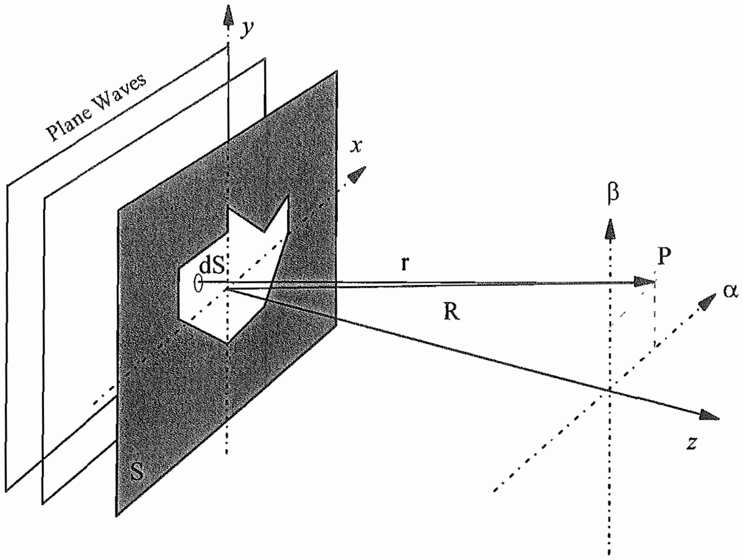
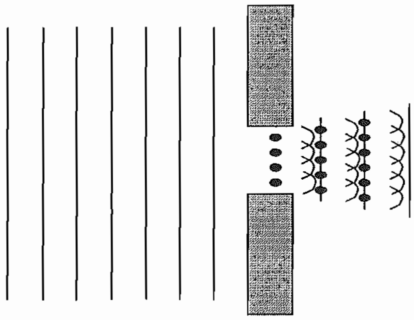
Dr T D Wilkinson

Q1 a)





If we consider an infinitely small differential of the aperture, *dS*, we can model this as a point source of light emitting spherical 'Huygens' wavelets with an amplitude of *A(x,y)dS*. The wavelet acts as a radiating point source, so we can calculate its field at the point *P*, a distance *r* from *dS*. The point source *dS* can be considered to radiate a spherical wave front of frequency ω.

We are assuming that no light is propagated in the reverse direction and that the light is in plane waves and the aperture is perfectly blocking. We are also assuming the light is monochromatic and has a high degree of coherence.

b) If the point *P* is reasonably coaxial (close to the *z* axis, relative to the distance *R*) and the aperture *A(x,y)* is small compared to the distance *R*, then the lower section of the equation for *dE* can be assumed to be almost constant and that for all intents and purposes, $r = R$.

$$dE = \frac{A(x,y)}{R} e^{j\omega t} e^{-kR\sqrt{1 - \frac{2\alpha x + 2\beta y}{R^2} + \frac{x^2+y^2}{R^2}}} dxdy$$

The similar expression in the exponential term in the top line of the original equation is not so simple. It can not be considered constant as small variations are amplified through the exponential. To simplify this section we must consider only the far field or Fraunhofer region where $R^2 >> x^2 + y^2$ In this case, the final term in the exponential $((x^2 + y^2)/R^2)$ can be considered negligible. To further simplify, we use the binomial expansion,

$$\sqrt{(1-d)} = 1 - \frac{d}{2} - \frac{d^2}{8} \dots$$

and keep the first two terms only to further simplify the exponential expression.

$$e^{jkR\left(1 - \frac{\alpha x + \beta y}{R^2}\right)}$$

Hence the simplified version of the field *dE*, can be expressed as:

$$dE = \frac{A(x,y)}{R} e^{j(\omega t - kr)} e^{jk\left(\frac{\alpha x + \beta y}{R}\right)} dxdy$$

The total effect of the dS wavelets can be integrated across *dE* to get an expression for the far field or Fraunhofer diffraction pattern.

$$E(\alpha,\beta) = \frac{1}{R} e^{j(\omega t - kR)} \iint_{Aperture} A(x,y) e^{jk(\alpha x + \beta y)/R} dxdy$$

