4B5 2009 Answers

1.

(a) We should use plane waves when dealing with a beam of particles, i.e. where there is no spatial localisation, and we should use wave packets when we are considering the behaviour of single particles.

(b) EM waves have the following E-k relationship: $E = \hbar ck$, whereas for matter waves: $E = \hbar^2k^2/2m$. As a result, the velocity of EM waves is a constant (speed of light), whereas the velocity of matter waves depends on k [Group velocity = $d\omega/dk$]. Therefore, EM wave packets retain their shape as they propagate through space, unless they are in a dispersice medium, whereas matter waves automatically spread out.

(c) Quantum factors which pose a problem to further miniaturisation of the transistor are **Tunneling** and **Coherence**.

(i) Tunneling is a purely quantum phenomenon whereby a particle of energy E can pass through (tunnel) a classically forbidden region, i.e. it can pass through a potential barrier with an energy greater than E. Tunneling is a problem as the gate oxide gets thinner, because it results in the leakage of signals from the gate, and hence reduces the transistor's gain. At the moment, the thickness of gate oxides is in the range 0.8 nm to a few nm. Further decreases in this dimension will increase tunnelling, and impair the gain of the transistor.

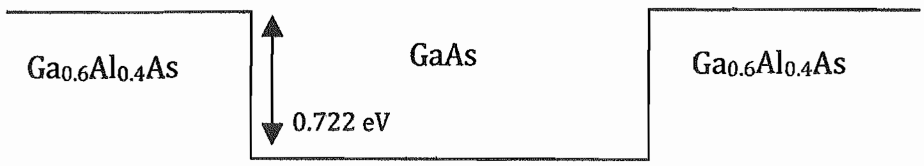
(ii) Coherence. As the electrons travel from the source to the drain in a conventional transistor, they are essentially incoherent as the distance they have to travel is significantly longer than the coherence length, so any quantum scattering is essentially smeared out. However, for 2eV electrons, the de Broglie wavelength $2\pi\sqrt{(\hbar^2/(2mE))}$, which is 0.9 nm. Thus, we can say that quantum coherence will give rise to interference effects which will alter the behaviour of the transistor once the gate length is comparable to 1 5 nm.

(d) (i) Quantum effects can be reduced by using a high-k dielectric as the gate oxide. This would mean lower gate voltages could be used for the same fields, and therefore there will be a lower probability of tunnelling.

(ii) Coherence effects cannot be overcome, so it makes more sense to try to use them to our advantage, eg in resonant tunnelling devices.

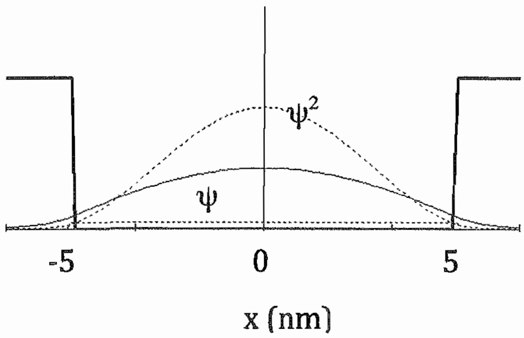
In fact, high –K dielectrics are being used to reduce the operational gate voltage as this reduces tunneling. Intel's new Atom processor uses a Hafnium-based dielectric for this very reason.

2(a) Energy of $Ga_{0.4}Al_{0.6}As$ is 2.172 eV, which is 0.722 eV greater than GaAs. Therefore, the quantum well, ignoring any band bending, is 0.722 eV deep, 10 nm wide, i.e (assuming all of the band-gap difference appears in the conduction band, although we could assume it is equally split between both the conduction and valence bands, and assume the well is therefore half as deep – the question does not specify which):



(b) Using infinite well energy formula, i.e. $E_n = h^2n^2/(8mL^2)$ to describe the nth energy level, we obtain 56 meV for the ground state energy. This is approximately 15 times lower than the depth of the well, so we can be *reasonably* confident that it is accurate to within a few percent. As long as the energy level of interest is less than or around 10% of the well depth, we can use the infinite well approximation. Our assumptions are that there is only 1 electron confined in an infinite quantum well, and that the problem is purely 1-D.

c) The wave-function and probability density:



The characteristic decay length is 1/k, or $\frac{\hbar}{\sqrt{2m(V-E)}} = 0.9$ nm

We could determine the probability of finding the electron outside the well either by looking at the plot of $|\Psi|^2$, and finding the area outside over the total area, or once we know the energy levels numerically, we can find the coefficients of the wave-functions and integrate to find the area. The second method would be more accurate.

3.

(a) Schrödinger's equation: $-(\hbar^2/2m)\partial^2\psi/\partial x^2 + V\psi = E\psi$

The form of V is $V(x) = ½kx^2 = ½m\omega x^2$

Therefore, $-(\hbar^2/2m)\partial^2\psi/\partial x^2 + ½ m\omega^2x^2\psi = E\psi$

If we change variables to let $y = (m\omega/\hbar)^{0.5}x$, and $\alpha = 2E/\hbar\omega$, we can re-write the above equation as:

$$\partial^2\psi/\partial y^2 + (\alpha - y^2)\psi = 0$$

as required. (2090)

(b) Starting with the equation from above, i.e.

$$\partial^2\psi/\partial y^2 + (\alpha - y^2)\psi = 0$$

If we say that $\psi(y) = F(y)e^{-y^2/2}$ we get

$$F'' - 2yF' + (\alpha - 1)F = 0$$

If we assume that F(y) is a power series, i.e.

