Q1(a) Based on notes.
Consider Structure as a whole
ULS – flexure (tower bending as whole cantilever)
– shear (shear wall requirements, transfer to each layer.
– torsion (twisting of cantilever)

– overall stability
– buckling of columns.

Must consider – self weight
– wind
– earthquakes.

SLS – vibration of cantilever in wind /sway.
– excessive deflection
– cracking of concrete (dry, hot environment).

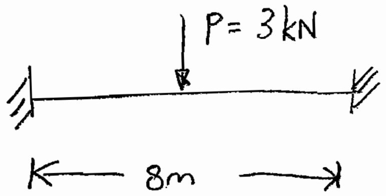
Mix design
Heat of hydration – large pours, warm climate
– use cooled water, cement replaceme
Pumpability – mix with high workability to permit staged pumping up tower
– superplasticizers, aggregate size/grading.
High strength mix for columns (with such high building nee strong base columns)

Different mixes

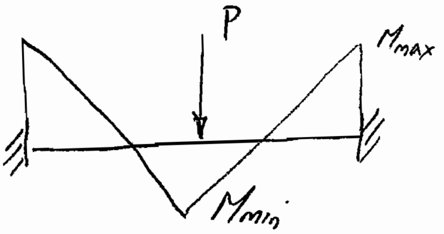
Yes – higher strength mix in columns at base – could reduce towards top.
– retardation to stop setting too early, particularly if need to pump a long way.
– some mixes will be for pumping, others not.

Robustness – confinement of columns – stirrups/links
– continuity at connections – tied together
– ductility – proportion steel + confine to permit rotation
– aim for slabs to fail before columns
– design to allow selective loss of columns not to lead to progressive collapse.

Q1(b)



$\gamma_m = 1.3$ $\quad\quad$ $\gamma_f = 1.2$



Load P $\mu_P = 3\,kN$.

Moments $|M_{MAX}| = |M_{min}| = \dfrac{PL}{8}$

Load effect $S = \dfrac{PL}{8} = \dfrac{3\times 8}{8} = 3\,kNm. = \mu_S.$

$CoV = \dfrac{\sigma}{\mu}$ $\quad \therefore \sigma_S = \mu_S . 0.1 = 3.0\times 0.1 = 0.3\ kNm.$

Resistance (Beam strength) $R = 6\ kNm = \mu_R$

$\sigma_R = \mu_R . 0.1 = 6\times 0.1 = 0.6\ kNm.$

(i) Characteristic load. $S_k = \mu_S + 1.645\times\sigma_S = 3 + 1.645\times 0.3$
$= \underline{3.494\ kNm}$ Ⓐ

Design load $S_d = S_k \times \gamma_f = 3.494\times 1.2 = \underline{4.19}\ kNm$ Ⓑ

Characteristic strength $R_k = \mu_R - 1.645\times\sigma_R = 6 - 1.645\times 0.6$
$= \underline{5.01\ kNm}.$ Ⓒ

Design strength $R_d = \dfrac{R_k}{\gamma_m} = \dfrac{5.01}{1.3} = \underline{3.86}\ kNm$ Ⓓ

(ii) $\beta = \dfrac{\mu_Z}{\sigma_Z} = \dfrac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \dfrac{6-3}{\sqrt{0.3^2 + 0.6^2}} = \dfrac{3}{0.67} = 4.47$

