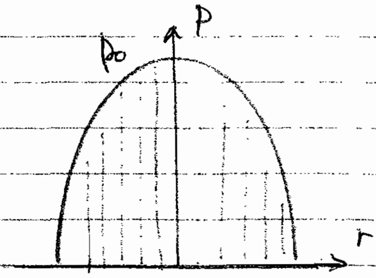
1.(a) Hertzian idealisations – bookwork, but essentially –

- smooth surfaces
- no friction
- elastic so small strains
- uniform isotropic materials
- no adhesion



(b)(i) Simple Hertz $R = 140\times10^{-9}\,m$

$P = 100\times10^{-9}\,N$

mica $E = 56.5\,GPa$ $\nu = 0.098$

Pt $E = 177\,GPa$ $\nu = 0.39$

$$\therefore E^* = \left\{\frac{1-.098^2}{56.5} + \frac{1-.39^2}{177}\right\}^{-1} = 44.8\,GPa$$

$$\therefore p_0 = \left\{\frac{6\times100\times10^{-9}\times(44.8\times10^9)^2}{\pi^3\times(140\times10^{-9})^2}\right\}^{1/3} = \underline{1.29\,GPa}$$

Data Sheet

$$a = \left\{\frac{3\times100\times10^{-9}\times140\times10^{-9}}{4\times44.8\times10^9}\right\}^{1/3} = \underline{6.17\,nm}$$

(ii)

$$\lambda = 2.05\times10^9\left\{\frac{9\times140\times10^{-9}}{2\pi\times0.4\times(44.8\times10^9)^2}\right\}^{1/3} = \underline{1.29}$$

(iii)

$$p_a(r) = -\frac{\sigma_0}{\pi}\arccos\left(\frac{a^2-r^2-2ad}{a^2-r^2+2ad}\right)$$

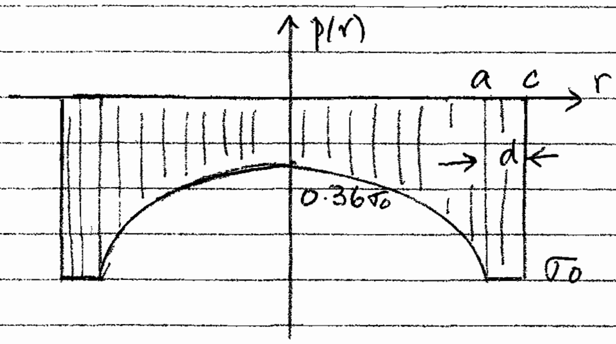
When $r=0$ $\quad p_a(0) = -\frac{\sigma_0}{\pi}\arccos\left(\frac{a-2d}{a+2d}\right)$ so if $d/a = 0.2$

$r=a$ $\quad p_a(a) = -\sigma_0$

$$p_a(0) = \frac{\sigma_0}{\pi}\cos^{-1}\frac{0.6}{1.4}$$

$$= \underline{-0.36\sigma_0}$$

ie. tension



(iv) $$a^3 \frac{4E^*}{3R} = P + 2\sqrt{2\pi E^* w}\, a^{3/2}$$

which is the JKR equation and is a quadratic in $a^{3/2}$.

Now using given quantities

$$\frac{4E^*}{3R} = \frac{4\times 44.8\times10^9}{3\times 140\times10^{-9}} = 4.27\times10^{17}$$

$$P = 100\times10^{-9} = 1\times10^{-7}\ N$$

$$2\sqrt{2\pi E^* w} = 2\{2\pi\times 44.8\times10^9\times0.4\}^{1/2} = 6.71\times10^5$$

So that $$4.27\times10^{17}\, a^3 = 1\times10^{-7} + 6.71\times10^5\, a^{3/2}$$

$$4.27\, a^3 = 1 + 6.71\times10^{12}\, a^{3/2}$$

a in m; let $a = 10x$ nm i.e. $10^{-8}x$ m

then $$4.27x^3 = 1 + 6.71x^{3/2}$$

or if $y = x^{3/2}$ $$4.27y^2 - 6.71y - 1 = 0$$

$$\therefore y = \frac{6.71 \pm \{6.71^2 + 4\times4.27\}^{1/2}}{2\times4.27} = 1.71$$

$\therefore x = 1.71^{2/3} = 1.43$ So $a = 14.3$ nm

Thus adhesion has caused contact patch to grow from radius of 6.17 nm to 14.3 nm.

