CRIB - 2008/9

1 (a).From Datasheet

$$\bar{S}_{11} = S_{11}c^4 + S_{22}s^4 + (2S_{12} + S_{66})c^2s^2$$

in which $c = \cos\phi$ and $s = \sin\phi$. Applied $\sigma_x \rightarrow \varepsilon_x = \bar{S}_{11}\,\sigma_x$

$$E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{\bar{S}_{11}}$$

Now, in the present case,

$$S_{11} = \frac{1}{E_1} = \frac{1}{200}, \qquad S_{22} = \frac{1}{E_2} = \frac{1}{7}$$

$$S_{12} = -\frac{\nu_{12}}{E_1} = -\frac{0.3}{200}, \qquad S_{66} = \frac{1}{G_{12}} = \frac{1}{3}$$

so that, for $\phi = 7°$,

$$\bar{S}_{11} = \frac{1}{200}\cos^4(7) + \frac{1}{7}\sin^4(7) + \left(\frac{1}{3} - \frac{0.6}{200}\right)\cos^2(7)\sin^2(7) = 0.00972\,\mathrm{GPa}^{-1}$$

$$\therefore E_x = 102.9 \text{ GPa}$$

This is just over 50% of the $\phi = 0°$ case (= 200 GPa).

The minimum value of E_ϕ (maximum $\bar{S}_{11}$) is found from the condition

$$\frac{\partial \bar{S}_{11}}{\partial \phi} = -S_{11}4c^3s + S_{22}4s^3c + (2S_{12} + S_{66})(-2cs^3 + 2sc^3) = 0$$

Divide through by $4cs$

$$\frac{\partial \bar{S}_{11}}{\partial \phi} = -S_{11}c^2 + S_{22}s^2 + (2S_{12} + S_{66})\left(\frac{-s^2 + c^2}{2}\right) = 0$$

$$\therefore c^2 = \frac{S_{12} - S_{22} + \frac{S_{66}}{2}}{-S_{11} - S_{22} + 2S_{12} + S_{66}} \qquad \text{(after substituting for } s^2 = 1 - c^2\text{)}$$

$$= \frac{-\frac{0.3}{200} - \frac{1}{7} + \frac{1}{2(3)}}{-\frac{1}{200} - \frac{1}{7} + \frac{2(0.3)}{200} + \frac{1}{3}} = 0.118$$

$$\therefore c = 0.344, \text{ ie } \phi = 69.8^o$$

$$\bar{S}_{11} = \frac{1}{200}\cos^4(69.9) + \frac{1}{7}\sin^4(69.8) + \left(\frac{1}{3} - \frac{0.6}{200}\right)\cos^2(69.8)\sin^2(69.8) = 0.1456\,\mathrm{GPa}^{-1}$$

$$\therefore E = 6.87\,\mathrm{GPa}$$

1(b). A *laminate* is made up of a stacked and bonded assembly of (unidirectional) plies, each having its fibre axis lying at a specified angle to a reference direction. They are used in preference to unidirectional composite material because they are more isotropic with regard to properties of interest (although they may not be fully isotropic).

If a laminate consists of three or more identical orthotropic laminae (i.e., all have the same material and geometric properties) which are oriented at the same angle relative to adjacent laminae, the lamina is called *quasi-isotropic* (or planar isotropic). The $[A]$ matrix is isotropic in form:

$$A_{ij} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & (A_{11} - A_{12})/2 \end{bmatrix}$$

By changing the lamina orientations while maintaining equal angles between adjacent laminae (e.g. $[-45/15/75]$, $[30/-30/-90]$), A_{ij} remains unchanged but the B_{ij} and D_{ij} don't. Thus, the laminate is isotropic with respect to its in-plane behaviour only.

