ENGINEERING TRIPOS PART IIB

Monday 4 May 2009 9 to 10.30

Module 4A10

FLOW INSTABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment
Data sheet for 4A10 (2 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A jet of water in air flows with velocity U in the x-direction and is initially of radius a. The surface tension of the water/air interface is σ. The surface of the jet is perturbed axisymmetrically to give a new jet radius $r = \alpha + \beta \cos kx$ as shown in Fig. 1.

(i) Use conservation of mass to determine α in terms of a and β^2, if terms of order higher than β^2 can be neglected. [30%]

(ii) Hence use an energy argument to determine for which wavenumbers k the jet is unstable. [30%]

(b) How would the range of unstable wavenumbers be changed for an air jet, also of initial radius a, in water? [20%]

(c) State with a reason whether you would expect the water droplets formed in (a) to be the same size as the air bubbles in (b). Detailed calculations are not required. [20%]

You may assume that the air has negligible density compared with the water. Note that the binominal theorem gives $(1+d)^p = 1 + pd$ for small d.

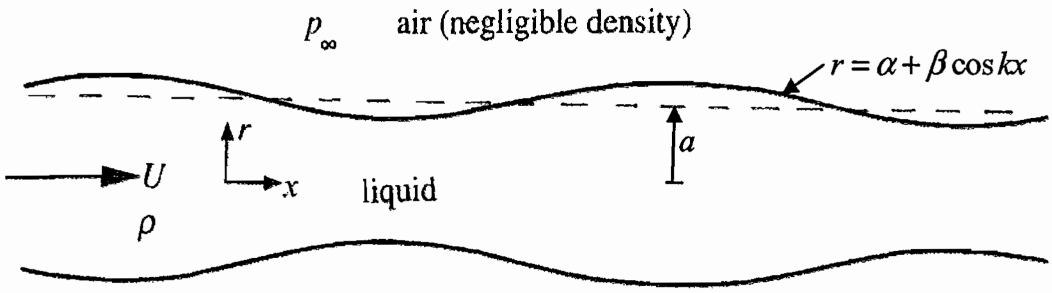

Fig. 1. Small perturbation of a water jet

2 (a) State Rayleigh's inflexion point theorem for parallel shear flows and give an example of

(i) an unstable

(ii) a stable

flow according to this theorem. [25%]

(b) Define the term spatial stability. [5%]

(c) An initially laminar boundary layer is formed as a steady stream of air flows over a thin flat plate as shown in Fig. 2. Describe the perturbations that would occur at different distances x from the leading edge of the plate. [50%]

(d) How would the mode of instability be changed if the plate in (c) were curved? [20%]

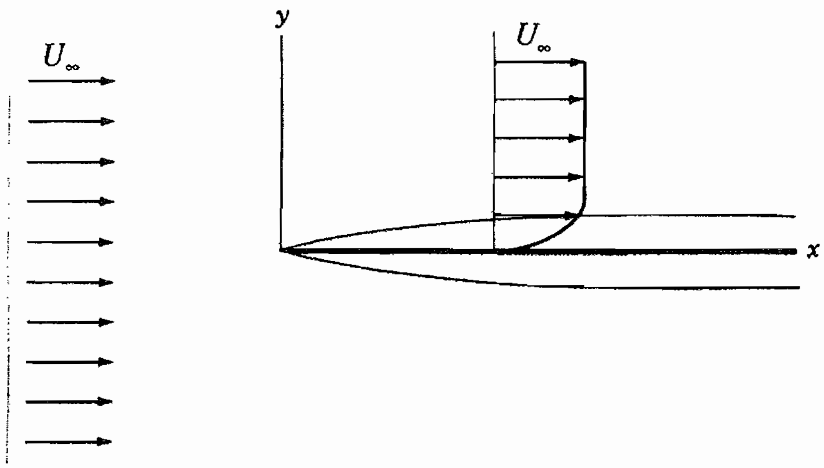

Fig. 2 Boundary layer growing on a flat plate

3 (a) A sphere of radius a is accelerated at a rate dU/dt in an otherwise stationary fluid, where ρ_f is the density of the fluid. Consequently the fluid accelerates around the sphere as it is displaced.