(v) $$d = \frac{3}{4\lambda^2}\left\{\frac{3\pi w R^2}{4E^*}\right\}^{1/3} = \frac{3}{4\times1.29^2}\left\{\frac{3\pi\times0.4\times(140\times10^{-9})^2}{4\times44.8\times10^9}\right\}^{1/3} = 3.36\text{nm}$$

$$\therefore d/a = 3.36/14.3 \simeq 0.235$$

For a further discussion of this distribution in an adhesive junction see K.L.Johnson (1997) Proc Roy Soc A 453 163-79

Numerical values for the Pt/mica contact taken from RWCarpick et al (1996) Langmuir 12 3334-40

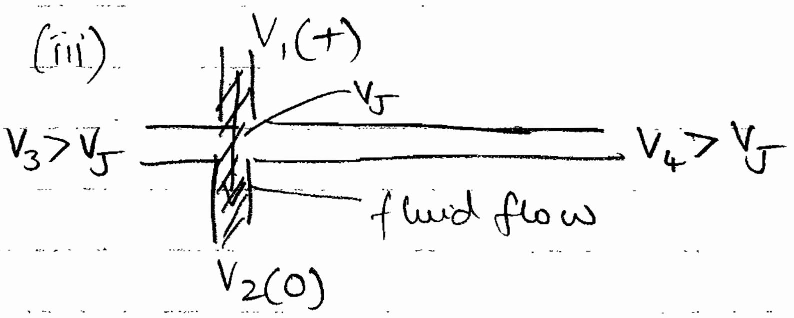
2/ (a) In electro-osmosis, electric fields are utilised to drive fluid flow. This typically occurs when electric fields can be applied tangential to the surface (e.g insulating materials forming a channel) so that the electrostatic body force on the double layer ions near the surface can be utilised to drag the fluid along thereby achieving pumping

(b) (i)

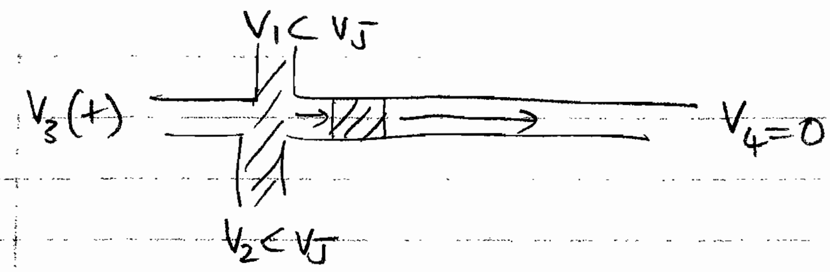
$$
\begin{aligned}
Q &= \text{volumetric flow rate} \\
&= A \cdot u \\
&= 100 \times 100 \times 10^{-12} \times \frac{0.1 \times 10^{4} \times 10^{-9}}{1.5 \times 10^{-3}} \\
&= 6.67 \times 10^{-12} \ m^3/s
\end{aligned}
$$

(ii) $t = \dfrac{L}{u} = \dfrac{10^{-2} \times 1.5 \times 10^{-3}}{0.1 \times 10^{4} \times 10^{-9}} = 15$ secs

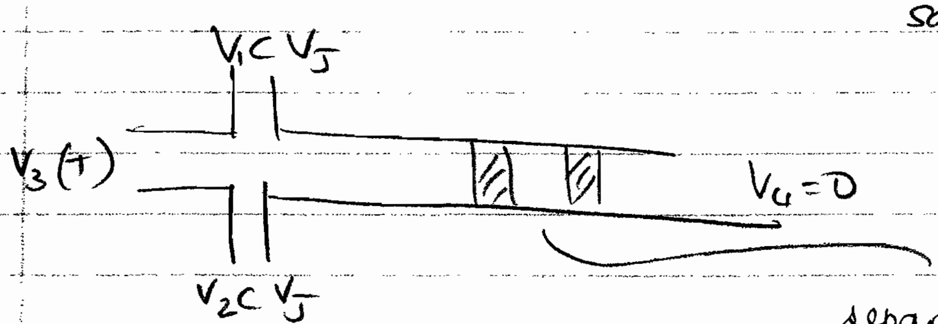
(iii)



Initially apply a field to drive fluid from ports 1 to 2.