1(c). (i) Axial Young's modulus of the composite material (equal strain model)

$$E_1 = fE_f + (1-f)E_m = (0.66)76 + (0.34)3 = 51.2\,\text{GPa}$$

Transverse Young's modulus of the composite material (equal stress model)

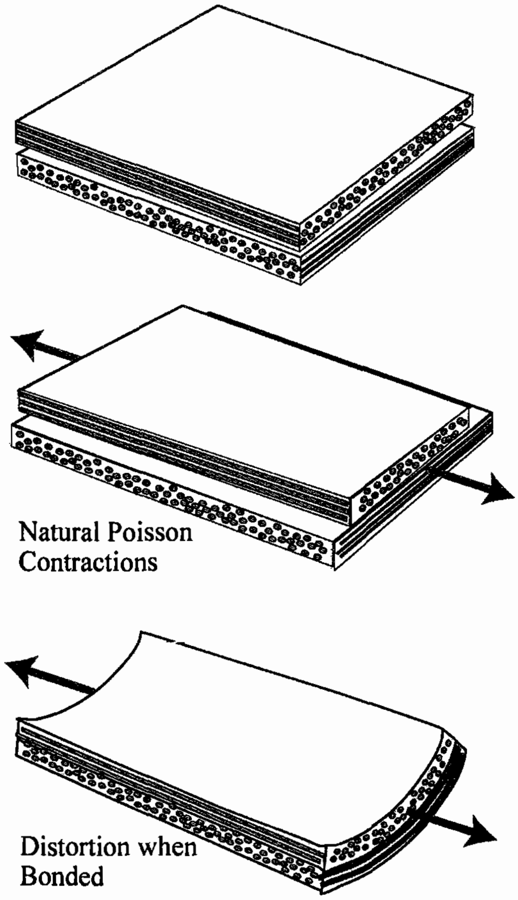
$$E_2 = \left\{\frac{f}{E_f} + \frac{(1-f)}{E_m}\right\}^{-1} = \left\{\frac{0.66}{76} + \frac{0.34}{3}\right\}^{-1} = 8.2\,\text{GPa}$$

Hence the Young's modulus of the cross-ply laminate (equal strain model)

$$E_{lam} = 0.5E_1 + 0.5E_2 = (0.5)51.2 + (0.5)8.2 = 29.7\text{GPa}$$

Therefore, the strain is given by stress/modulus = 0.1/29.7= 3.4 millistrain.

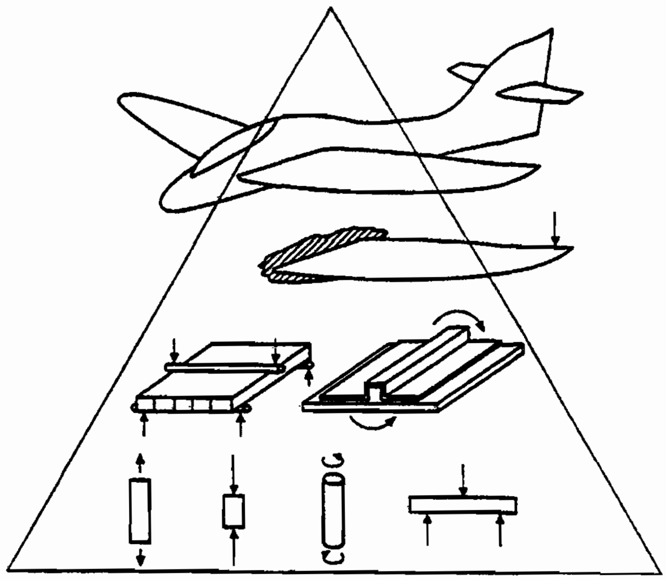
When the laminate is loaded in uniaxial tension, the difference in Poisson contractions of the two plies will cause the laminate to distort in the manner shown below (the students are not expected to draw this). Because the laminate is not symmetric, axial loading will generate bending and twisting curvatures which will cause out-of-plane distortion.