The initial exponential term $e^{j(\omega t - kR)}$ refers the wave to an origin at $t = 0$, but we are only interested in the scaling of relative points at P with respect to each other, so it is safe to normalise this term to 1.

$$\frac{1}{R}e^{j(\omega t - kR)} = 1$$

Thus, our final expression for the far field diffraction pattern becomes:

$$E(\alpha,\beta) = \iint_A A(x,y)e^{jk(\alpha x+\beta y)/R}dxdy$$

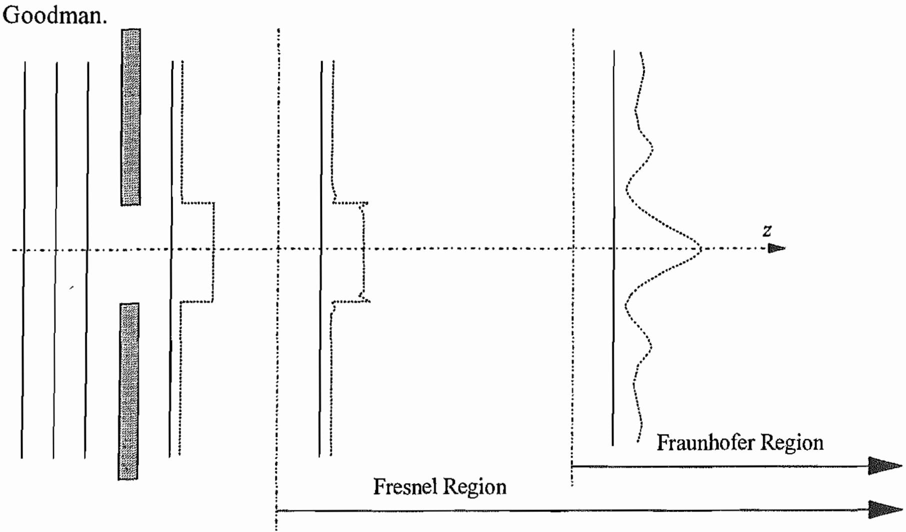
Hence the far field diffraction pattern at the point P is related to the aperture function $A(x,y)$, by the Fourier transform.

Fraunhofer region = Far field pattern = FT{ Aperture function}

iii) The exact validation of these approximations is difficult to analyse directly, so it is safer to assume a sufficiently large value of R to get the far field diffraction pattern of the aperture. One useful guideline proposed by Goodman is to assume that the far field pattern occurs when.

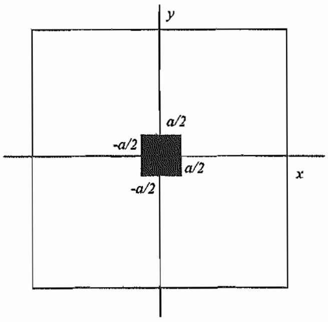
$$R >> \frac{k\left(x_{\max}^2 + y_{\max}^2\right)}{2}$$

where x_{max} and y_{max} are the maximum dimensions of the aperture $A(x,y)$. The regions of the approximation are defined such that in the far field or Fraunhofer region, the approximations are accurate, hence the field distribution $E(x,y)$ only changes in size with increasing z, rather than changes in structure. In the case where the approximation is bearably accurate, we are in the Fresnel region. Before the Fresnel region, the evaluation of E is extremely difficult and is defined as the near field diffraction pattern. The exact boundary of the Fresnel region will depend on the acceptable accuracy and can be found in Goodman.

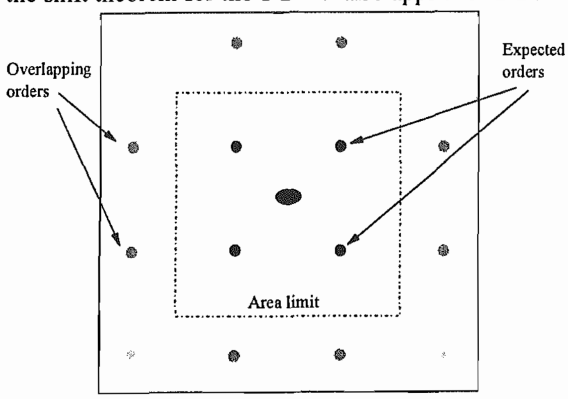


b) We want to calculate the far field or Fraunhofer region for a square aperture. This aperture can be represented in two dimensions as a 'block' function.
The far field of this aperture is its Fourier transform:

$$F(u,v) = Aa^2\text{sinc}(\pi au)\text{sinc}(\pi av)$$



The far field is a 2-D sinc function Now we will look at what happens when we shift the aperture by a distance $a/2$ from the origin of the plane. From this result we can see that the shift theorem for the 1-D FT also applies in 2-D. Hence an aperture shifted x_o, y_o from the origin adds an exponential phase term to the original 2-D sinc function of the centred aperture.

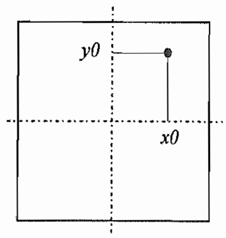


The original rectangular aperture is defined as a single pixel. By combining an array of these pixels at various positions on a regular grid, it is possible to generate a complex amplitude function in the far field. Such a 2-D combination of these pixels in various positions is defined as a Hologram and the pattern generated by the hologram if the far field is the Replay Field.

By altering the value of the amplitude A of each pixel, centred on a grid of interval b (in the example of two pixels above a was equal to b, but may not always be so), it is possible to add up the 2-D sinc functions and create an arbitrary 2-D distribution in the far field region. By superimposing all the exponential phase terms due to the shift and varying the amplitude A, it is possible to create useful patterns in the far field. In general terms, the broader the feature or combination of pixels, the smaller or more delta function-like the replay object. Also, repetitive pixel patterns in the hologram leads to repetitive features in the replay field.

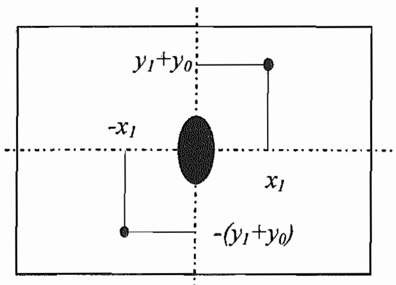
The exact structure of this distribution depends on the shape of the 'fundamental' pixel and the number and distribution of these pixels in the hologram. The pattern we generate with this distribution of pixels is repeated in each lobe of the sinc function from the fundamental pixel. The lobes can be considered as spatial harmonics of the central lobe, which contains the desired 2-D pattern.

Q2) a) With the MF, the FT of the reference is done off line on a computer and is defined as the matched filter *R(u,v)* for that particular reference *r(x,y)*. In fact, the generation of the filter may be more complicated (to include invariances) and it is advantageous to use only the phase information of the reference FT rather than the full complex amplitude and phase. The object in the reference *r(x,y)* is centred in the process of generating the filter *R(u,v)*, so that if a correlation peak occurs, its position is directly proportional to the object in the input image, with no need for any decoding. Unlike in the JTC, there is only one correlation peak and there are no DC terms to degrade the correlator output.

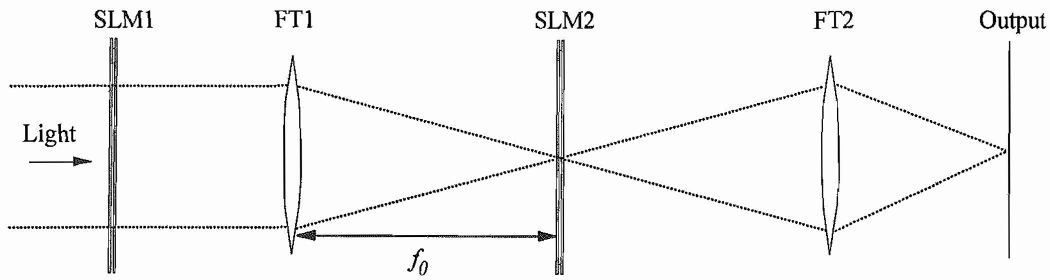


The main limitation of the MF is that it is optically and mechanically complicated and there are some critical alignment issues with the positioning of the Ft of the input and the MF. The JTC is a much simpler architecture mechanically and has very few restrictive tolerances. It is however technically more complicated as it requires the non-linearity to generate the correlation peaks and it is not as flexible as the reference and image have to be displayed on the same SLM in the input plane.