$\Phi(4.47) = .9^5 6089$

$\Phi(-4.47) = 1 - 0.9^5 6089 = \underline{3.91\times 10^{-6}}.$

$\therefore$ Prob. Fail $\underline{P_f = 3.91\times 10^{-6}}$

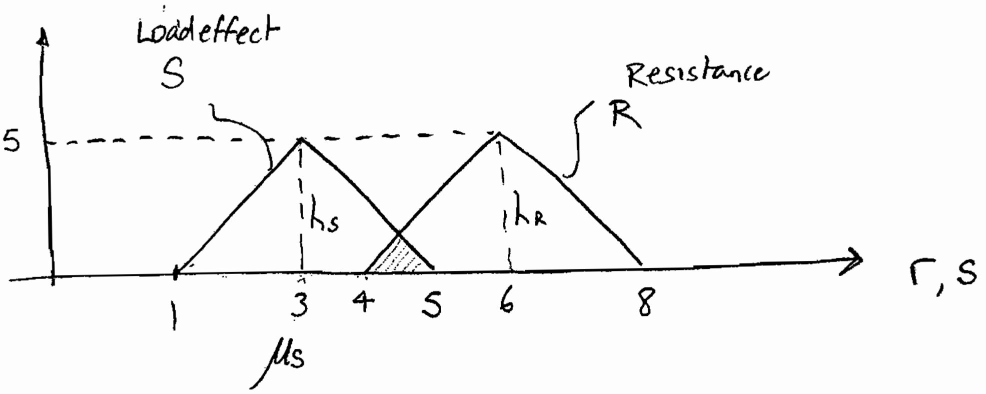
Q1(b) cont.

(iii) Here $R_d < S_d$ $\Rightarrow$ Not safe since require design strength > design load effect.

However $\beta = 4.47$ is greater than Eurocode target for ULS for the design working life of $\beta = 3.8$. So based on reliabity analysis this would be safe.

Since there is a discrepancy in the results, it would be conservative to assume the design at present is <u>NOT</u> safe.

(iv)



For S: Area under graph = 1.0 $\Rightarrow \frac{1}{2} \times 4 \times h_s = 1.0 \Rightarrow h_s = 0.5$

For R: " " " " $\Rightarrow \frac{1}{2} \times 4 \times h_R = 1.0 \Rightarrow h_R = 0.5$

For S: Want eqn of graph in region $3 < s < 5$.

$y = mx + c$ Slope $m = -\frac{0.5}{2} = -0.25$

At $x = 3$, $y = -0.25 \times 3 + c = 0.5$

$\therefore c = 0.5 + 0.75 = \frac{5}{4}$

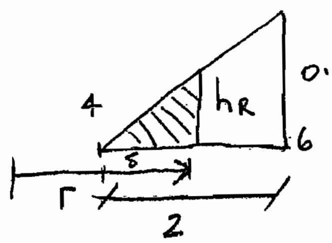
$\therefore y = f_S(s) = -\frac{1}{4}s + \frac{5}{4} = \frac{1}{4}(5 - s) \ \forall\ 3 < s < 5.$

For R: Want eqn of graph in region $4 < r < 6$.

$F_R(r)$ = Area under graph in region $r \leq 6$.

$= \frac{1}{2} \times (r-4) \times h_R$ where $h_R = \left(\frac{r-4}{4}\right)$

$= \frac{1}{8}(r-4)^2 = \frac{1}{8}(r^2 - 8r + 16).$



$\frac{h_R}{0.5} = \frac{\delta}{2} = \frac{(r-4)}{2}$

Q1(b)(v) cont.

Convolution integral.

$$P_f = \int_{-\infty}^{\infty} f_S(x)\, F_R(x)\, dx.$$

$$= \int_4^5 f_S(x)\, F_R(x)\, dx.$$

$$= \int_4^5 \tfrac{1}{4}(5-x)\,\tfrac{1}{8}\,(x^2-8x+16)\,dx.$$

$$= \int_4^5 \frac{1}{32}\left[5(x^2-8x+16) - x(x^2-8x+16)\right] dx.$$

$$= \int_4^5 \frac{1}{32}\,(5x^2-40x+80-x^3+8x^2-16x)\,dx.$$

$$= \frac{1}{32}\int_4^5 (-x^3+13x^2-56x+80)\,dx.$$

$$= \frac{1}{32}\left[-\frac{x^4}{4}+\frac{13x^3}{3}-\frac{56x^2}{2}+80x\right]_4^5$$