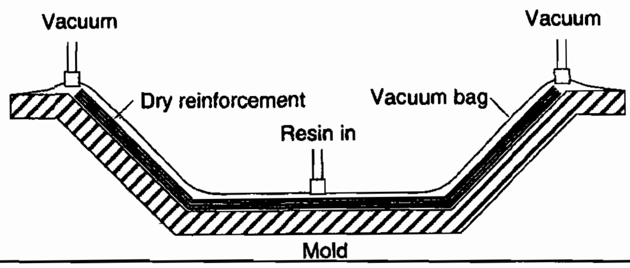
(ii) A cross-ply laminate which would not exhibit such curvature could be made by converting the stacking sequence to a symmetric one (eg 0/90/90/0). ie the stacking sequence in the top half reflects that in the bottom half. A symmetric laminate is less likely to exhibit out-of-plane distortions (bending and twisting curvatures) under in-plane loads. This is because B_{ij}.=0 ie bending-stretching coupling will not be present in such laminates.

2. (a) (i) As with all structures joints are a cause of stress concentration. The problem is particularly exacerbated in composites on account of their anisotropic nature, so that getting the load into the fibres is more difficult. Shear lag models of joints show how the load can get in via shear at the interface, with some slip at the ends. In general this strategy of tapering the load in is a good one, for example using tapered thicknesses of laminate. An alternative is to apply overlays of composite. Adhesive joints are preferred, not least because there is damage typically found at drilled holes. Finally metal inserts can be used where absolutely necessary.

(ii) The main point here is that air certification requirements force a very expensive testing pyramid as illustrated, from resin to coupon to sub part to full-scale prototype. Late in the design process much of this testing will already have been done, and the delays and cost involved in starting again are prohibitive.



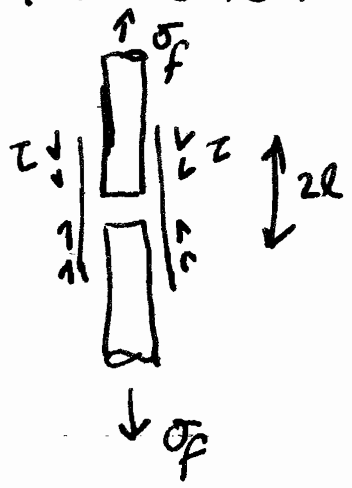
(b) The following taken from the notes:
Open or closed mould
Use vacuum bag with open mould
Vacuum forces flow of thermoset resin through reinforcement, typically hand lay-up
Injection and outlet ports control flow
Slow process, but capable of large parts
SCRIMP - (Seeman composites resin infusion moulding process) carrier layer allows fast path for resin.



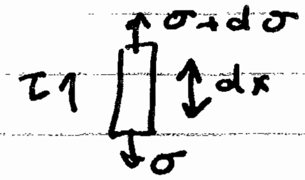
Schematic of vacuum injection moulding. Astrom, 1997

This is an attractive route for boat hulls because it is well suited to large structures, with the vacuum allowing relatively fast injection of the resin (relative to hand lay-up at least). The properties are not as good as in autoclaving (which might be a preferred route for the most expensive racing yachts) but still the vacuum gives acceptable porosity.

2(c) The shear lag phenomenon:



Under a remote stress σ_f some fibres fail. They shorten due to the stress drop and slip past the surrounding matrix.



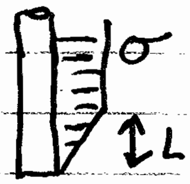
Fibre radius a

$$\frac{d\sigma}{dx} \cdot \pi a^2 = 2\pi a \tau$$

$$\Rightarrow \quad \sigma = \frac{2\tau x}{l}$$

So, the axial stress in the fibre increases linearly with distance over a length

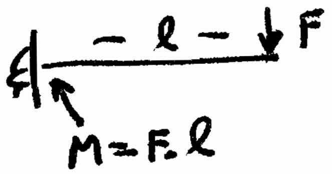
$l = 2\tau x / \sigma_f$, the shear lag length.

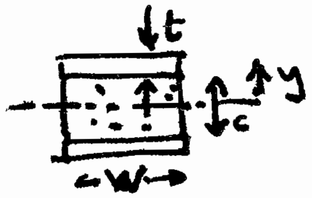


The shedding of axial load over a length of 2L leads to a local stress concentration and the increased likelihood of tensile failure in neighbouring fibres. This gives rise to a domino effect and to the formation of a macroscopic crack.