The output plane from the JTC contains a large central DC term which is an unwanted source of noise, and it degrades the output and detectability of the peaks from the optical system. There are always a symmetric pair of correlation peaks, so only half the output plane needs to be considered but the position of the correlation peak has to be decoded to gain the position of the reference object in *r(x,y)* if it correlates with *s(x,y)*.



b) The choice of binary phase modulation allows the use of FLC SLMs to display the information in an optical system. Hence, with the right choice of lenses, we can build a working BPOMF correlator. The input SLM can be slower than the filter SLM so it could be a nematic device for video rate detection systems.



The basic optical layout for a BPOMF follows directly from the theoretical expectations. The input light illuminates SLM1 which is used to display the input image. SLM1 is also a FLC SLM, but it is used in intensity mode (black and white). The SLM is an N_IxN_I array of square pixels, with a pitch of Δ_I, we are assuming that there is no pixel deadspace. The modulated light then passes through lens f_0 which performs the FT of the input image. The FT is formed in the focal plane of the lens and will have a finite resolution (or 'pixel' pitch) given by.

$$\Delta_0 = \frac{f_0 \lambda}{N_1 \Delta_1}$$

There are N_1 'pixels' in the FT of the input image on SLM1, hence the total size of the FT will be $N_1\Delta_0$. The BPOMF is displayed on SLM2 in binary phase mode. SLM2 is also a FLC device with $N_2 x N_2$ pixels of pitch Δ_2. The FT of SLM1 must match pixel for pixel with the BPOMF on SLM2 in order for the correlation to occur. For this reason we must choose f_0 such that.

$$N_1\Delta_0 = N_2\Delta_2$$

Hence we can say.

$$f_0 = \frac{N_2\Delta_2\Delta_1}{\lambda}$$

Once the FT of the input (matched in size to SLM2) has passed through SLM2, the product of the input FT and the BPOMF has been formed. This is then FT'ed again by the final lens and the output is imaged by a CCD camera.

c) The filter is generated by the FT of the of the refernce image where left 'A' is r1 and right A is r2

$$R_1(u,v)e^{j2\pi x_o u} + R_2(u,v)e^{-j2\pi x_0 u}$$

The spectrum is generated by the FT of the input image where s1 and s2 represent each A

$$S_1(u,v)e^{j2\pi y_o v} + S_2(u,v)e^{-j2\pi y_0 v}$$

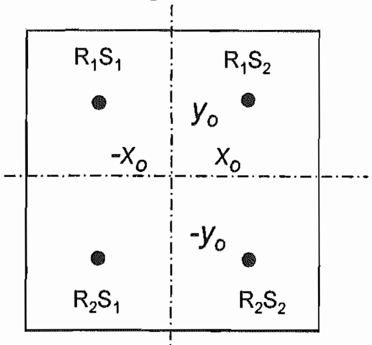
After the MF plane, take the product of the 2 will give us 4 correlation terms and 4 correlation peaks

