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ENGINEERING TRIPOS PART IIB 2009

Paper 4C4

DESIGN METHODS

1. Child resistant containers

(a) Task abstraction – for example: [10%]

(i) Modify an existing child resistant container to improve accessibility for older adults;

(ii) Design a new child resistant container to improve accessibility for older adults;

(iii) Design a safe, yet accessible means for storing oral medication;

(iv) Design a safe, yet accessible means for administering medication.

Solution neutral problem statement – for example:

Provide a means for safe, yet accessible storage of oral medication, suitable for adults of all ages

(b) Key requirements – for example: [10%]

(i) Clear identification of medication type, dosage and instructions;

(ii) Inhibit opening by children (<14 years of age?);

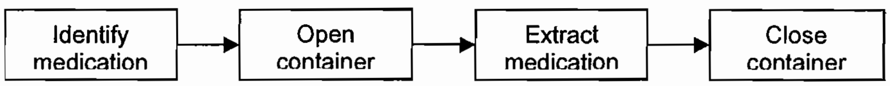
(iii) Enable opening by adults with range of vision and dexterity;

(iv) Enable secure closing by all;

(v) Low cost < 1p per unit;

(vi) Use of food grade materials.

(c) Function structure – for example: [20%]




(d) Various possibilities using mechanical/electro-mechanical solutions. [40%]

(e) Mention of verification and validation with 'real' users – checking for accessibility and lack of errors arising from the design. [20%]

2. Generally book work – answer should include reference to: [100%]

Risk management – contingency planning;

Phased development – use of prototyping;

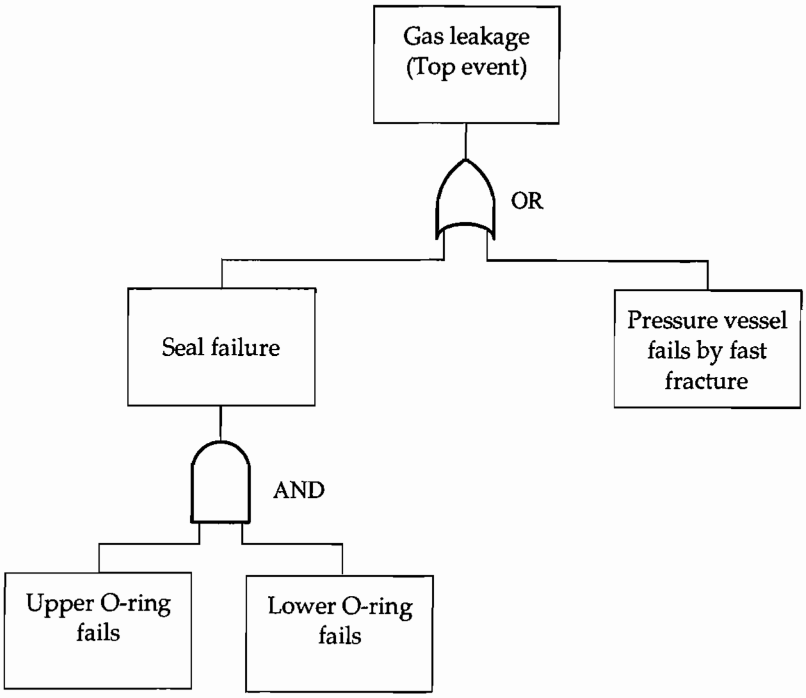
Management of Rework – look early for problems;

Partitioning of project – device and manufacturing equipment;

Verification and validation – appropriate evaluation throughout;

Concurrent (parallel) development – not serial!

3. (a) Fault tree




(b) Estimate of mean stress intensity:

$$\mu_K = E(K) \approx K(\mu_p, \mu_a) + \frac{1}{2}\left\{\left[\frac{\partial^2 K}{\partial p^2}\right]_\mu \sigma_p^2 + \left[\frac{\partial^2 K}{\partial a^2}\right]_\mu \sigma_a^2\right\}$$

$$\frac{\partial K}{\partial p} = \frac{r}{t}\sqrt{\pi a} \quad \text{and} \quad \frac{\partial^2 K}{\partial p^2} = 0, \quad \frac{\partial K}{\partial a} = \frac{1}{2}\frac{pr}{t}\sqrt{\pi}\, a^{-\frac{1}{2}} \quad \frac{\partial^2 K}{\partial a^2} = -\frac{1}{4}\frac{pr}{t}\sqrt{\pi}\, a^{-\frac{3}{2}}$$