3. (a) In general the rules of thumb proposed are designed to help a less experienced designer get a reasonable first design without making significant errors. This is true particularly of the 'simplify' and 'same material' rules. A balanced symmetric layup prevents odd coupling effects, which may or may not cause unpredictable problems. Again a more experienced designer might be able to check whether these actually matter. For axi-symmetric structures the symmetric rule is not relevant, though balanced laminates will prevent twist. Having at least 10% off-axis plies is suggested to prevent splitting. Only where there is material elsewhere in the structure to protect against splitting is a substantial unidirectional laminate likely to be sensible. The suggestion of using $\pm\theta$ or a sequence of 0°, 90° and ±45° plies is again to simplify. The $\pm\theta$ reflects filament winding while the 0°, 90° and ±45° set is more relevant to hand layup. But again these rules are only as guidance and should be relaxed to optimise the layup.

3.(b) (i)





Assume the face sheets are under uniform stressing $\pm\sigma$. $M = Fl = (\sigma t w)c$

Strain in face sheets $\varepsilon_{max} = \dfrac{\sigma}{E_x} = \dfrac{Fl}{cwtE_x}$.

(ii) Assume cost & profit of the core are fixed.

<u>Mass</u> of face sheets, $m = 2wlt\rho$

<u>Cost</u> of face sheets, $C = 2wlt\,C_m\rho$

where C_m = cost per kg

<u>Lost profit</u>, $LP = 2wlt\rho\,(C_m + p)$

where p = premium = £20/kg.

The task is to minimise the lost profit subject to the beam surviving the load F.

3 (b)(ii) contd.

So, eliminate the free variable t,

using $t = \dfrac{F\ell}{c w E_x \varepsilon_{max}}$.

$$LP = 2w\ell\rho\,(C_m + p)\,\frac{F\ell}{c w E_x \varepsilon_{max}}.$$

So minimise LP by minimising $\dfrac{(C_m + p)\rho}{E_x\ \varepsilon_{max}}$.

Now use the data sheet to obtain values for E_x, C_m and ε_{max} (= min. of e^+, e^-), with $p = £\,20/kg$.

mat$^\ell$.	C_m	ρ	E_x	ε_{max}	$\left(\dfrac{C_m + 20}{E_x\ \varepsilon_{max}}\right)\rho$	
CFRP	100	1500	140	0.4	3214	← best
GFRP	5	1900	45	0.3	3518	
Kevlar	25	1400	80	0.1	7875	

So, choose CFRP.

3 (c)(iii) Assume $\pm 45°$ layers carry N_{xy} load
$0°$s carry N_x load.

For $0°$s : $E_x t = t_o E_1$ (assumes stiffness only of $0°$s)

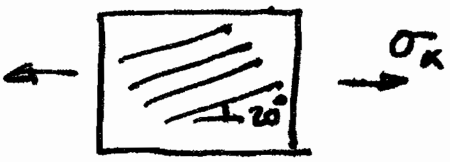
$0°$s $\Rightarrow$ $t_o = \dfrac{F\ell}{\varepsilon_{max}\, c w E_1} = \dfrac{100 \cdot 3}{0.3\times10^{-2} \cdot 0.02 \cdot 0.1 \cdot 45\times10^9}$
↖ use ε^+ $= 1.1\,mm$

$\pm 45°$ $t_{45} = \dfrac{N_{xy}}{G\ e_{LT}} = \dfrac{50 \times 10^3}{0.5\times10^{-2} \cdot 11\times10^9} = 0.91\ mm$

Add 10% 90°, although not required by loads.

4 (a) Hot wet environmental conditions will most affect the matrix material, causing it to soften. This will have a particular influence on microbuckling compressive failure, which is controlled by matrix strength. We could also expect other matrix-dominated properties such as transverse strength and stiffness to be affected. Effects on toughness are less predictable. Perhaps the increased ductility of the matrix will help enhance toughness.

4(b)



First, find $(\sigma_1, \sigma_2, \sigma_{12})$ in material axes.