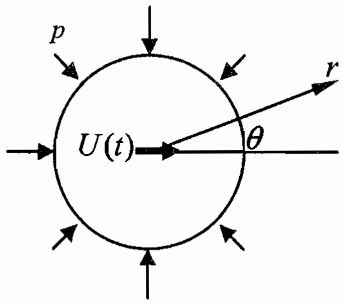

Fig. 3

Show that the force the fluid exerts on the sphere can be expressed as $f = -\frac{2}{3}\rho_f \pi a^3 dU/dt$,

where the pressure force (see Fig. 3) is: $f = \int_0^{\pi} p 2\pi a^2 \cos\theta \sin\theta \, d\theta$

Note that:

the velocity potential for the flow due to the sphere is: $\phi = \frac{Ua^3}{2r^2}\cos\theta$

surface pressure is given by: $p = -\rho_f \left[\frac{\partial \phi}{\partial t} + \frac{1}{2}u^2\right]$

and $u^2 = u_r^2 + u_\theta^2 = \left(\frac{\partial \phi}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial \phi}{\partial \theta}\right)^2$ [50%]

(b) Briefly explain the physical significance of added mass and determine the added mass of the sphere in (a). [20%]

(c) The added mass per unit length for a cylinder also of radius a is $m_a = \rho_f \pi a^2$, where ρ_f is the density of the fluid and a is the radius of the cylinder. By comparing the added mass and the mass of the displaced fluid for this cylinder with those for the sphere in (a) and (b), what might you infer about their relationship? [15%]

(cont.

(d) Estimate the ratio of the added mass to the actual mass of a submerged ping pong ball when it is released under water assuming the mass of the ping pong ball is entirely made up of air. [15%]

The density of water and air are $1000\ \text{kgm}^{-3}$ and $1.2\ \text{kgm}^{-3}$ respectively.

4 (a) Describe how vortex shedding behind a cylinder can generate a flow-structure interaction. Include an explanation of how the flow-structure interaction responds as the flow velocity is increased or decreased. [30%]

(b) If a simple harmonic model of vortex-induced oscillations behind a cylinder of circular cross-section is represented by a mass-spring-damper system, derive a suitable equation of motion in terms of its aerodynamic and mechanical properties. [30%]

(c) The cylinder in (b) is replaced with a square beam. Derive an equation for the flow speed at which galloping will occur. [40%]

END OF PAPER

Module 4A10 Data Card

EQUATIONS OF MOTION

For an incompressible isothermal viscous fluid:

Continuity $\nabla \cdot \boldsymbol{u} = 0$

Navier Stokes $\rho \dfrac{D\boldsymbol{u}}{Dt} = -\nabla p + \mu \nabla^2 \boldsymbol{u}$

D/Dt denotes the material derivative, $\partial/\partial t + \boldsymbol{u} \cdot \nabla$

IRROTATIONAL FLOW $\nabla \times \boldsymbol{u} = \boldsymbol{0}$

velocity potential ϕ,

$$\boldsymbol{u} = \nabla \phi \text{ and } \nabla^2 \phi = 0$$

Bernoulli's equation

for inviscid flow $\dfrac{p}{\rho} + \tfrac{1}{2}|\boldsymbol{u}|^2 + gz + \dfrac{\partial \phi}{\partial t} =$ constant throughout flow field.