$R_1S_1e^{j2\pi(x_o u+y_0 v)}$

$R_1S_2e^{j2\pi(x_o u-y_0 v)}$

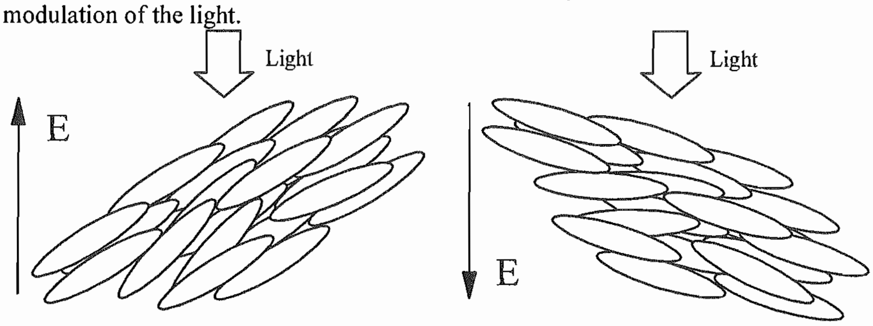
$R_2S_1e^{j2\pi(-x_o u+y_0 v)}$

$R_2S_2e^{-j2\pi(x_o u+y_0 v)}$

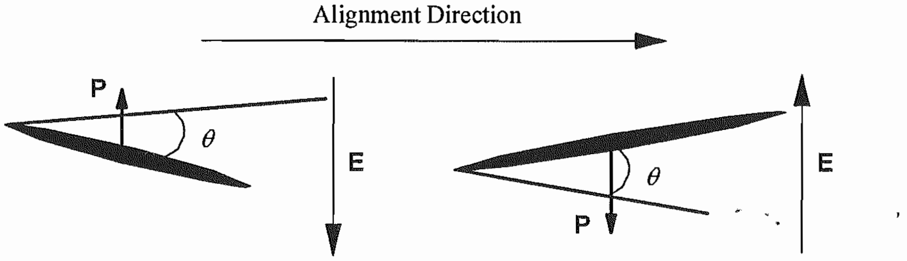


d) If the input SLM is HDTV resolution then there are several problems due to the rectangular shape of the SLM (1900x1080). The HDTV format forms the apodisation limits for the spectrum of the spectrum of the input. Hence the spatial frequencies in the input spectrum will be convolved with the FT of the aperture which is not square, therefore it will not be a symmetric sinc function. This will distort the spectrum in 1 dimension. Also the short edge will remove more of the input illumination source leading to more sidelobes in the spectrum This problem can be minimised by only using 1080x1080 pixels in the input SLM or by illuminating it with a non-circularly symmetric light source.

Q3 a) There are two main geometries for liquid crystal modulation of light, in plane and out of plane, based on the orientation of the LC molecules and how they move with an applied field. In the case of nematic LCs in planar alignment, it is an out of plane effect. The existence of this dielectric anisotropy means that we can move the molecules around by applying an electric field across them. This combined with the flow properties means that a nematic molecule can be oriented in any direction with the use of an electric field. This is a very desirable feature as it leads to their ability to perform greyscale modulation of the light.

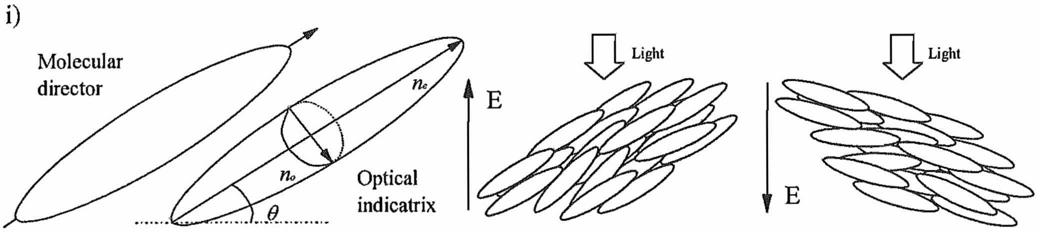


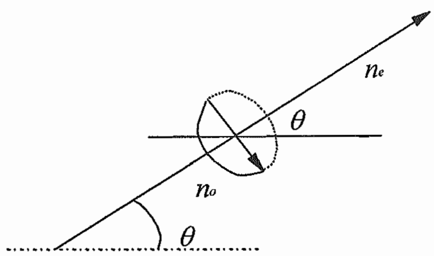
In the case of a smectic such as a SSFLC the molecular motion is in plane as it is restricted by the cell thickness of 2-5μm and the molecules are bounded into two stable states either side of the director cone. The angle between these two states is defined as the switching angle *θ*. This is referred to as a surface stabilised FLC geometry an creates a high degree of ferroelectricity and creates a large birefringent electro-optical effect. The penalty for doing this is that the molecules are only stable in the two states an therefore the modulation will only be binary. The up side to this binary modulation is that it can be very fast (~10μsec) and that the stability can lead to the molecules remaining in the two states in what is known as bistable switching.