Next switch the field off between ports 1 and 2 and drive a plug of sample from 3 to 4.



as the sample flows down the separation column, constitu separate by electrophoresis

(c) The ability of a charged species (e.g. charged biomolecule) to drift in an electric field relative to the bulk solution is termed as electrophoresis. The mobility of the species is usually a function of the charge and the viscous drag coefficient experienced by this species as it moves in solution

[20%]

(d) bulk solution travels 1mm in 1.5 secs (t_1)

$v = \mu E$

v = electrophoretic drift velocity

$V_{rel} = \mu_{rel} \cdot E$

$= 10^{-9} \times 10^{4} = 10^{-5}$ m/s

$\therefore$ in 1.5 secs, 2 species would be separated by $V_{rel} \times t_1 = 1.5 \times 10^{-5}$ m $= 15\,\mu$m

[20%]

3/ (a) Test structures play a key role in process characterisation as well as MEMS design by:

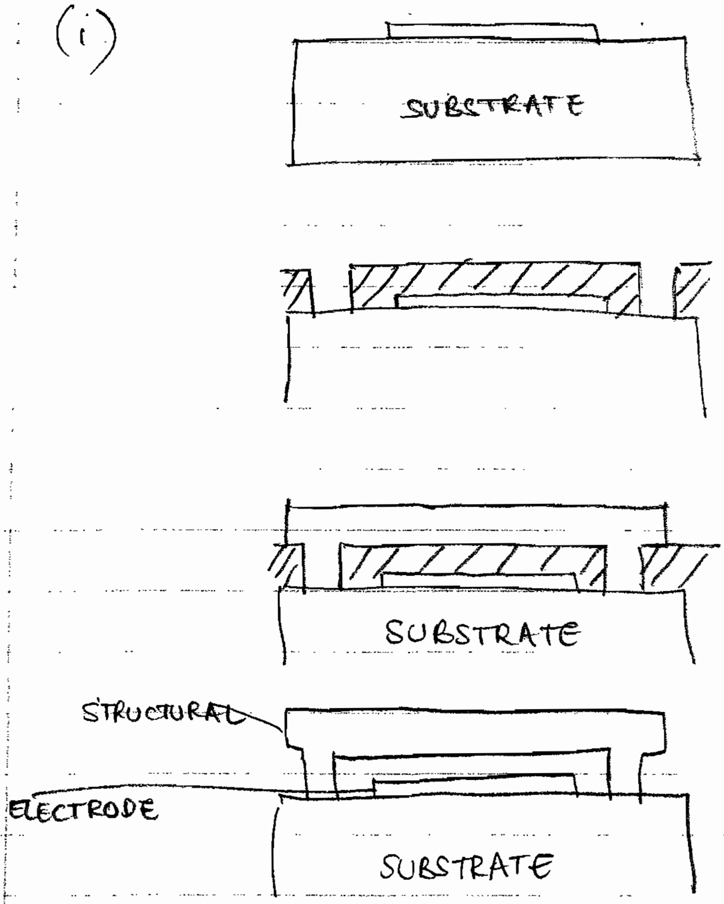
— METROLOGY: measurement of geometric tolerances in manufacturing

— BETTER MODELS: the ability to ~~test~~ validate subsets of device operation and feedback into better models

— CONSTITUTIVE PROPERTIES: measure electro-mechanical properties (e.g. Young Modulus, thermal conductivity etc) useful for design

(b)

(i)



DEPOSIT AND PATTERN ELECTRODE

DEPOSIT AND PATTERN SACRIFICIAL MATERIAL

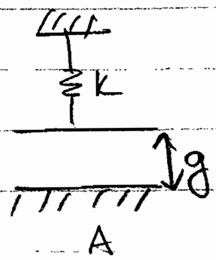
DEPOSIT AND PATTERN STRUCTURAL MATERIAL

RELEASE SACRIFICIAL MATERIAL

(ii)

$$W^* = \frac{1}{2} C V^2$$

$$= \frac{1}{2} \frac{\varepsilon_0 A}{g} V^2$$



$$F_{mech} = k(\overset{\text{original gap @ } V=0}{g_0} - g) \qquad F_{elec} = -\frac{\partial W}{\partial g} = \frac{\varepsilon_0 A V^2}{2g^2}$$

$F_{NET} = F_{mech} - F_{elec}$ (NET FORCE)

CONDITION FOR PULL-IN

$$\frac{\partial F_{NET}}{\partial g} = 0 \quad \text{or} \quad -k + \frac{\varepsilon_0 A V^2}{g^3} = 0 \qquad \text{—} (1)$$

$F_{NET} = 0$ (equilibrium)

$$k(g_0 - g) = \frac{\varepsilon_0 A V^2}{2g^2} \qquad \text{—} (2)$$

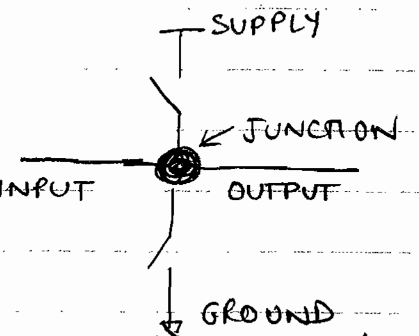
From (1) and (2) $k(g_0 - g) = \frac{kg}{2}$ or $g = \frac{2g_0}{3}$

