2009 PART IIB 4C16 ADVANCED MACHINE DESIGN
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1 a) $$\frac{u(z)}{U} = \frac{1}{2\eta U}\frac{dp}{dx} z(z-h) + \left(1 - \frac{z}{h}\right)$$

First term is due to pressure gradient, gives parabolic distribution, called Poisseuille flow.

Second term is due to linear shear, called Couette flow.

b) consider volumetric flow rate per unit width.

$$q = \int_0^h u(z)dz = \frac{1}{2\eta}\int_0^h z(z-h)\frac{dp}{dx}\,dz + U\int_0^h\left(1-\frac{z}{h}\right)dz$$

$$= \frac{1}{2\eta}\left[\frac{h^3}{3} - \frac{h^3}{2}\right]\frac{dp}{dx} + U\left\{h - \frac{h^2}{2h}\right\}$$

$$\therefore\ q = -\frac{h^3}{12\eta}\frac{dp}{dx} + \frac{Uh}{2}$$

In entry section of bearing (first half)

$$q = -\frac{h_1^3}{12\eta}\left(\frac{dp}{dx}\right)_1 + \frac{Uh_1}{2}.$$

In outlet section (second half)

$$q = \frac{-h_o^3}{12\eta}\left(\frac{dp}{dx}\right)_o + \frac{Uh_o}{2}$$

$\frac{dp}{dx}$ is constant in each half (h is constant)

Let pressure at step be p_0, then $\left(\frac{dp}{dx}\right)_1 = \frac{2p_0}{B}$

$$\left(\frac{dp}{dx}\right)_2 = \frac{-2p_0}{B}$$

Then $$q = \frac{-h_1^3}{12\eta}\cdot\frac{2p_0}{B} + \frac{Uh_1}{2} = \frac{h_o^3}{12\eta}\frac{2p_0}{B} + \frac{Uh_o}{2}$$

to give $\quad \frac{p_0}{3\eta B}\left\{h_1^3 + h_0^3\right\} = U(h_1 - h_0)$

but $h_1 - h_0 = d$, so $h_1 = h_0 + d$

and $P' = \frac{1}{2} p_0 B \rightarrow$ (force per unit width of bearing)

so $\quad P' = \frac{B}{2} \times \overbrace{3\eta BU \times \frac{d}{(h_0+d)^3 + h_0^3}}^{p_0}$

$$\underline{P' = \frac{3\eta B^2 U}{2} \frac{d}{(h_0+d)^3 + h_0^3}}$$

c) in the outlet region

$$\frac{u(z)}{U} = \frac{1}{2\eta U} z(z-h_0)\left(\frac{-2p_0}{B}\right) + \left(1 - \frac{z}{h_0}\right)$$

$$= \frac{-2}{2\eta UB} z(z-h_0) \underbrace{3\eta BU \frac{d}{(h_0+d)^3 + h_0^3}}_{p_0} + \left(1 - \frac{z}{h_0}\right)$$

$$= -3z(z-h_0)\frac{d}{(h_0+d)^3 + h_0^3} + \left(1 - \frac{z}{h_0}\right)$$

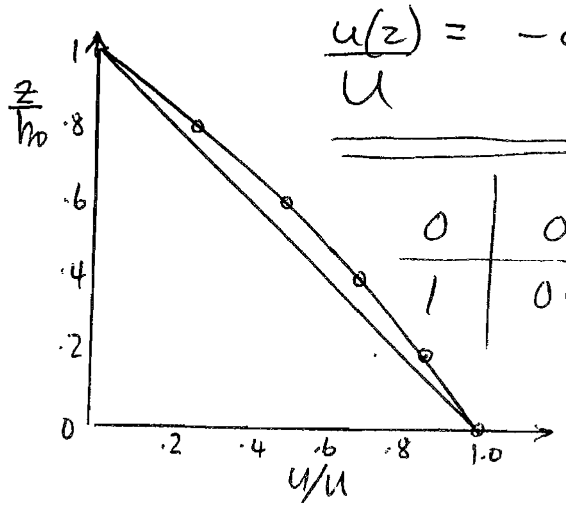
now $d = h_0\, 0.68$

$$\frac{u(z)}{U} = -3z(z-h_0)\frac{h_0\, 0.68}{(h_0 + 0.68h_0)^3 + h_0^3} + \left(1 - \frac{z}{h_0}\right)$$

$$= -0.683\, \frac{z(z-h_0)\, h_0}{h_0^3\left((1+0.68)^3 + 1\right)} + \left(1 - \frac{z}{h_0}\right)$$

