ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Monday 27 April 2009 2.30 to 4

Module 4A1

NUCLEAR POWER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4A1 datasheet (8 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) What basic principles are used to minimize exposure when working with sources of ionizing radiation? What type of radiological protection would be required when working with α, β and γ radiation? [30%]

(b) The activity of a small laboratory cobalt-60 source is rated at $18.5\,\mathrm{GBq}$. Estimate the equivalent absorbed dose rate in man from the source at a distance of 1 m in air, stating any assumptions. Data on Co-60 can be found on page 3 of the datasheet. Take the energy absorption cross-section for γ radiation in human tissue to be $3\,\mathrm{m}^{-1}$. [30%]

(c) It is required to shield the source so that casual users of the laboratory receive a dose of less than $3\,\mu\mathrm{Sv\,hr}^{-1}$. Choosing from either concrete or lead, outline, with reasons, a suitable design of shield.

You should use the conservative 'linear build-up' model to evaluate the effectiveness of shielding materials, so that the effect of a shield of thickness t varies as $(1+\mu t)\exp(-\mu t)$ rather than simply as $\exp(-\mu t)$.

Data at appropriate energies are given below. [40%]

	Lead	Concrete	Water	Air
Total linear coefficient, μ / m^{-1}	45.2	9.42	3.85	0.0497
Density, ρ / $\mathrm{kg\,m}^{-3}$	11300	2400	1000	1.29

2 (a) A nuclear reactor in the form of a bare, uniform cube of side length $2L$ has regularly spaced fuel channels and is modelled in one-group diffusion theory. If extrapolation distances can be neglected, show that the ratio of the power in the central channel to the mean power per channel is $\pi^2/4$. [You can quote the solution to the diffusion equation for this familiar case without proof.] [20%]

(b) It is proposed to reduce the coolant flow rate in a channel for which the power is equal to the mean channel power so that the temperature rise in the coolant is the same as that in the central channel. Assuming no change in heat transfer coefficients or specific heat capacities, use the steady flow energy equation to find the required percentage reduction in coolant flow rate in the mean channel. [15%]

(c) The reactor power is limited by a hot spot within the fuel in the central channel. Explain why the hot-spot limitation will not apply to the mean channel after the coolant flow rate is reduced as in (b).

For analysing this situation, you may find the following form of Ginn's equation more convenient than the one given on the datasheet:

$$\theta = 2\left(\frac{T - T_{ci}}{T_{co} - T_{ci}}\right) - 1 = \sin\frac{\pi x}{2L} + Q\cos\frac{\pi x}{2L}$$

where T_{ci} is the coolant inlet temperature and the other symbols are as defined on the datasheet. [15%]

(d) The coolant flow rate in the mean channel is reduced further until the hot-spot limitation is reached. Find an expression from which the ratio of the coolant flow rates in the mean and central channels can be found in this case. You may assume without proof that the maximum value of θ is related to the value of Q by $\theta_{\max}^2 = 1 + Q^2$.

If the value of Q for the central channel is $\sqrt{8}$ show that the required ratio of flow rates is then $\dfrac{2\pi^2}{\pi^4 - 8}$. [50%]

3 (a) Describe the phenomenon of xenon poisoning and explain its effect on the reactivity requirements for the steady-state operation of a thermal nuclear reactor and for restarting the chain reaction after reactor shutdown. [15%]

The equations governing the behaviour of xenon-135 in a 'lumped' reactor model can be written as

$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I$$

$$\frac{dX}{dt} = \lambda_i I - \lambda_x X - \phi \sigma X$$

where all symbols have their usual meanings.

(b) Show that the steady-state loss of reactivity ρ_0 due to xenon poisoning in a high power reactor approaches $-\gamma_i/\nu$ where ν is the mean number of neutrons per fission. State any assumptions made. [20%]

(c) A thermal reactor fuelled with U-235 is shut down after a long period of steady operation at a neutron flux of 5×10^{17} m^{-2}s^{-1}. If the reactor is to be restarted after a 3 hour outage, how much additional reactivity is required? The relevant physical data can be found on pages 4 and 5 of the datasheet. [65%]

4 (a) Describe the basic steps in the reprocessing of nuclear fuel. List the main waste streams arising and describe how they are handled. [40%]

(b) A large utility operating a number of pressurised water reactors requires 500 tonnes (as U metal) fuel per year at an enrichment of 3.5% U-235. Taking the U-235 content of natural uranium to be 0.7% and assuming an enrichment plant tails of 0.3% U-235, calculate the amount of natural uranium and the number of separation work units (SWU) required. Losses in processing can be neglected. [20%]

(c) If the cost of natural uranium is $200 per kg, the cost of a SWU is $100 per kg, the cost of reprocessing is $1000 per kg (based on reactor feed and including waste disposal costs) and the cost of spent fuel disposal is $400 per kg U (based on reactor feed), decide if it is worth reprocessing and recycling the uranium.