For the application of phase modulation, both geometries work, but have flaws. A pure phase modulator will retard the wave in the same way, irrespective of its polarisation state. An example of this would be an etched piece of glass. Both in plane and out of plane are polarisation sensitive techniques. Out of plane will modulate the polarisation that is parallel to n_e in a perfect fashion, but will have no effect the orthogonal state. In plane will modulate the phase by rotating the polarisation state in some way and can be converted into phase modulation through the addition of external polarisor. This is an example of polarisation modulation as would be found from a FLC material.

i)





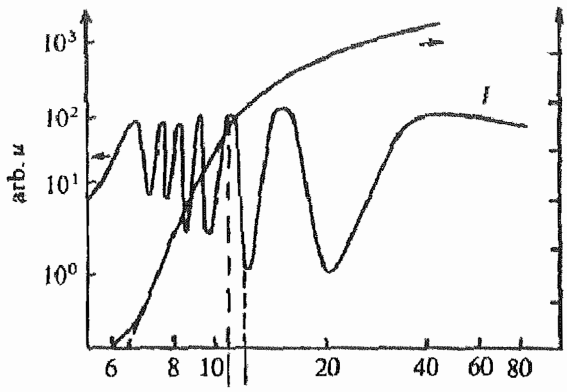
ii) Based on the geometry of the optical indicatrix, the light (polarised parallel to n_e) passing from above sees an average of the two components of the refractive index.

Hence based on the trigonometry we have two relative components $n_e\sin(\theta)$ and $n_o\cos(\theta)$. The light passing through will effectively see an RMS average of the two components based on the geometric mean.

$$n(\theta) = \frac{n_o n_e}{\left(n_e^2 \sin^2\theta + n_o^2 \cos^2\theta\right)^{1/2}}$$

iii) The retardance G is a function of the birefringence of the state ($n(\theta) - n_o$) and also the wavelength of the light λ and the thickness of the cell containing the LC, *d*. The retardance is the phase difference between a wave passing through the short axis and the wave passing through the material oriented and an angle θ.

$$\Gamma = \frac{2\pi d\left(n(\theta) - n_o\right)}{\lambda}$$

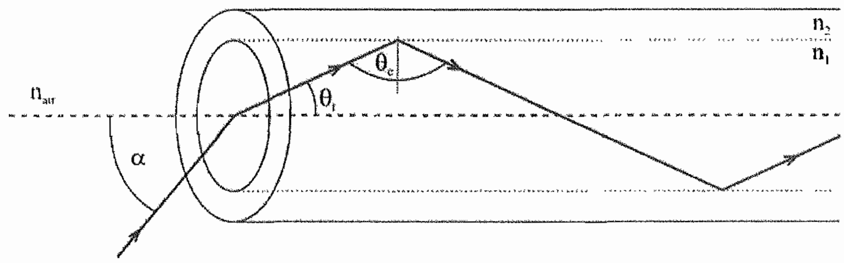


This will go through many cycles of 2pi and is shown by the non oscillating curve in the figure on the left.

iv) With a nematic LC, only the polarisation state that is parallel to n_e will see any retardance. The perpendicular state will only see no and will mot change as the molecules tilt out of plane. This is by definition polarisation dependant phase modulation.

It can be remedied by adding a half wave plate between 2 NLC cells at 90deg to one another. Hence the SOP will be rotated after the first NLC and then the orthogonal state will be modulated by the 2nd NLC. It cal also be done by adding a quarter waveplate to a reflective NLC device between the NLC and the mirror.

Q4 a)



i) $NA = \sin\alpha$

ii) Consider critical angle light propagation within the optical fibre. Applying Snell's Law for critical internal reflection, we get:

$$\sin\theta_c = \frac{n_2}{n_1}$$

Applying Snell's Law for the refraction of light at the end-face of the fibre (and also assuming $n_{air} = 1$), we get the following expression for the acceptance angle α:

$$\begin{aligned} \sin\alpha &= n_1 \sin\theta_t \\ &= n_1 \sin\left(90^o - \theta_c\right) \\ &= n_1 \cos\theta_c \\ &= n_1\sqrt{1 - \sin^2\theta_c} \end{aligned}$$