Substituting for g in (1) :-

$$V_{PI} = \sqrt{\frac{8k g_0^3}{27 \varepsilon A}}$$

(iii) Increased tensile stress would make it harder for the beam to deflect and pull-in voltage would be increased. The opposite holds for compressive film stress

(c) Represent a circuit schematic where represents a switch

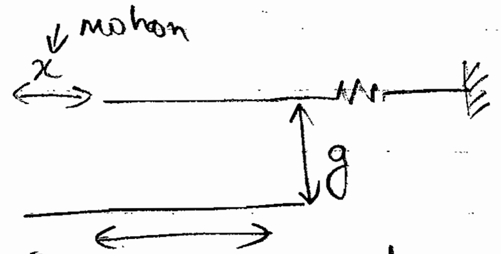


In this case, the relative resistance from INPUT TO JUNCTION, OUTPUT TO JUNCTION & FROM these points to supply and ground define switching together with the magnitude of V_{PI}. V_{PI} needs to small for low voltage switching and devices can switch quickly if their dimensions are reduced

Q4 (a) A micromachined vibratory rate gyroscope is typically driven to large resonant motion in one direction. If a rotation rate is applied along an orthogonal direction, the coriolis effect couples the driven resonant motion to motion along a third orthogonal direction – this motion is proportional to the applied rotation rate and can be measured to construct a gyroscope.

Mode matching involves tuning the resonant frequencies of driven and sensed modes to increase the modal coupling and thereby the sensitivity.

(b) $$W^* = \frac{1}{2}\frac{\varepsilon_0 (a-x) \cdot b \cdot V^2}{g}$$



$$F = \frac{-\partial W^*}{\partial g} = \frac{\varepsilon_0 b V^2}{2g}$$

a → nominal overlap
b → width into page

This force is independent of displacement. For an an array of comb fingers, the force is multiplied. Comb drives are not limited by pull-in instability as parallel plate actuators, and can be used to generate large displacements in MEMS devices in a controlled manner.

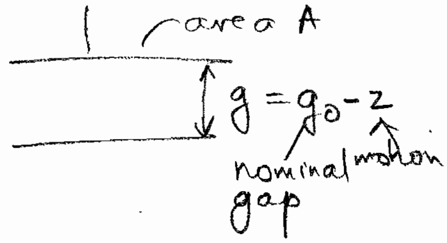
Two vibratory masses are utilised to provide for the cancellation of common mode effects by driving these masses out of phase.

(c) $$I = \frac{d(CV)}{dt} = V_{DC}\frac{dC}{dx} \cdot \frac{dx}{dt}$$

$$= V_{DC} \cdot \frac{\varepsilon_0 b}{g} \cdot \dot{x}$$

(d)

$$C = \frac{\varepsilon_0 A}{g_0 - z}$$



$$C_0 = \frac{\varepsilon_0 A}{g_0}$$

$$\frac{\Delta C}{C_0} = \frac{\dfrac{\varepsilon_0 A}{g_0 - z} - \dfrac{\varepsilon_0 A}{g_0}}{\dfrac{\varepsilon_0 A}{g_0}}$$

$$\frac{\Delta C}{C_0} = \frac{z}{(g_0 - z)}$$

If $z \ll g_0 \Rightarrow \dfrac{\Delta C}{C_0} \simeq \dfrac{z}{g_0}$

(e) (i) $F = \dfrac{\varepsilon_0 b}{2g}(2 V_{AC} \cdot V_{DC}) = \left(\dfrac{\varepsilon_0 b}{g}\right) \cdot (V_{AC} \cdot V_{DC})$

for N fingers $F = \dfrac{N \varepsilon_0 b}{g} \cdot V_{AC} V_{DC} = \dfrac{kx}{Q}$

spring Quality fac

$$V_{AC} \cdot V_{DC} = \left(\frac{kx}{Q} \cdot \frac{g}{N \varepsilon_0 b}\right)$$

to minimize product, need large Q, large N,
for given kx
large (b/g), low k
aspect ratio

(ii) $F_{coriolis} = 2 m \Omega v$

$= 2 \times 10^{-9} \times 1 \times 2\pi \times 10^{4} \times 5 \times 10^{-6}$

$= 6.28 \times 10^{-10}\,N = 628\,pN$

JAW/AAS April 09