$$= \frac{1}{32}\left[\left(-\frac{625}{4}+\frac{13\times125}{3}-\frac{56\times25}{2}+400\right)-\left(-\frac{256}{4}+\frac{832}{3}-\frac{896}{2}+320\right)\right]$$

$$= \frac{1}{32}\left[-\frac{369}{4}+\frac{793}{3}-\frac{504}{2}+\frac{80}{1}\right]$$

$$= \frac{1}{32}\left(\frac{-3\times369+4\times793-6\times504+12\times80}{12}\right)$$

$$= \frac{1}{32\times12} = \frac{1}{384}.$$

$$P_f = \frac{1}{384} = \underline{0.0026} \qquad (2.6\times10^{-3})$$

Q2 (a)

(i) GBFS - granulated blast furnace slab - produced by grinding slab from steel production mills.

PFA - pulverized fuel ash - produced by grinding ash waste from coal burning power stations.

SILICA FUME - fine power produced as waste in carbide production process.

(ii) Directly from list in notes.

(iii) Gypsum retards set ($CaSO_4$)

(iv) C_3A — Highly exothermic } Mitigate by ↓ quantity
Rapid set } by using cement replacements to reduce % C_3A

Q2(b)

Expenditure

$$WLC = 2500k + \frac{250}{(1.03)^{15}} + \frac{250}{(1.03)^{30}} + \frac{250}{(1.03)^{45}} + \frac{250}{(1.03)^{60}}$$

(A) (B) (C) (D)

$$+ \frac{100}{(1.03)^{25}} + \frac{100}{(1.03)^{50}} + 50$$

(E) (F)

$$+ \int_0^{75} \frac{1}{\exp(r_c t)}\,dt \Big\} = \frac{1}{r_c}\left[1-\exp(-r_c t)\right]_0^{75} = \frac{1}{0.0296}\left[1-\exp(-0.0296 \times 75)\right.$$

where $r=3\%$ $\ln(1+r) = r_c$ $\Rightarrow r_c = 0.0296$

$$= 2500 + 160.5 + 102.0 + 66.1 + 42.4$$

(A) (B) (C) (D)

$$+ 47.8 + 22.8$$

(E) (F)

$$+ 50$$

$$+ \underbrace{\frac{0.891}{0.0296}}_{30}$$

(G)

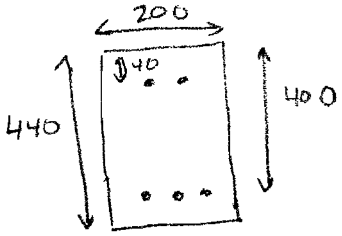
$$= \underline{\underline{3022.69}}$$ (H)

<u>INCOME</u> $I = \int_0^{75} \frac{100}{\exp(r_c t)}\,dt. = (E) \times 100 = \underline{\underline{3011}}$

i.e. Loss $\boxed{£11,235}$

On economic basis <u>DO NOT</u> recommend this project be included.

3a)