From data sheet,

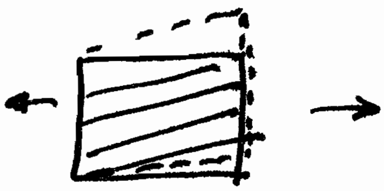
$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} \cos^2 20° & \times & \times \\ \sin^2 20° & \times & \times \\ -\sin 20° \cos 20° & \times & \times \end{pmatrix} \begin{pmatrix} \sigma_x \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.883 \\ 0.117 \\ -0.321 \end{pmatrix} \sigma_x$$

$$\text{T-H} : \frac{\sigma_1^2}{S_L^2} - \frac{\sigma_1 \sigma_2}{S_L^2} + \frac{\sigma_2^2}{S_T^2} + \frac{\sigma_{12}^2}{S_{LT}^2} \geq 1$$

$$\Rightarrow \left(\frac{0.883}{1379}\right)^2 - \left(\frac{0.883 \times 0.117}{1379^2}\right) + \left(\frac{0.117}{27.6}\right)^2 + \left(\frac{0.321}{60}\right)^2 = \frac{1}{\sigma_x^2}$$

$$\Rightarrow \quad \underline{\sigma_x = 146 \text{ MPa}}$$

4 (c) (i)



The 20° layer tends to shear, but ~~the~~ the −20° layer shears in the opposite direction.

However, the layers are <u>constrained</u> to share ~~the~~ same in-plane strain state, and shear <u>stresses</u> develop in each layer. According to the T-H criterion this will cause a drop in the allowable value of σ_x.

4. (k)(ii) $\bar{Q}_{20} = \begin{pmatrix} 60 & 9 & 20 \\ 9 & 7 & 3 \\ 20 & 3 & 9 \end{pmatrix}$ $\bar{Q}_{-20} = \begin{pmatrix} 60 & 9 & -20 \\ 9 & 7 & -3 \\ -20 & -3 & 9 \end{pmatrix}$

$\Rightarrow$ $A = 4t \begin{pmatrix} 60 & 9 & 0 \\ 9 & 7 & 0 \\ 0 & 0 & 9 \end{pmatrix}$ GPa where t = ply thickness

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = A^{-1} \begin{pmatrix} N_x \\ N_y \\ N_{xy} \end{pmatrix} = \frac{1}{4t.\det} \begin{pmatrix} 7 & -9 & 0 \\ -9 & 60 & 0 \\ 0 & 0 & x \end{pmatrix} \begin{pmatrix} \sigma_x . 4t \\ 0 \\ 0 \end{pmatrix}$$

$\det = 420 - 81 = 339$

Hence $\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} 7 \\ -9 \\ 0 \end{pmatrix} \dfrac{\sigma_x}{339\,\text{GPa}}$

Now resolve to strains in the $+20°$ ply.

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = T^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{pmatrix} 0.883 & 0.117 & 0.321 \\ 0.117 & 0.883 & -0.321 \\ -0.643 & 0.643 & 0.766 \end{pmatrix} \begin{pmatrix} 7 \\ -9 \\ 0 \end{pmatrix} \frac{\sigma_x}{(339\,\text{GPa})}$$

$$\Rightarrow \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} 0.0151 \\ -0.0210 \\ -0.0303 \end{pmatrix} \frac{\sigma_x}{\text{GPa}}$$

Max. strain criterion: $e_1 = \dfrac{S_L^+}{E_1} = \dfrac{1379\ \text{MPa}}{76\ \text{GPa}} = 0.018$

$e_2 = \dfrac{S_T^-}{E_2} = \dfrac{648\,\text{M}}{5.5\,\text{G}} = 0.012$

$e_{12} = \dfrac{S_{LT}}{G} = \dfrac{60\,\text{M}}{2.3\,\text{G}} = 0.026$

Put $\varepsilon = e$ $\Rightarrow$ $\sigma_x = \begin{pmatrix} \frac{0.018}{0.015} \\ 0.012/0.021 \\ 0.026/0.03 \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.57 \\ 0.866 \end{pmatrix}$ GPa

critical

So, failure occurs at $\underline{\sigma_x = 0.57\ \text{GPa}}$