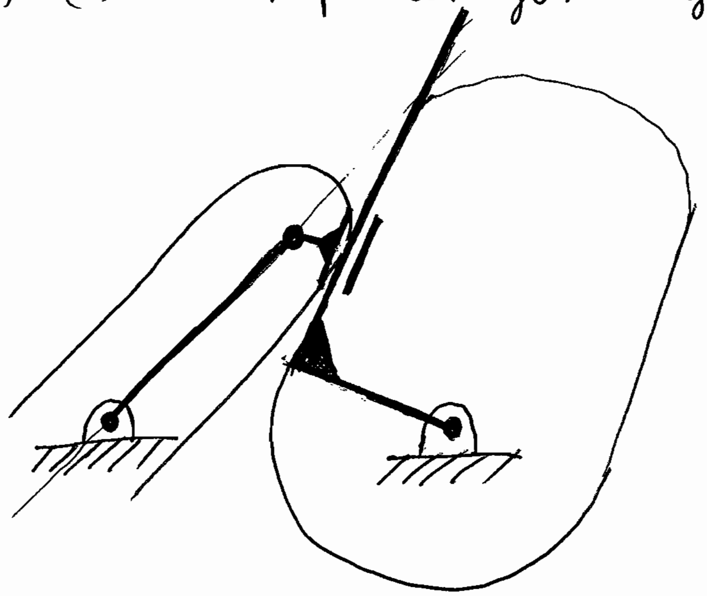
$$= -\frac{2.04\; z(z-h_0)}{5.74\; h_0^2} + 1 - \frac{z}{h_0}$$

$$\underline{\frac{u(z)}{U} = -0.355\, \frac{z}{h_0}\left(\frac{z}{h_0} - 1\right) + 1 - \frac{z}{h_0}}$$

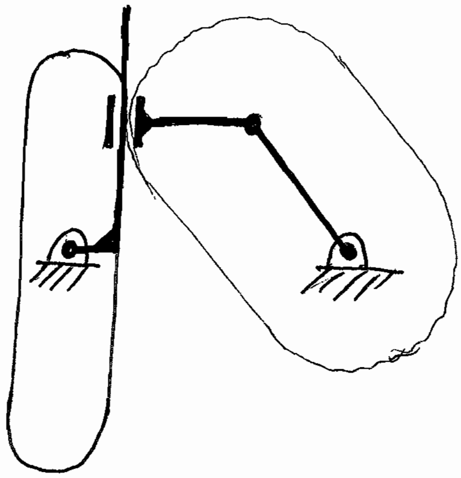


0	0.2	0.4	0.6	0.8	1.0	z/h_0
1	0.857	0.685	0.485	0.257	0	u/U

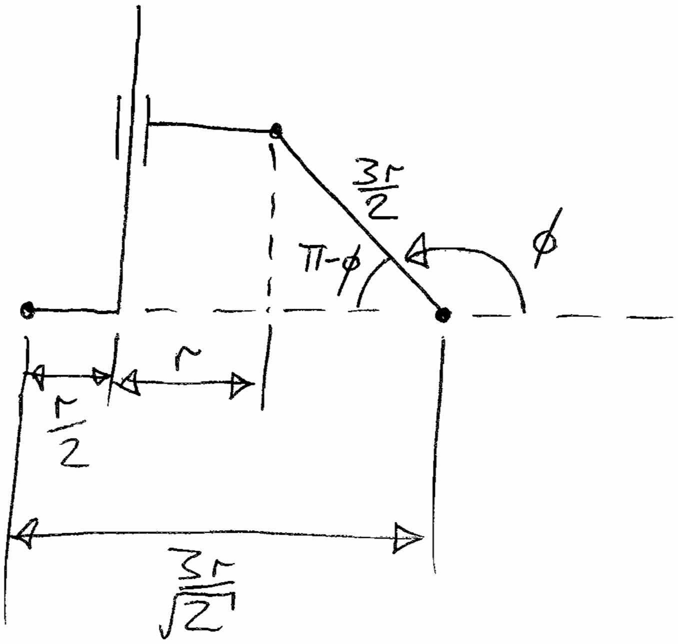
2 (a) (i) trap door just begins to open.