Hence $\mu_K \approx \frac{\mu_p r}{t}\sqrt{\pi\mu_a} + \frac{1}{2}\left\{0 + \left[-\frac{1}{4}\frac{\mu_p r}{t}\sqrt{\pi}\,\mu_a^{-\frac{3}{2}}\right]\sigma_a^2\right\}$

$$\approx 500\times10^5\times10\sqrt{\pi\times0.3\times10^{-3}} + \frac{1}{2}\left\{0 + \left[-\frac{1}{4}500\times10^5\times10\sqrt{\pi}\left(0.3\times10^{-3}\right)^{-\frac{3}{2}}\right]\left(0.1\times10^{-3}\right)^2\right\}$$

$$=15.35\times10^6 - 0.21\times10^6 \quad =15.14\times10^6 \text{ Pa m}^{1/2}$$

Estimate of variance of stress intensity:

$$\sigma_K^{\ 2} \approx \left[\frac{\partial K}{\partial p}\right]_\mu^2 \sigma_p^2 + \left[\frac{\partial K}{\partial a}\right]_\mu^2 \sigma_a^2 = \left[\frac{r}{t}\sqrt{\pi\mu_a}\right]^2 \sigma_p^2 + \left[\frac{1}{2}\frac{\mu_p r}{t}\sqrt{\pi}\,\mu_a^{-\frac{1}{2}}\right]^2 \sigma_a^2$$

$$= \left[10\sqrt{\pi \times 0.3\times10^{-3}}\right]^2 \left(30\times10^5\right)^2 + \left[\frac{1}{2}500\times10^5\times10\sqrt{\pi}\left(0.3\times10^{-3}\right)^{-\frac{1}{2}}\right]^2 \left(0.1\times10^{-3}\right)^2$$

$$= 0.848\times10^{12} + 6.545\times10^{12} = 7.393\times10^{12}$$

${\sigma_K}^2 \approx 7.393\times10^{12}$ and so the estimate of the st. dev. of $K$ is $\sigma_K \approx 2.72\times10^6$ Pa m$^{1/2}$

The probability of fast fracture is given by P( $K > K_{IC}$ )

Substitute $z = \dfrac{K-\mu_K}{\sigma_K}$ hence when $K = K_{IC}$ $z = \dfrac{K_{IC}-\mu_K}{\sigma_K} = \dfrac{22-15.14}{2.72} = 2.52$

and $P(z > 2.52) = 1 - P(z < 2.52) = 1 - 0.9941 = 0.0059 = 0.59\%$

Note: Numerical analysis shows the actual probability is ~0.35%. The approximate method adopted is less accurate in the tail of what is a moderately skewed distribution.

The probability of the upper AND lower O-rings failing is 0.01 × 0.01 = 0.0001 = 0.01%.

Hence the probability of gas leakage is ~ 0.59% + 0.01% = 0.60%. It is clear that the high probability of fast fracture of the pressure vessel dominates the problem.

(c) $K$ is not actually normally distributed so this assumption leads to inaccuracy (see above). Use of a weibull distribution may be more appropriate for this sort of problem.

(d) A tougher material for the pressure vessel would be the most beneficial change. A relatively small increase in $K_{IC}$ would give a very large decrease in the probability of failure. Considering the variance of $K$ it seems that the large range of crack size $a$ is the main problem. Increasing wall thickness, and better control of the fill pressure would also help. If O-ring leakage becomes the dominant issue then one strategy might be to add a 3rd O-ring to reduce the probability of leakage by another factor of 100. Alternatively look for a more reliable O-ring.

4 (a) The volume of material used $M$ is the volume of the two 'ends', $\pi(R+t)^2 t$ each, plus the volume of the cylinder $\pi\left[(R+t)^2 - R^2\right]H$. Hence

$$M = 2\pi(R+t)^2 t + \pi\left[(R+t)^2 - R^2\right]H$$

So the optimization problem is to minimize

$$f(R,H) = 2\pi(R+t)^2 t + \pi\left[(R+t)^2 - R^2\right]H$$

subject to $$V = \pi R^2 H$$ [10%]

(b)

