_0 e^{2\alpha x}}}$

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$\int dy = \int \dfrac{\sin\theta_0 e^{\alpha x}}{\sqrt{1-\sin^2\theta_0 e^{2\alpha x}}}dx$, let $\sin\theta_0 e^{\alpha x} = u$

$\rightarrow \alpha \sin\theta_0 e^{\alpha x} dx = du$

$\therefore y + \text{constant} = \dfrac{1}{\alpha}\int \dfrac{du}{\sqrt{1-u^2}} = \dfrac{1}{\alpha}\left(\sin^{-1}u + c\right)$

$\therefore y = \dfrac{\sin^{-1}(\sin\theta_0 e^{\alpha x})}{\alpha} + c$

when $x=0, y=0$ $\quad \therefore c = -\dfrac{\sin^{-1}(\sin\theta_0)}{\alpha} = -\dfrac{\theta_0}{\alpha}$

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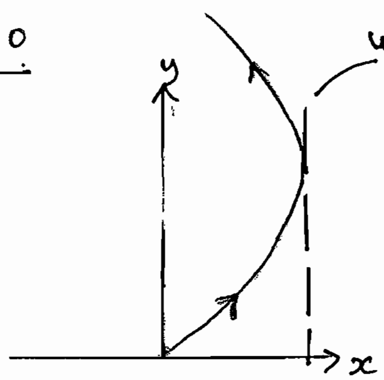
$\therefore y = \dfrac{\sin^{-1}(\sin\theta_0 e^{\alpha x})}{\alpha} - \dfrac{\theta_0}{\alpha}$

$\underline{\alpha > 0}$

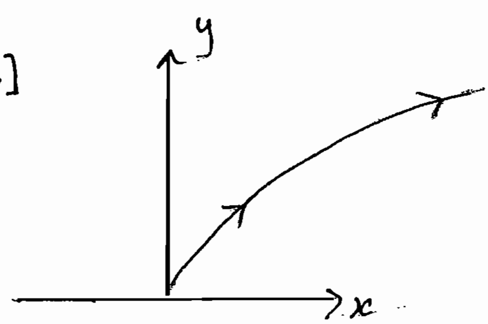


where $y' = \infty$

$\Rightarrow \sin\theta_0 e^{\alpha x} = 1$

$\Rightarrow x = -\dfrac{1}{\alpha}\ln[\operatorname{cosec}\theta_0]$

$\underline{\alpha < 0}$



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