(ii) fully open.



b)



$$\pi-\phi=\cos^{-1}\frac{\frac{3}{\sqrt{2}}-\frac{1}{2}-1}{3/2}$$

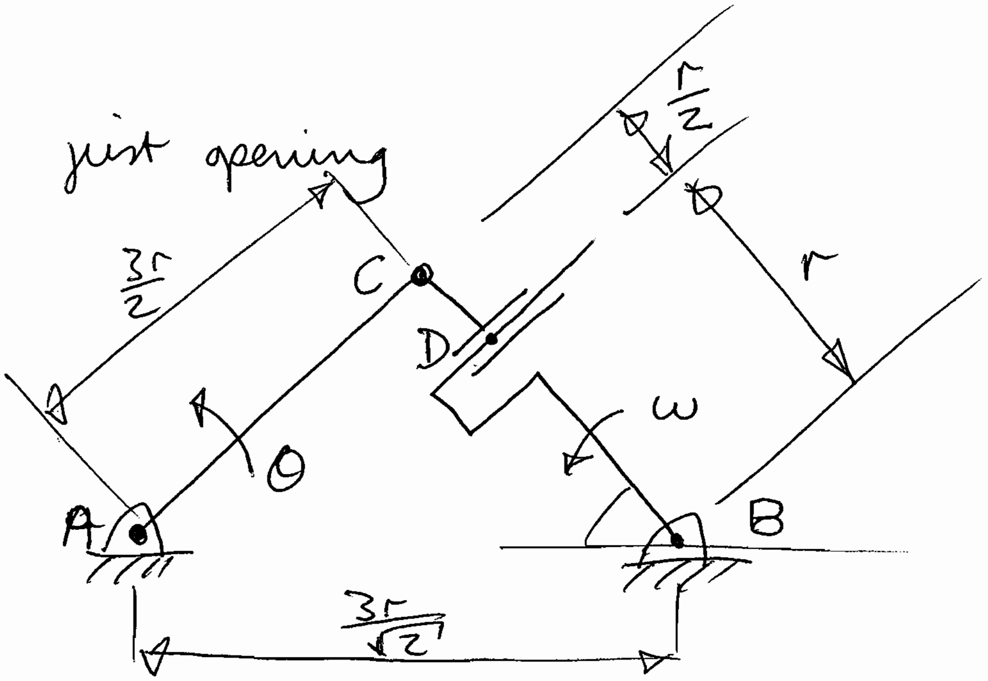
$$\phi=\pi-\cos^{-1}\frac{\frac{2}{3}\cdot\frac{3}{\sqrt{2}}-1}{1}$$