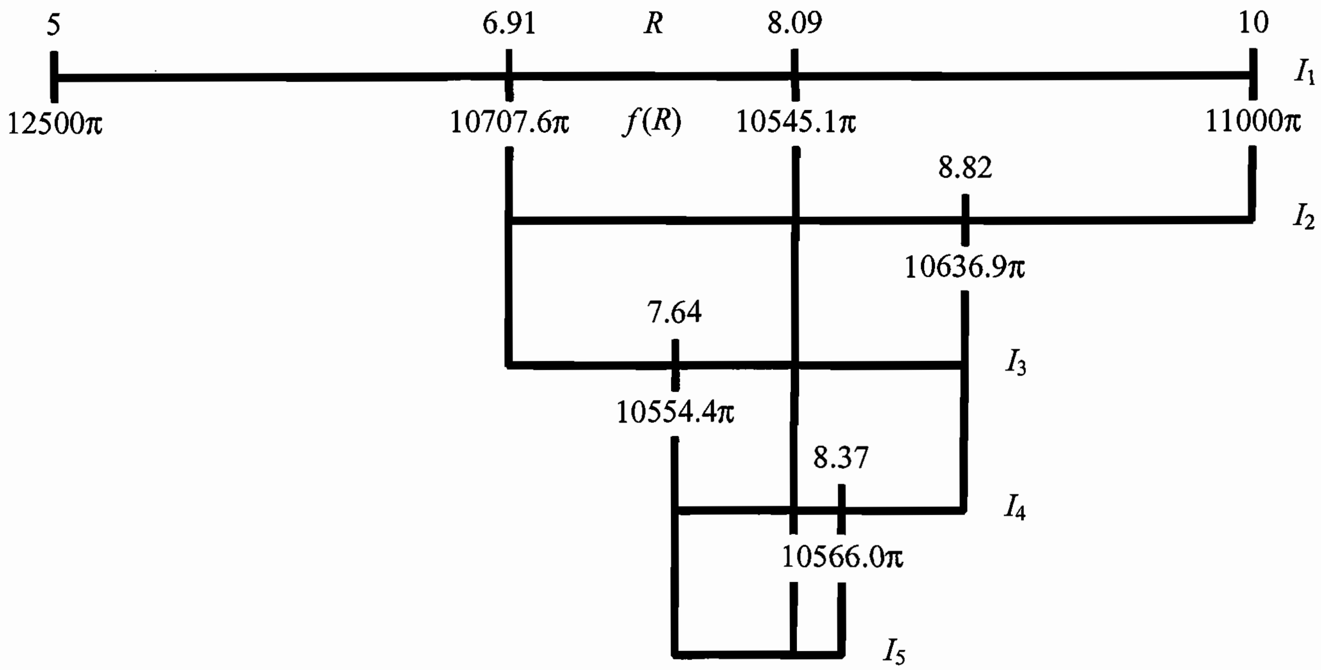
Substituting $H = \dfrac{V}{\pi R^2}$ $$f(R) = 2\pi(R+t)^2 t + \pi\left[(R+t)^2 - R^2\right]\frac{V}{\pi R^2}$$

$$\therefore \qquad f(R) = 2\pi(R+t)^2 t + V\left[\left(1+\frac{t}{R}\right)^2 - 1\right]$$ [10%]

(c)

For the values specified $$f(R) = 20\pi(R+10)^2 + 1000\pi\left[\left(1+\frac{10}{R}\right)^2 - 1\right]$$

For the GSLS method $\dfrac{\Delta R}{I} = 0.382$




Hence $I_5 = 7.64\text{ cm} \le R \le 8.37\text{ cm}$ [40%]

(d)

$$f(R) = 2\pi(R+t)^2 t + V\left[\left(1+\frac{t}{R}\right)^2 - 1\right] = 2\pi(R+t)^2 t + V\left[\frac{2t}{R} + \frac{t^2}{R^2}\right]$$

$$\therefore \quad \frac{df}{dR} = 4\pi(R+t)t - V\left[\frac{2t}{R^2} + \frac{2t^2}{R^3}\right] = 4\pi(R+t)t - 2Vt\left[\frac{1}{R^2} + \frac{t}{R^3}\right] \qquad (1)$$

$$\therefore \quad \frac{d^2 f}{dR^2} = 4\pi t + 2Vt\left[\frac{2}{R^3} + \frac{3t}{R^4}\right] \qquad (2)$$

For a minimum $\frac{df}{dR} = 0$ and $\frac{d^2 f}{dR^2} > 0$.

From (1) $\quad \frac{df}{dR} = 0 \Rightarrow 4\pi(R+t)t - 2Vt\left[\frac{1}{R^2} + \frac{t}{R^3}\right] = 0$

$$\therefore \quad 4\pi(R+t)t - \frac{2Vt}{R^3}[R+t] = 0$$

$$\therefore \quad R^3 = \frac{V}{2\pi} \Rightarrow R = \sqrt[3]{\frac{V}{2\pi}} \qquad (3)$$

From (2) it is clear by inspection that $\dfrac{d^2 f}{dR^2} > 0$ for any positive value of $R$.

For the values given in part (c) $\quad R = \sqrt[3]{\dfrac{1000\pi}{2\pi}} = \sqrt[3]{500} = 7.937$ cm

Hence
$$H = \frac{V}{\pi R^2} = \frac{V}{\pi}\left(\frac{2\pi}{V}\right)^{\frac{2}{3}} = \sqrt[3]{\frac{4V}{\pi}} \qquad (4)$$

$$\therefore \qquad H = \sqrt[3]{\frac{4 \times 1000\pi}{\pi}} = \sqrt[3]{4000} = 15.874 \text{ cm}$$

This confirms that the GSLS is converging on the correct value. The order of convergence of GSLS is 1.618 (the golden ratio) – hence the method's name.

As (3) and (4) show, perhaps surprisingly, the optimal value of $R$ does not depend on $t$. [40%]