2×12mm bars

$A_{sc} = 2 \times \pi \times 12^2 / 4 = 226 \text{mm}^2$

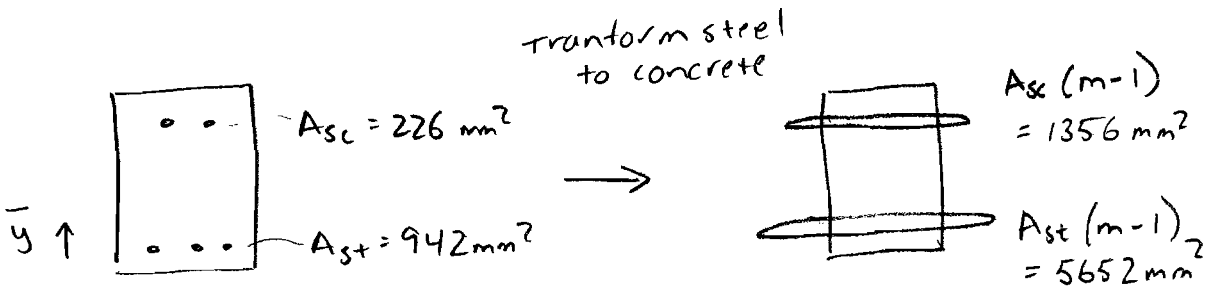
3×20mm bars

$A_{st} = 3 \times \pi \times 20^2 / 4 = 942 \text{mm}^2$

Steel $f_{yd} = 400 \text{MPa}$, $E_s = 210 \text{GPa}$

Conc $f_{cd} = 25 \text{MPa}$, $E_c = 30 \text{GPa}$, $f_t = 4 \text{MPa}$

i)



$$m = \frac{E_s}{E_c} = \frac{210}{30} = 7$$

Find $\bar{y}$

$$\bar{y} = \frac{\Sigma Ay}{A} = \frac{200 \times 440 \times 220 + \overbrace{1356}^{A_{sc}(m-1)}(400) + \overbrace{5652}^{A_{st}(m-1)} \times 40}{200 \times 440 + 1356 + 5652}$$

$$= \frac{19360000 + 542400 + 226080}{95008} = \frac{20128480}{95008}$$

$$= 211.86 \text{ mm}$$

$$I_{UN} = \frac{200 \times 440^3}{12} + (220 - 211.86)^2 \times 200 \times 440$$

$$+ (211.86 - 40)^2 \times 5652 + (400 - 211.86)^2 \times 1356$$

$$= 1.4197 \times 10^9 + 5.83 \times 10^6 + 166.94 \times 10^6 + 48.00 \times 10^6$$

$$= 1.640 \times 10^9 \text{ mm}^4$$

3a i) continued

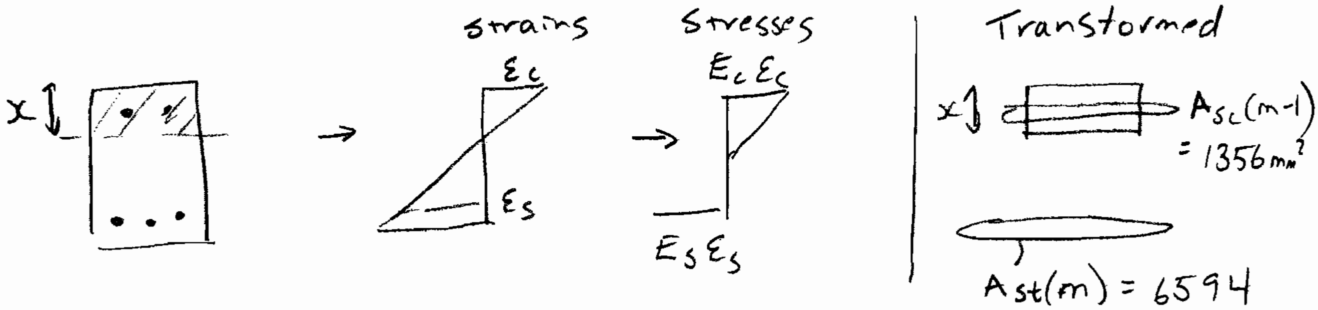
first cracking:

$$\sigma = \frac{My}{I} \quad \therefore \quad M_{cr} = \frac{I f_t}{y} = \frac{1.640 \times 10^9 \times 4}{211.86}$$

$$= 30.97 \times 10^6 \text{ N.mm}$$

$$= 30.97 \text{ kN m}$$

ii) cracked elastic behaviour



find x

$$\frac{200 . x . x}{2} + 1356(x - 40) = 6594(400 - x)$$

$$100x^2 + 1356x + 6594x - 54240 - 2637600 = 0$$

$$100x^2 + 7950x - 2691840 = 0$$

$$x = \frac{-7950 \pm \sqrt{(7950)^2 - 4(100)(-2691840)}}{2 \times 100}$$

$$= \frac{-7950 \pm 33763}{200} = 129.06 \text{ mm}$$

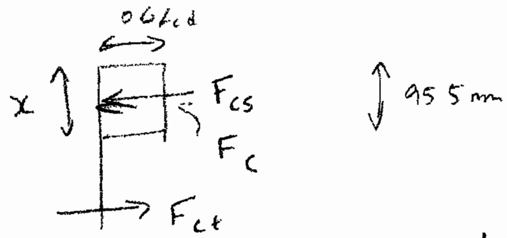
$$I_{cr} = \frac{200 \times 129.1^3}{12} + 200 \times 129.1 \times \left(\frac{129.1}{2}\right)^2$$

$$+ 1356 \times (129.1 - 40)^2 + 6594(400 - 129.1)^2$$

$$= 35.86 \times 10^6 + 107.58 \times 10^6 + 10.77 \times 10^6 + 483.91 \times 10^6$$

$$= 638.1 \times 10^6 \text{ mm}^4$$

3 iii)



Ultimate

Longitudinal Equilibrium

$$x \cdot 25 \times 200 \times 0.6 + 226 \times 400 = 942 \times 400$$

$$3000x + 90400 = 376800$$

$$x = 95.5 \text{ mm}$$

$$M_U = \frac{95.5^2}{2} \times 0.6 \times 25 \times 200 + 226 \times 400 \times (95.5 - 40)$$

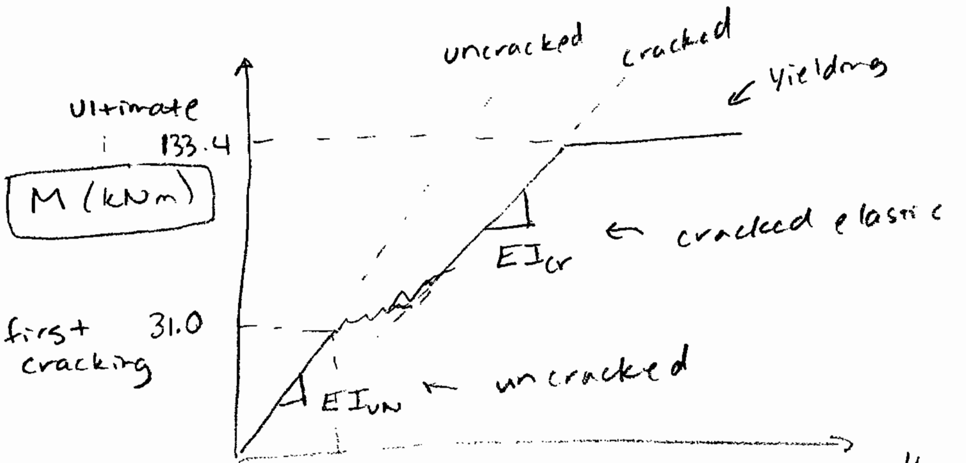
$$+ 942 \times 400\,(400 - 95.5)$$

$$= 13.67 \times 10^6 + 5.017 \times 10^6 + 114.7 \times 10^6$$

$$= 133.4 \times 10^6 \text{ Nmm}$$

$$M_U = 133.4 \text{ kNm}$$

iv)



$$\kappa_{cr} = \frac{31.0 \times 10^6}{30000 \times 1.640 \times 10^9}$$

$$= 6.30 \times 10^{-7} \text{ 1/mm}$$

EI_{UN} 4.920×10^{13} N.mm²

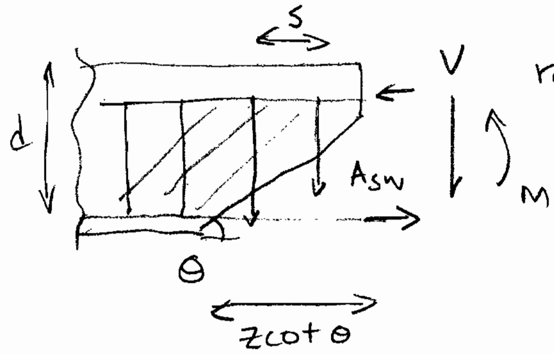
EI_{CR} : 1.914×10^{13} N.mm²

4a) Bookwork - refer also to lecture notes

The variable angle truss analogy is based on the lower bound theory of plasticity. The concrete is assumed to be cracked, at an angle θ, and carries load in compression. The angle can be selected to vary between certain limits (limits required to avoid other failures). The steel is designed to carry the resultant tensile loading.

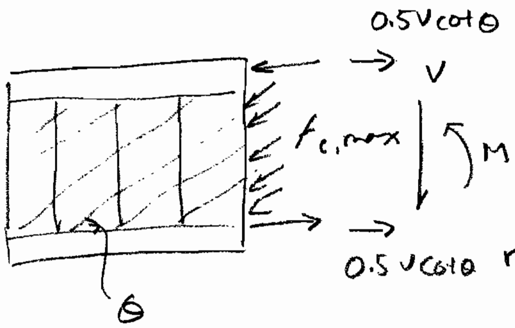
i) shear

- steel contribution



resolve vertically to give $V_{Rd,s} = \frac{A_{sw} f_y}{\gamma_m} \cdot \frac{z\cot\theta}{s}$

- concrete contribution



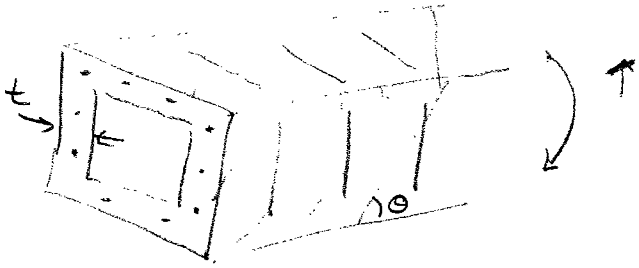
resolve vertically to give $V_{Rd,max} = f_{c,max}(z\cos\theta) b_w \sin\theta$

resolving longitudinally gives additional forces $0.5V\cot\theta$

The shear resistance is controlled by the smaller of the two contributions

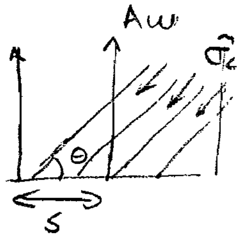
4 a) continued

(ii) torsion



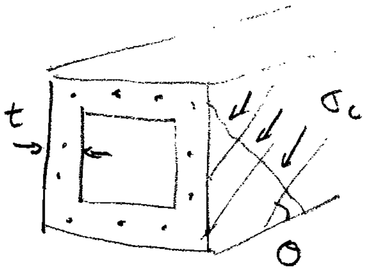
torsion is resisted by diagonal compressive struts at angle θ

• FBDs



equilibrium in transverse direction gives

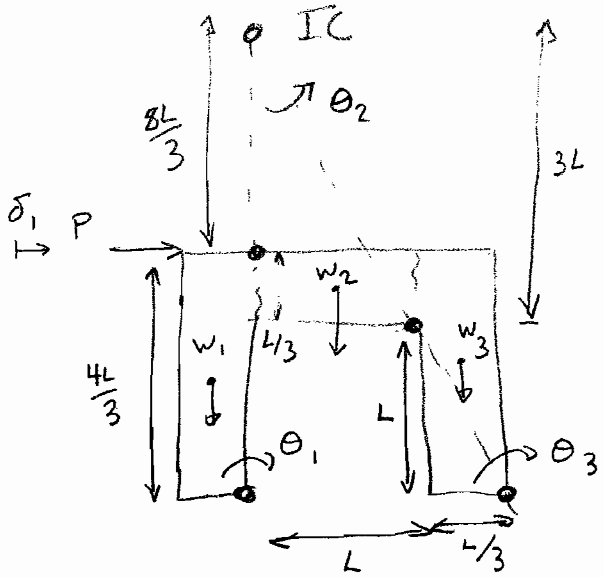
$$\frac{A_w f_{yd}}{s} = \sigma_c t \sin^2\theta$$