$$\phi=\pi-\cos^{-1}(\sqrt{2}-1)$$

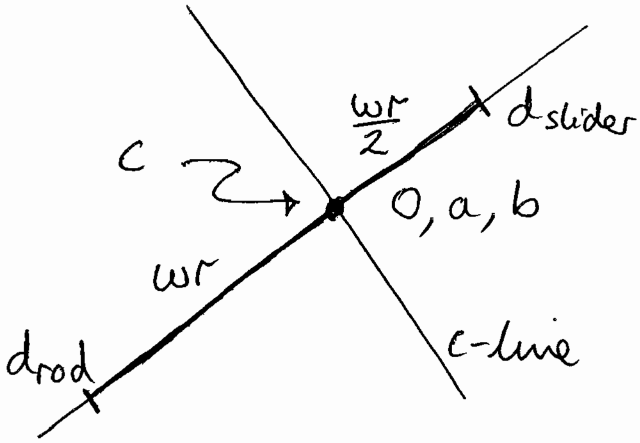
$$\phi=\pi-1.1437$$

$$\underline{\underline{\phi=2\text{ rad}}}$$

c) just opening



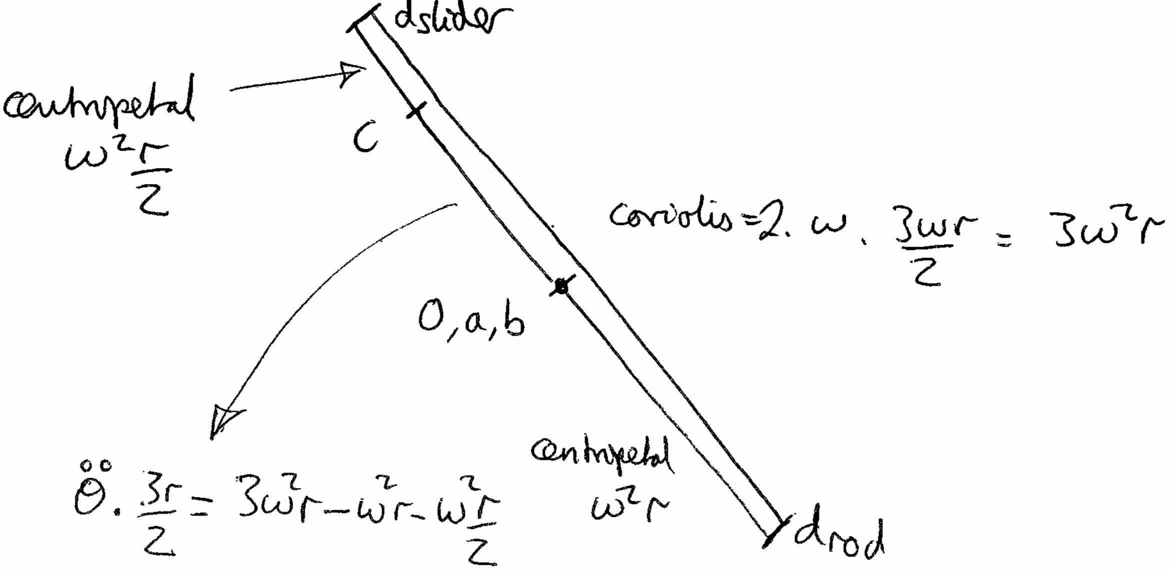
velocity diagram :



velocity at C is zero, so $\dot{\theta} = 0$

sliding velocity is $\frac{3\omega r}{2}$

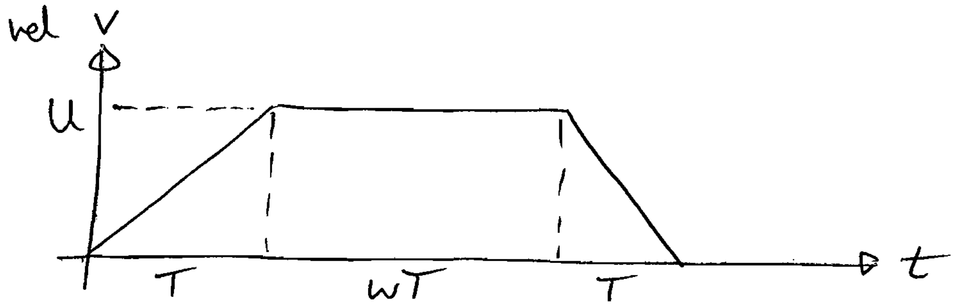
acceleration diagram :



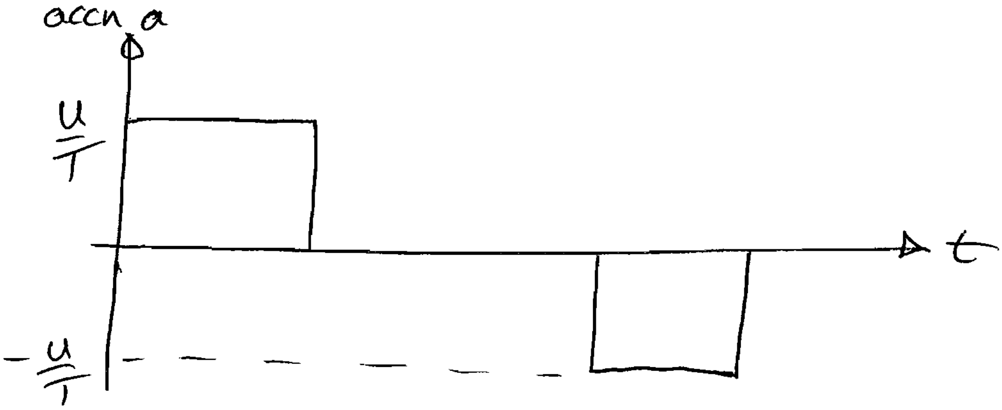
$\ddot{\theta} . \frac{3r}{2} = 3\omega^2 r - \omega^2 r - \omega^2 \frac{r}{2}$

$\ddot{\theta} = \omega^2$

3. a)