Assume that the fuel leaves the reactor containing 96% uranium at an enrichment of 0.8% U-235. Take total losses in the reprocessing plant to be 1%. [40%]

END OF PAPER

Data Sheet S/15

Revised January 2003

MODULE 4A1

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	299.792458×10^6 m s^{-1}
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Planck constant	h	6.626176×10^{-32} J s
Boltzmann constant	k	1.380662×10^{-23} J K^{-1}
Elementary charge	e	$1.6021892 \times 10^{-19}$ C

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}$ kg (931.5016 MeV)
Electron volt	eV	$1.6021892 \times 10^{-19}$ J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	10^{-28} m^2

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Yt$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

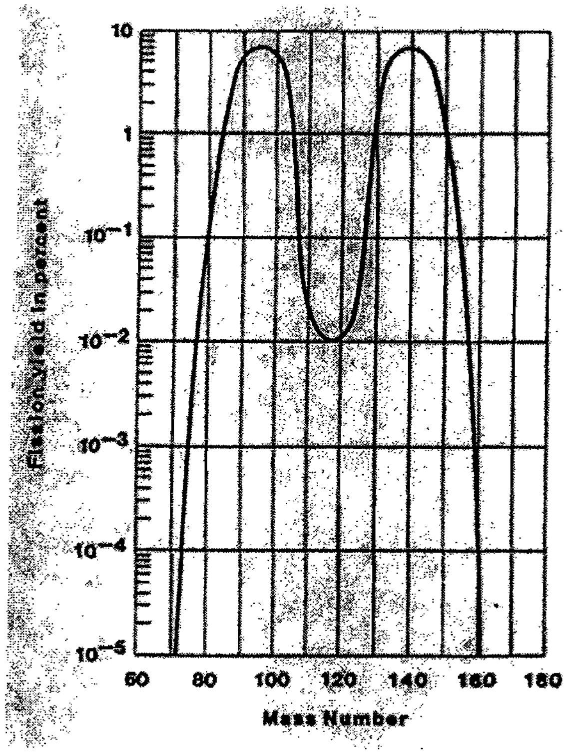
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90	$\rightarrow$	Rb-90	$\rightarrow$	Sr-90	$\rightarrow$	Y-90	$\rightarrow$	Zr-90
	33 s		2.7 min		28 yr		64 hr	

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131	$\rightarrow$	Te-131	$\rightarrow$	I-131	$\rightarrow$	Xe-131
	23 min		24 min		8 days	

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135	$\rightarrow$	Xe-135	$\rightarrow$	Cs-135 (nearly stable)	Fission yield $\gamma = 0.064$
	6.7 hr		9.2 hr		

Xe-135 is a strong absorber of thermal neutrons, with $\sigma_a = 3.5$ Mbarn.

Pm-149	$\rightarrow$	Sm-149	Fission yield $\gamma = 0.014$
	54 hr		

Sm-149 is a strong absorber of thermal neutrons, with $\sigma_a = 53$ kbarn.

Se-87	$\rightarrow$	Br-87	$\rightarrow$ either {Kr-86 + n} or {Kr-87	$\rightarrow$	Rb-87}
	17 s		55 s	78 min	

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy $\Rightarrow$ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where $n =$ neutron concentration

$c =$ precursor concentration

$\beta =$ delayed neutron precursor fraction $= \sum \beta_i$

$\lambda =$ average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu} =$ the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $$R(r)=A_1J_0(r)+A_2Y_0(r)$$

$J_0(0)=1$; $Y_0(0)=-\infty$;

The first zero of $J_0(r)$ is at $r=2.405$.

$J_1(2.405)=0.5183$, where $J_1(r)=\frac{1}{r}\int_0^r xJ_0(x)dx$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $g\,cm^{-3}$	Σ_a cm^{-1}	D cm	$L^2=D/\Sigma_a$ cm^2
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $$\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$$

Enrichment of Isotopes

Value function: $$v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)\text{ for small }x$$

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $$R_n\equiv\frac{L''_n}{P}=\frac{x_p-x'_{n+1}}{x'_{n+1}-x''_n}$$

Stripping section reflux ratio: $$R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x'_{n+1}-x_w}{x'_{n+1}-x''_n}\right]$$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length.

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity