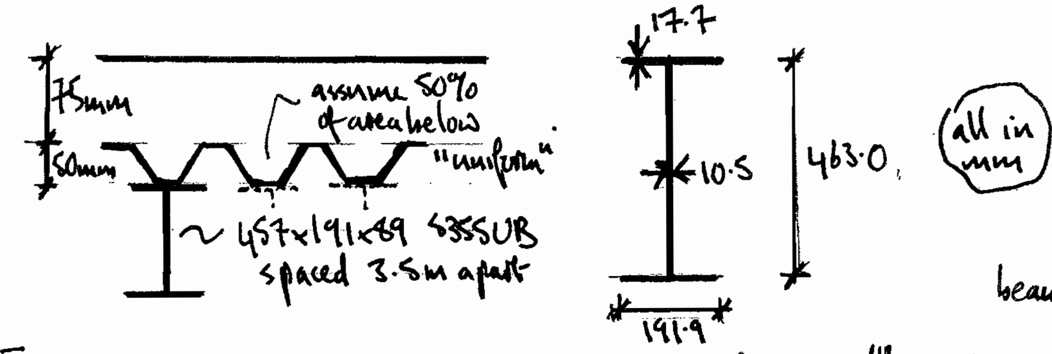
1a)



Floor spans 14 m, simply supported ⇒ effective width $= \frac{14}{4}$ or 3.5m (beam spacing)

∴ both same ⇒ $b_e = 3.5\,m$.

Compactness check: web, $\lambda = \frac{b}{t}\sqrt{\frac{\sigma_y}{355}} \sim 355 = \frac{463.0 - 17.7}{10.5} = 42.4 < 56$

flange outstands, $\lambda = \left(\frac{191.9}{2} - 10.5\right)/17.7 = 4.8 < 8$

Compactness OK.

Load intensity: conc. floor $24\,kg/m^3 \times [0.075 + 0.05/2]\,m \times 3.5\,m = 8.4\,kN/m$

UB self-wt $= 89.3\,kg/m \times 9.81\,m/s^2 = 0.88\,kN/m$

services $= 1\,kN/m^2 \times 3.5\,m = 3.5\,kN/m$

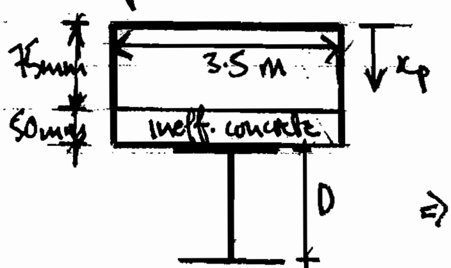
total dead load $= 12.78\,kN/m$

Imposed loading (given) $= 6\,kN/m^2 \times 3.5\,m = 21\,kN/m$

Total factored load $= 1.4 \times 12.78 + 1.6 \times 21.0 = 51.5\,kN/m = w$

Moment at midspan (effective width) $= \frac{wL^2}{8} = \frac{51.5 \times 14^2}{8} = 1262\,kNm$

Design moment. Assume neutral axis in concrete, at depth x_p



Axial eqlm: $x_p \cdot (0.6 f_{cu})\, b_e = A_s \sigma_y$ (area of steel)

30MPa, given $1.14 \times 10^{-2}\,m^2$

$$\Rightarrow x_p = \frac{1.14 \times 10^{-2}\,m^2 \times 355 \times 10^6\,Pa}{30 \times 10^6\,Pa \times 0.6 \times 3.5\,m} = 64.2\,mm$$

M_d, design moment; $A_s \sigma_y \left[\frac{D}{2} + h_e - \frac{x_p}{2}\right]$ (effect. slab depth)

$$M_d = 1.14 \times 10^{-2}\,m^2 \times 355 \times 10^6\,Pa\left[0.\frac{463}{2} + 0.125 - \frac{0.064}{2}\right] = 1313\,kNm$$

c.f. applied moment 1262 kNm, thus M_d OK.

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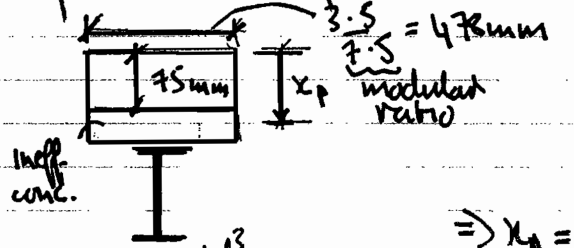
1b) Axial force in concrete $= A_s\sigma_y = 1.14\times10^{-2}\times355\times10^6 = \underline{4047\,kN}$

P_d for 65×13 mm studs = 47 kN via DS6 ∴ #studs in ½-span $= \dfrac{\text{axial force}}{P_d}$

$\Rightarrow 4047/47 = 87$ studs $\Rightarrow$ total $= \underline{174 \text{ studs}}$

spacing $= 14\,m/174 = \underline{80\,mm}$, sufficient; studs placed in troughs

1c) Imposed load deflection: use a transformed section.



$$\Big[\underbrace{\frac{3500}{7.5}\times75}_{\text{area eff. conc}} + \underbrace{11400}_{A_s}\Big]x_p = \Big[\frac{3500}{7.5}\times75\times\frac{75}{2} + 11400\left[125+463/2\right]\Big]$$

$\Rightarrow x_p = \underline{114.5\,mm}$

$$I_{xx} = \overbrace{478\times75^3/12}^{bd^3/12\ \text{conc}} + \underbrace{(478\times75)\times\left(114.5-\tfrac{75}{2}\right)^2}_{/\!/\ \text{axis}} + \overbrace{41020\times10^4}^{I_{xx}\ \text{steel}} + \underbrace{114\times10^2}_{A_s}\times\left[\tfrac{463}{2}+125-114.5\right]$$

$= \underline{1.307\times10^9\,mm^4}$.

Short term load deflection uses unfactored live load = 21 kN/m

$$\delta = \frac{5wL^4}{384EI} = \frac{5\times21\times10^3\times14^4}{384\times(205\times10^9)\times1.307\times10^{-3}} = \underline{0.0392\,m}$$

Allowable deflection = span/250 = 14 m/250 = $\underline{0.056\,m}$

Thus, short term application of load $\underline{\text{within deflection limits.}}$

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2a) In practice, columns are not perfectly straight. There are residual stresses, material inhomogeneities, different fixity conditions, cross-sectional depth effects (which precipitate extreme fibre yielding). All reduce the buckling load compared to Euler's prediction, $P_E = \pi^2 EI/L^2$.

2b) A circular hollow section with outside diameter 200mm and a wall thickness of 10mm; use $I = \Sigma \frac{\pi r^4}{4}$, $A = \Sigma \pi r^2$

$$\Rightarrow I = \frac{\pi}{4}\left[100^4 - 90^4\right] = 27\times10^6\,mm^4 \;;\; A = \pi\left[100^2 - 90^2\right] = 5969\,mm^2$$

$$\Rightarrow r = \sqrt{I/A} = \sqrt{\frac{27\times10^6}{5969}} = 67.3\,mm$$

r/y or extreme fibre = 67.3/100 = 0.67 ∴ this in between limits of welding in DS1 of C and B, thus; either choose lowest or interpolate between curves. Choose lowest: C

(i) Pin ends. $k = 1$, $L_E = 15m$, $\lambda = \frac{L_E}{r}\sqrt{\frac{\sigma_y}{355}}$ ~ 275MPa, given

$$\lambda = \frac{15}{0.0673} \times \sqrt{\frac{275}{355}} = 196 \Rightarrow \bar{\sigma}_c = 0.13 \text{ via C, DS1}$$

$$P_c = A.\bar{\sigma}_c.\sigma_y = 5969\times10^{-6}\,m^2 \times 0.13 \times 275\times10^6 = 213.39\,kN$$

(ii) If restrained in the middle, but pin-ended, $L_E = 7.5m$, $\lambda = 98$

$$\Rightarrow \bar{\sigma}_c = 0.38,\; P_c = 623.4\,kN$$

∴ 623.4/213.4 fold increase = 2.92 : Euler says a fourfold increase since effective length is halved (load $\propto 1/L_{eff}^2$).

(iii) Stiffness of intermediate bracing, from DS1, has to be at least

$$16 P_E/L = 16.\frac{\pi^2 EI}{L^2}\Big/L = \frac{16\times\pi^2\times205\times10^9\times27\times10^{-6}}{15^3}$$

$$= 259\,kN/m$$

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Suggested 457×152×82 UB, S355 ⇒ $I_{yy} = 1185\,cm^4$, $A = 105\,cm^2$
$J = 89.2\,cm^4$, $Z_p = 1811\,cm^3$

$L = 15m$, given: $D = 465.8 - \underbrace{18.9}_{\text{flange thick}} = 446.9\,mm$

3a) $M_1 = \frac{\pi}{L}\left[GJEI_{yy}\right]^{1/2} = \frac{\pi}{15}\left[81\times10^9\times(89.2\times10^{-8})\times205\times10^9\times(1185\times10^{-8})\right]^{1/2}$
$= \underline{\underline{87.74\,kNm}}$

$M_2 = \frac{\pi^2}{L^2}E\frac{I_{yy}}{2}D = \frac{\pi^2}{15^2}\cdot205\times10^9\times\frac{1185\times10^{-8}}{2}\times0.4469 = \underline{\underline{23.81\,kNm}}$

$M_E = \left[M_1^2 + M_2^2\right]^{1/2} = \left[87.74^2 + 23.81^2\right]^{1/2} = \underline{\underline{90.91\,kNm}}$

$M_p = Z_p\sigma_y = 1811\times10^{-6}\times(355\times10^6) = \underline{\underline{642.9\,kNm}}$

$\lambda_{LT} = 75\sqrt{M_p/M_E} \simeq \underline{\underline{200}} \Rightarrow \bar{M}_c = \underline{\underline{0.125}} = M_c/M_p$

$\Rightarrow \underline{\underline{M_c = 80.4\,kNm}}$

$\underline{\beta = 1}$ since equ. + opp. end moments specified; either

$M_u = m < M_c$, stability; $m < M_p$, strength.

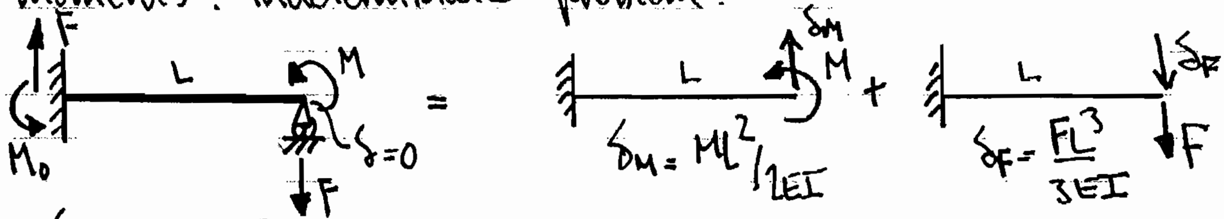
Stability governs; $m = M_c = \underline{\underline{80.4\,kNm}}$

No need to check for shear since $V = 0$ [equ+opp. moments].

Compactness: $\lambda_{web} = \frac{465.8 - 2\times18.9}{10.5}\sqrt{1} = 40.8 < 56$ in bending, OK

$\lambda_{flange} = \frac{(155.3 - 10.5)/2}{18.9}\sqrt{1} = 3.8 < 8$, OK.

3b) One end of the beam is clamped but can still warp ∴ treat the unsupported span as the equivalent length of beam between end moments: indeterminate problem:



$\delta_M = \delta_F \Rightarrow F = 3M/2L$; $M_0 = M/2$, $\underline{\underline{\beta = -1/2}}$ ← same sense

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M_1, M_2, M_E and M_c all apply from part (a) since the beam length is unchanged : however M_u changes since $\beta = -1/2$ and not $\beta = 1$.

From BS2 : $M_u = 0.4m$ ∴ stability $0.4m < M_c$

$$\Rightarrow m < \frac{80.4}{0.4} = 201.0\,kNm$$

Or, in strength case $m < M_p = 642.9\,kNm$: stability governs.

$\Rightarrow$ $M = m$ increases from 80.4 to 201 kNm.

Applied shear force, V, equals $F = 3M/2L = \frac{3 \times 201 \times 10^3}{2 \times 15} = 20.1\,kN$

V_c, critical shear, $= A_{web} \times \tau_y$

$$= (0.4658 - 2 \times 0.0189) \times 0.0105 \times \frac{355 \times 10^6}{\sqrt{3}} = 921\,kN$$

∴ $V \ll V_c/2$ and OK to presume limiting moment as M_p rather than M_y when using BS2.

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Suggested $356 \times 171 \times 67$ UB S355, $I_{xx} = 19460\,cm^4$, $r_{xx} = 15.1\,cm$, $I_{yy} = 1362\,cm^4$, $Z_p = 1211\,cm^3$, $J = 55.7\,cm^3$, $A = 85.5\,cm^2$

Compactness in bending: $\lambda_{web} = \frac{\overset{\text{overall depth}}{363.4} - 2\times 15.7}{\underset{\text{thickness}}{9.1}} \sqrt{\frac{\sigma_y}{355}} = 36.5 < 56$, ok to use CDC.

4a) $P_p = A\sigma_y = 85.5\times10^{-4} \times 355\times10^6 = 3035.2\,kN$; $M_p = Z_p\sigma_y = 1211\times10^{-6}\times355\times10^6 = 429.9\,kNm$

To use CDC (DS3), first $r/y = \frac{151}{363.4/2}$ in x dirn → bending about x-axis

$r/y = 0.83 > 0.7$ via DS1: use curve A, DS1, hot rolled

$\sigma_c/\sigma_y = \frac{A\sigma_c}{A\sigma_y} = \frac{P_c}{P_p} = \frac{1000\,(\text{given})}{3035} = 0.33 = \bar{\sigma}_c \Rightarrow \lambda = 120$ via curve A

$$\lambda = \frac{L_c}{r}\sqrt{\frac{\sigma_y}{355}} = 120 \Rightarrow L_c = \frac{r\times120}{\sqrt{\sigma_y/355}} = 0.151\times120 = 18.12\,m$$

$L/L_c = \frac{15\,(\text{given})}{18.12} = 0.81$; $\beta = 0$ (since only single end moment is applied)

⇒ via DS3, UB major axis bending:

$M_c/M_p' \simeq 0.42$

M_p' is the reduced plastic moment $= M_p$ - effects of compressive core

Compressive core due to $P = A\times\sigma_y$: $A = d\times 9.1\,mm$ ⇒ $d = 0.31\,m$ (doesn't extend into flanges)

$$M_p' = 429.9 - \frac{bd^2}{4}\sigma_y = \overset{kNm}{429.9} - \frac{0.0091\times0.31^2}{4}\times355\times10^6$$

$= 352.3\,kNm \Rightarrow M_c = 148.0\,kNm.$

4b) Interaction equation approach.

(i) Strength $\frac{P}{P_p} + \frac{M_{max}}{M_p} \leq 1$: $\underbrace{\frac{1000}{3035.2} + \frac{M_{max}}{429.9}}_{\text{all kN, m}} \leq 1$

$\Rightarrow M_{max} \simeq 288.3\,kNm$

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(ii) Stability $\frac{P}{P_c} + \frac{M_u}{M_c} \leq 1$

P_c via DS1, curve A, as before $\therefore$ $\lambda = \frac{L_E}{r}\sqrt{\frac{\sigma_y}{355}} = \frac{15}{0.151}\sqrt{1} \simeq \underline{100}$

$\Rightarrow \underline{\bar{\sigma}_c = 0.47}$, $P_c = \bar{\sigma}_c \cdot \sigma_y A = 0.47 \times (355\times10^6) \times 85.5\times10^{-4}$

$\Rightarrow \underline{\underline{P_c = 1427\,kN}}$

M_c, via DS2.

$M_1 = \frac{\pi}{L}\left[GJEI_{yy}\right]^{1/2} = \frac{\pi}{15}\left[81\times10^9 \times (55.7\times10^{-8}) \times 205\times10^9 \times (1362\times10^{-8})\right]^{1/2} = \underline{\underline{74.3\,kN}}$

$M_2 = \frac{\pi^2}{L^2} \cdot E\frac{I_{yy}}{2}D = \frac{\pi^2}{15^2} \times 205\times10^9 \times \left(\frac{1362\times10^{-8}}{2}\right) \times (0.3634 - 0.0157) = \underline{\underline{21.3\,kN}}$

$M_E = \left[M_1^2 + M_2^2\right]^{1/2} = \left[74.3^2 + 21.3^2\right]^{1/2} = \underline{\underline{77.3\,kNm}}$

$\Rightarrow \lambda_{LT} \simeq 75\sqrt{\frac{M_p}{M_E}} \simeq 177 \Rightarrow \bar{M}_c = 0.15 \Rightarrow \underline{\underline{M_c = 64.5\,kNm}}$

$\frac{P}{P_c} + \frac{M_u}{M_c} = 1 \Rightarrow \frac{1000}{1426} + \frac{M_u}{64.5} = 1 \Rightarrow \underline{\underline{M_u = 19.3\,kNm}}$

$\beta = 0$, $M_u = 0.6m$ via DS2 $\Rightarrow m < M_u/0.6 = 19.3/0.6 = \underline{\underline{32.2\,kNm}}$

$\therefore$ $\underline{m = 32.2\,kNm}$ governs (stability) [and this is the end moment carried]

N.B. much lower than CDC.

Check for shear: applied $= \frac{32.2\,kNm}{15\,m} = 2.15\,kN$

$V_c = A_{web} \cdot \tau_y = (0.3634 - 2\times0.0157)\times0.0091 \times \frac{355\times10^6}{\sqrt{3}} = \underline{618.5\,kN}$

$\underline{V \ll V_c/2}$, so M_p in DS2 fine to use.