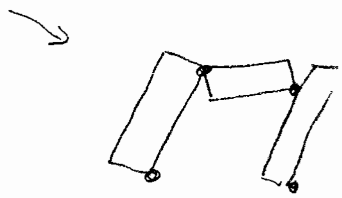
longitudinal equilibrium gives

$$\Sigma A_l f_{yd} = \sigma_c t \cos^2\theta\, u$$

4 b)



Find instantaneous centres



relative rotations

$$\theta_3 \cdot L = \theta_2 \cdot 3L \qquad \therefore \theta_3 = 3\theta_2$$

$$\theta_1 \cdot \frac{4}{3}L = \theta_2 \cdot \frac{8L}{3} \qquad \therefore \theta_1 = 2\theta_2$$

$$\delta_1 = \frac{4L}{3}\theta_1 = \frac{8L}{3}\theta_2$$

Upper bound

$$P.\delta_1 - W_1 \cdot \theta_1 \cdot \frac{L}{3} \cdot \frac{1}{2} - W_2 \cdot \theta_2 \cdot \frac{L}{2} - W_3 \cdot \theta_3 \, \frac{L}{3} \cdot \frac{1}{2} = 0$$

$$P.\frac{8\cancel{L}}{3}\cancel{\theta_2} - W_1 \cdot 2\cancel{\theta_2} \cdot \frac{\cancel{L}}{6} - W_2 \cancel{\theta_2} \frac{\cancel{L}}{2} - W_3 \cdot 3\cancel{\theta_2} \cdot \frac{\cancel{L}}{6} = 0$$

$$P\,\frac{8}{3} = \frac{W_1}{3} + \frac{W_2}{2} + \frac{W_3}{2} \qquad \text{where } W_1 = W_3 = \frac{4}{3}W_2$$

$$P = \frac{3}{8}\left(\frac{W_1}{3} + \frac{3}{4}\frac{W_1}{2} + \frac{W_1}{2}\right) = \frac{29}{64}W_1$$

$$P = \frac{29}{64}W_1 \qquad W_1 = \rho g \frac{L}{3} \cdot \frac{L}{3} \cdot \frac{4L}{3} = \rho g L^3 \frac{4}{27}$$

$$\therefore P = \frac{29}{64} \cdot \frac{4}{27} \rho g L^3 = \frac{29}{432} \rho g L^3 = 0.067 \rho g L^3$$

ENGINEERING TRIPOS PART IIB 2009

MODULE 4D7: Concrete and masonry structures

Q1 Design considerations, reliability

A popular question and generally answered well. The main problems arose in connection with the identification of the correct convolution integral.

Q2 Cement properties, whole life costing

Parts 2(a(i)) and 2(a(ii)) were related to the properties of cements and cement replacement materials but a number of students did not address these aspects and instead wrote about concretes in general e.g. fibre reinforced concrete. Students demonstrated a good understanding of whole life costing but a few had trouble with elements of continuous discounting.

Q3 Reinforced concrete behaviour

The main difficulties were in part 3(b) where students assumed a rectangular stress block and yielding of the steel rather than considering the cracked elastic behaviour. The moment-curvature diagrams were either sketched very well or rather poorly.

Q4 Reinforced concrete in shear, analysis of masonry frame

A number of students did not draw the requested free bodies when describing the variable angle truss analogy. The responses to the masonry part of the question were generally good but small mistakes were common when calculating the relative rotation angles, weights etc.

J. Lees, May 14, 2009