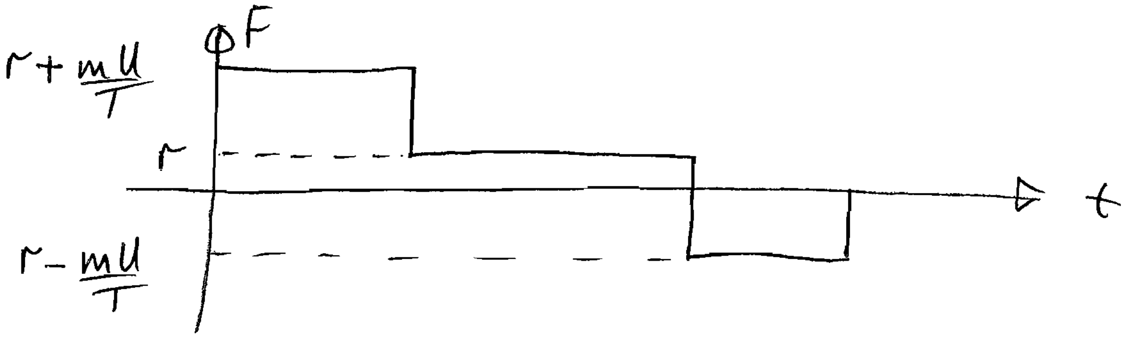


vel v
U
T
wT
T
t

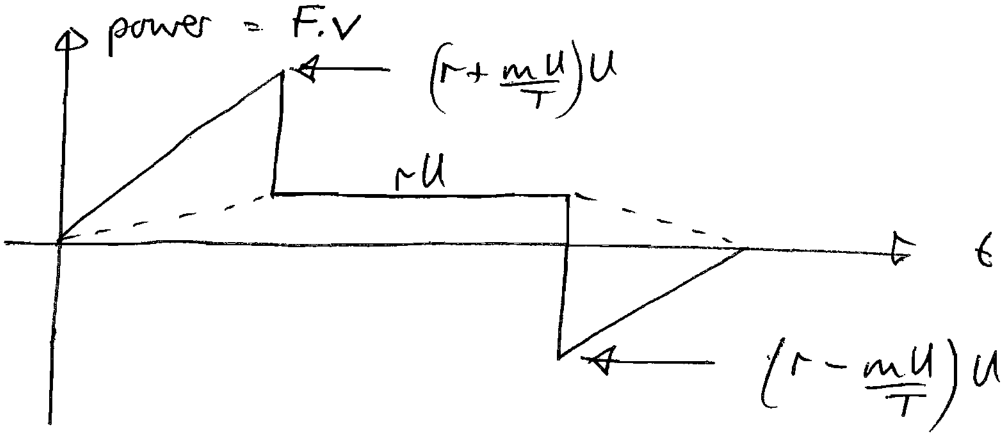


accn a
U/T
−U/T
t

$F = ma + r$:



F
r + mU/T
r
r − mU/T
t



power = F.V
(r + mU/T)U
rU
(r − mU/T)U
t

b)

$$P_{mean} = \frac{\frac{T}{2}\left(r + \frac{mU}{T}\right)U + wTrU + \frac{T}{2}\left(r - \frac{mU}{T}\right)U}{(2+w)T}$$

$$=$$

$$= \frac{U\left(\frac{r}{2} + \cancel{\frac{mU}{T}} + rw + \frac{r}{2} - \cancel{\frac{mU}{T}}\right)}{2+w}$$

$$P_{mean} = rU\frac{(1+w)}{(2+w)}$$

$$P_{peak} = \left(r + \frac{mU}{T}\right)U$$

hence $$\frac{P_{peak}}{P_{mean}} = \frac{\left(r + \frac{mU}{T}\right)U}{rU}\frac{(2+w)}{(1+w)}$$