(Sub. for θ_c): $$= n_1\sqrt{1 - n_2^2/n_1^2}$$

$$= \sqrt{n_1^2 - n_2^2}$$

Numerical Aperture: $NA = \sin\alpha = \sqrt{n_1^2 - n_2^2} = n_1\sqrt{2\Delta}$ [

where, $\Delta = \dfrac{n_1^2 - n_2^2}{2n_1^2} \cong \dfrac{n_1 - n_2}{n_1}$

b) i) If $n_1 = 1.52$ and $n_2 = 1.50$, then: $\Delta \approx (1.52 - 1.50)/1.52 = 0.0132$

$$\sin\alpha = n_1\sqrt{2\Delta} = 1.52 \times (2 \times 0.0132)^{1/2}$$

$$\alpha = 14.3\,°$$

Alternatively, students may use: $\sin\alpha = \sqrt{n_1^2 - n_2^2}$, leading to the answer, $\alpha = 14.2\,°$

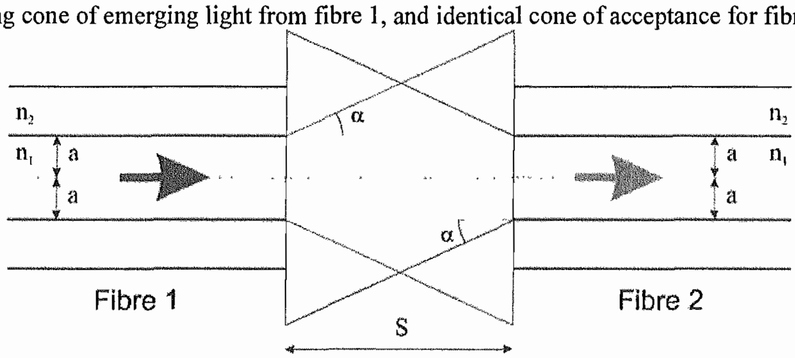
ii) Bandwidth-Length Product: $BL = \dfrac{c}{2(n_1 - n_2)}$

$$= 3\times10^8\ \text{ms}^{-1} / (2 \times (1.52 - 1.50))$$

$$= 7.5\times10^9\ \text{bit/s . m}$$

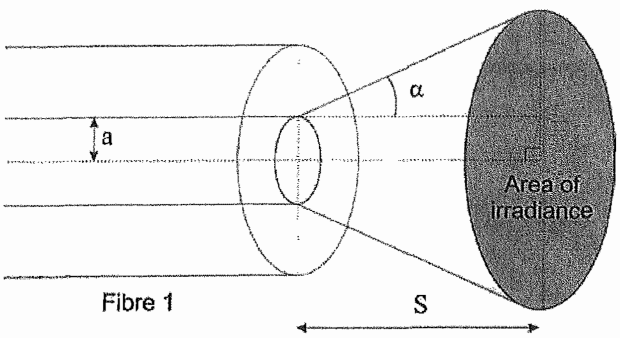
$$= 7.5\ \text{Gbit/s . m}$$

c) Consider a diverging cone of emerging light from fibre 1, and identical cone of acceptance for fibre 2:



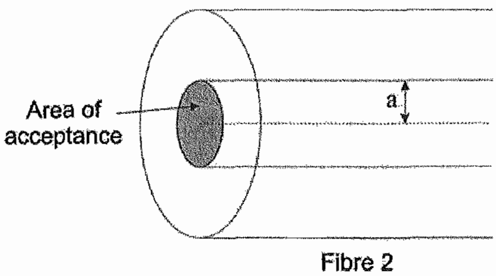
Assuming no reflection losses at the either fibre end facet, the coupling efficiency is simply the ratio of:

η = area of acceptance / area of irradiance



Area of irradiance $= \pi(a + S\tan\alpha)^2$

Area of acceptance $= \pi a^2$



Therefore coupling efficiency, $\eta = \dfrac{\pi a^2}{\pi (S \tan\alpha + a)^2}$

$$\eta = \frac{a^2}{(S \tan\alpha + a)^2}$$

S = 100 µm, a = 6 µm, n_1 = 1.52, n_2 = 1.50, From before, α = 14.3 °

Substituting into the equation therefore gives: $\eta = \dfrac{(6\mu m)^2}{((100\mu m \times \tan 14.3^\circ) + 6\mu m)^2} = 0.0366$ Coupling efficiency is 3.66 %.