$$F = \sum_{p=0}^{\infty} a_p y^p$$

Then

$$F' = \sum_{p=0}^{\infty} p a_p y^{p-1} \quad \text{and} \quad F'' = \sum_{p=0}^{\infty} p(p-1) a_p y^{p-2}$$

Now, y can never have a negative power, as then the solution would have a singularity at y = 0. therefore, in the expansion for F'' we can let p -> p+2. That then gives us the following:

$$\sum_{p=0}^{\infty}\left[(p+2)(p+1)a_{p+2} - (2p+1-\alpha)a_p\right]y^p = 0$$

For a non-trivial solution then, we must have:

$$\frac{a_{p+2}}{a_p} = \frac{(2p+1-\alpha)}{[(p+1)(p+2)]}$$

Now, this series essentially goes as 1/p, the sum of which diverges to infinity. Therefore, we must artificially truncate the power series at some value of p, say n. Because a_p is related to a_{p+2}, we can split the solution into two power series, one with even and the other with odd powers of y. Depending on whether n is even or odd, we then set the other power series equal to zero, so in other words, the solution is truncated at some value of p which we call n, and if n is even the series only contains even terms, and if n is odd, it only contains odd terms. Then that gives us the following relationship:

$$2n + 1 - \alpha = 0$$

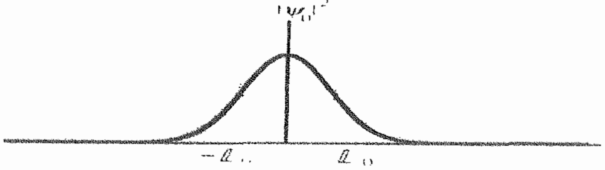
which means $\alpha = 2n + 1$. But, $a = 2E/\hbar\omega$ which means

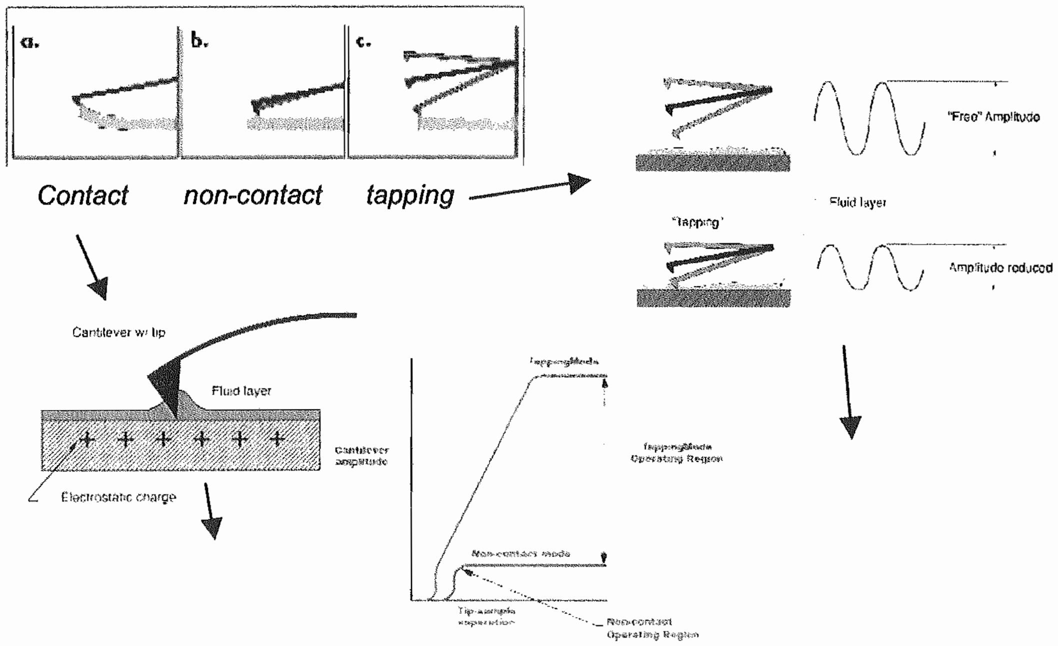
$$E_n = (n + ½)\hbar\omega, \text{ as required.}$$

(b) Well, $\omega = (k/m)^{0.5}$, = 2×10^{14} rad/s, which corresponds to an energy, $(h\omega/2\pi)$ of 0.132 eV, which means the ground state energy is 0.066 eV.

(c)

Only discrete values of energy are allowed as this is a quantum system which has *modes.* If we visualise the potential profile of a QSHO, it is a parabolic well, so only those quantum states whose wavelength is a half-integer divisor of the well length are allowed. This is not a discrepancy with classical mechanics, as for highly excited states or macroscopic objects, the Quantum and classical theories converge.





(d) answer should include a description of conducting AFM- sources of current flow (Ohmic current, Tunneling current, field emission, and then time-dependent currents such as displacement current etc). This is a contact-mode technique wherby the tip is placed in mechanical and hopefully Ohmic contact with the sample of interest. A potential difference between the tip and sample causes a current to flow between the two, which can be measured, and the sample potential can be determined. This has a spatial resolution comparable with the tip contact radius, which is on the order 1 nm. The tip should be conductive and this can be achieved by coating a conventional tip with metal. One should be careful when scanning to not wear this coating off too quickly.

C. Durkan, May 2009