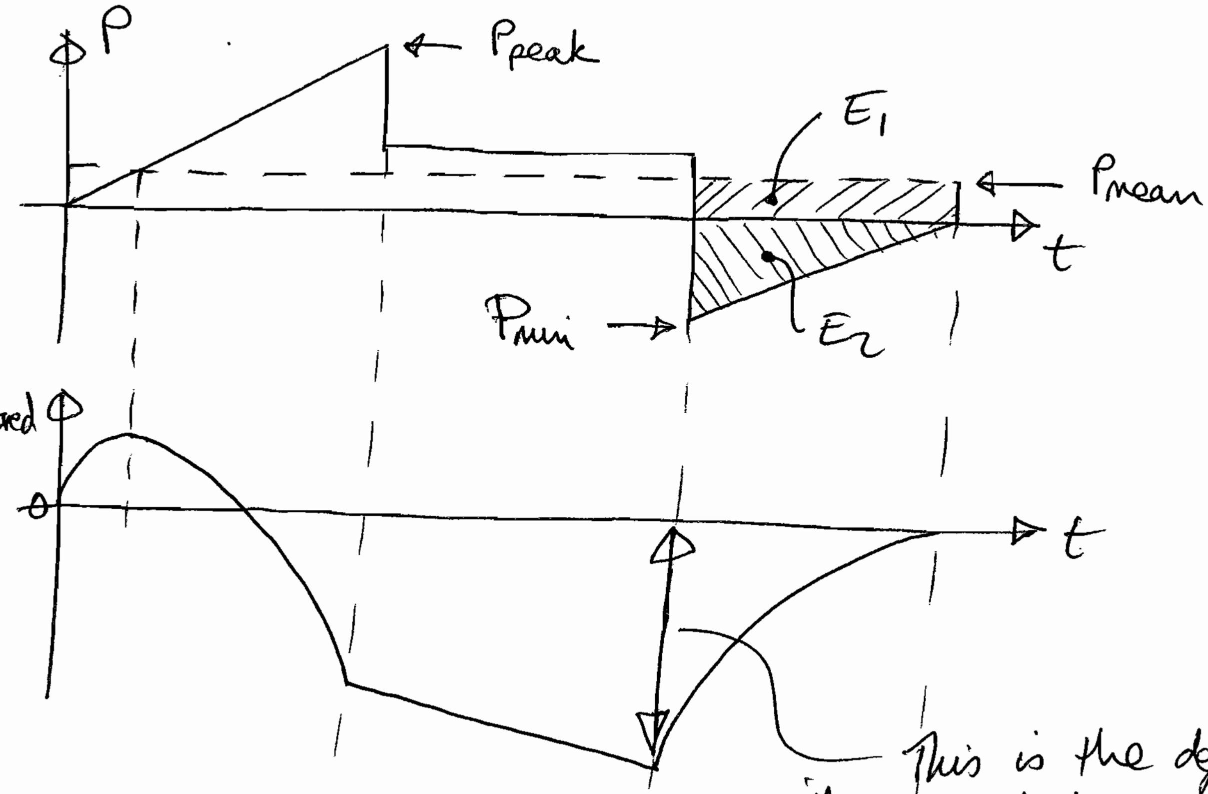
$$\underline{\underline{\frac{P_{peak}}{P_{mean}} = \left(1 + \frac{mU}{rT}\right)\left(\frac{2+w}{1+w}\right)}}$$

Hybrid drive is beneficial when $\frac{P_{peak}}{P_{mean}}$ is large,

so small w (urban driving)

large U/T (high acceleration)

large $\frac{m}{r}$ (vehicle mass large compared to rolling resistance, drag) c)



This is the deficit with zero start energy. Hence provide this amount in the store at $t=0$.

required energy is $E_1 + E_2$

$$= (P_{mean})T - \frac{1}{2}(P_{min})T$$

$$= rU\left(\frac{1+w}{2+w}\right)T - \left(r - \frac{mU}{T}\right)u\frac{T}{2}$$

$$E_{start} = \frac{1}{2}mU^2 + rUT\left(\frac{1+w}{2+w} - \frac{1}{2}\right)$$

Note, if r is zero, mean power is zero, hence energy in store at $t=0$ is simply that required to accelerate the vehicle to speed U, i.e. $\frac{1}{2}mU^2$.

4. Full dynamic similarity requires all relevant non-dimensional groups to have same numerical values. These might include Reynolds' number, Sommerfeld Number, eccentricity, or inlet:outlet film thickness ratio etc. depending on whether journal or thrust bearings are being considered. Even if values of non-dimensional groups can be kept similar, absolute factors such as film thicknesses, shear rates, temperatures, cooling rates etc. cannot so easily all be made comparable at linear scales which differ by a factor of 10. Consequently, there may be a change in physical or chemical mechanisms on which some important aspects of tribological behaviour depend. A good answer would highlight some of the compromises that would have to be made.

5. *(a) Ahn and Kim pay particular attention to modelling of friction associated with the cam shaft rotation. Discuss why friction should affect the lift curve, and how the model for such a frictional element would depend on the details of the mechanism.*

Friction affects the lift curve due to the change in contact force. This in turn affects the displacement associated with the contact stiffness and also possibly any elastic deformation of the linkages. In the model of Ahn and Kim these elements are found to be important in affecting the closing velocity. Depending on whether the linkage is soft or stiff, and whether the loads on the contact give significant contact deformation, we could expect to see differences in how important this element of the model is.

In the work of Ahn and Kim, they model friction via a sort of Stribeck curve. After a sticking element to the curve there is a reduction in friction with increasing velocity, associated with an increase in lubrication efficacy. This might either be due to EHL or perhaps due to hydrodynamic lubrication. At increasing velocities the friction increases again, as the dissipation due to viscous work increases. The appropriateness of these models would depend on the regime of lubrication. An EHL map would be appropriate to identify the regime of operation and so an appropriate lubrication model.

(b) What factors need to be taken into account to optimise the lift curve for practical high speed cams?

The results of Ahn and Kim show that good models of lift can be obtained by including (i) the actual profile, (ii) friction, (iii) contact stiffness, (iv) impact and dynamics. So in turn we can modify these to optimise the lift. The actual profile is the most important variable. Here we can adjust the shape to optimise the actual motion, paying attention to the effect that this has on the acceleration and hence the forces generated on the contacts and in the linkages. Steps in the profile and early starts can improve the performance. Ahn and Kim explore dynamic effects to improve the performance, including the cam inertia and the spring stiffness. These work, as with friction, by influencing the dynamics of the cam contact, affecting the contact loads and hence the contact stiffness. At the same time as optimising the lift curve, attention needs to be paid to stressing.

(c) Cam mechanisms commonly contain springs. What factors govern the choice of spring element?

Springs are critical in ensuring that the cam profile is faithfully followed. In particular too small a spring force will result in loss of contact between the cam and the follower. Where there are form errors in the form of waviness in the cam even too low a contact force is likely to result in dynamic problems (bouncing of the follower on the cam). Hence control of this force is critical. Conversely too high a spring force will result in higher stresses in the mechanism, either leading to excessive wear or fatigue, and to the dynamic errors associated with contact stiffness that Ahn and Kim allude to.

In the work of Ahn and Kim springs also provide the energy for the actuation. With their dynamic model the spring stiffness has a significant role to play in controlling the opening and closing velocities, via changes in the elastic deformations. High speed photography shows that, in fact, the springs deform out of their axis during the deformation, so that the dynamic behaviour of the spring itself is important.

6. *(a) What are the main objectives of the power management strategy of a parallel hybrid vehicle? What are the problems associated with implementing an ideal strategy, and what approaches can be taken to overcome them?*

A comprehensive answer should include the following points:

- Main objective is to deliver the vehicle acceleration/deceleration demanded by the driver at the same time as minimising emissions and/or fuel consumption.
- An important feature is the regenerative braking.
- Power management strategies fall roughly into three categories: heuristic or rule-based control; static optimization; and dynamic optimization.
- Dynamic optimization is ideally the best, but real-time implementation is difficult because it is not possible to predict accurately the future trajectory of the vehicle.
- Lin and others in their paper use dynamic optimization of a simulated vehicle over a number of drive cycles to design a rule-based power controller. The controller was found to be near optimal and robust when tested with previously unseen drive cycles.

(b) The vehicle studied in the paper used a lead-acid battery for energy storage. What other storage devices might be considered? What criteria should be used in selecting an energy storage device? In terms of these criteria and the truck application, how does a lead-acid battery compare to other storage devices?

A comprehensive answer will include the following points:

- Suitable storage devices other than chemical batteries include: flywheel (driven either mechanically or electrically); compressed gas; supercapacitor (perhaps in combination with a chemical battery).

- Important criteria are: maximum energy capacity (J) and maximum power (charging and discharging) (kW); the relationship of energy and power to cost (£), mass (kg) and volume (m^3); allowable variation of state of charge (SOC); lifetime (number of charge/discharge cycles); safety; compatibility with rest of drive system.
- Lead-acid batteries rank poorly compared to other chemical battery technologies in terms of energy stored per unit mass and per unit volume. Battery technology is undergoing continual improvement, but currently flywheels offer high energy and power per unit mass and per unit volume.