KINEMATIC CONDITION AT A MATERIAL INTERFACE

A surface $z = \eta(x,y,t)$ moves with fluid of velocity $\boldsymbol{u} = (u, v, w)$ if

$$w = \frac{D\eta}{Dt} = \frac{\partial \eta}{\partial t} + \boldsymbol{u} \cdot \nabla \eta \quad \text{on } z = \eta(x,t).$$

For η small and $\boldsymbol{u}$ linearly disturbed from $(U,0,0)$

$$w = \frac{\partial \eta}{\partial t} + U \frac{\partial \eta}{\partial x} \quad \text{on } z = 0.$$

SURFACE TENSION σ AT A LIQUID-AIR INTERFACE

Potential energy

The potential energy of a surface of area A is σA.

Pressure difference

The difference in pressure Δp across a liquid-air surface with principal radii of curvature R_1 and R_2 is

$$\Delta p = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right).$$

For a surface which is almost a circular cylinder with axis in the x-direction, $r = a + \eta(x,\theta,t)$ (η is very small so that η^2 is negligible)

$$\Delta p = \frac{\sigma}{a} + \sigma\left(-\frac{\eta}{a^2} - \frac{\partial^2 \eta}{\partial x^2} - \frac{1}{a^2}\frac{\partial^2 \eta}{\partial \theta^2}\right),$$

where Δp is the difference between the internal and the external surface pressure.

For a surface which is almost plane with $z = \eta(x,t)$ (η is very small so that η^2 is negligible)

$$\Delta p = -\sigma \frac{\partial^2 \eta}{\partial x^2}$$

where Δp is the difference between pressure at $z = \eta^+$ and $z = \eta^-$.

ROTATING FLOW

In steady flows with circular streamlines in which the fluid velocity and pressure are functions of radius r only:

Rayleigh's criterion

The flow is unstable / stable to inviscid axisymmetric disturbances if Γ^2 decreases / increases with r.

$\Gamma = 2\pi r V(r)$ is the circulation around a circle of radius r.

Navier Stokes equation simplifies to

$$0 = \mu\left(\frac{d^2 V}{dr^2} + \frac{1}{r}\frac{dV}{dr} - \frac{V}{r^2}\right)$$

$$-\rho \frac{V^2}{r} = -\frac{dp}{dr}.$$

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STABILITY OF PARALLEL SHEAR FLOW

Rayleigh's inflexion point theorem

A parallel shear flow with profile $U(z)$ is only unstable to inviscid perturbations if

$$\frac{d^2U}{dz^2}=0 \quad \text{for some } z.$$

CONVECTIVE FLOW

The Boussinesq approximation leads to

$$\nabla\cdot\boldsymbol{u}=0$$

$$\frac{D\boldsymbol{u}}{Dt}=-\frac{1}{\rho_0}\nabla p+(1-\alpha(T-T_0))\boldsymbol{g}+\upsilon\nabla^2\boldsymbol{u}$$

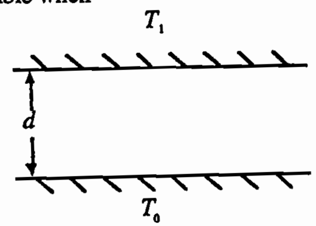
and

$$\frac{DT}{Dt}=\kappa\nabla^2 T$$

Rayleigh-Bénard convection

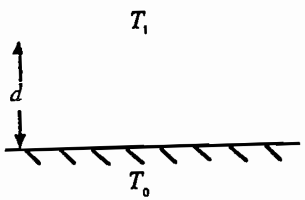
A fluid between two **rigid** plates is unstable when

$$\text{Ra}\geq 1708$$



A liquid with a **free** upper surface is unstable when

$$\frac{\text{Ra}}{\text{Ra}_c}+\frac{\text{Ma}}{\text{Ma}_c}\geq 1$$



where

$$\text{Ra}=\frac{g\alpha(T_0-T_1)d^3}{\upsilon\kappa},\quad \text{Ma}=\frac{\chi(T_0-T_1)d}{\rho\upsilon\kappa}\quad \text{with } \chi=-\frac{d\sigma}{dT}$$

$\text{Ra}_c \approx 670$ $\quad$ $\text{Ma}_c \approx 80.$

USEFUL MATHEMATICAL FORMULA

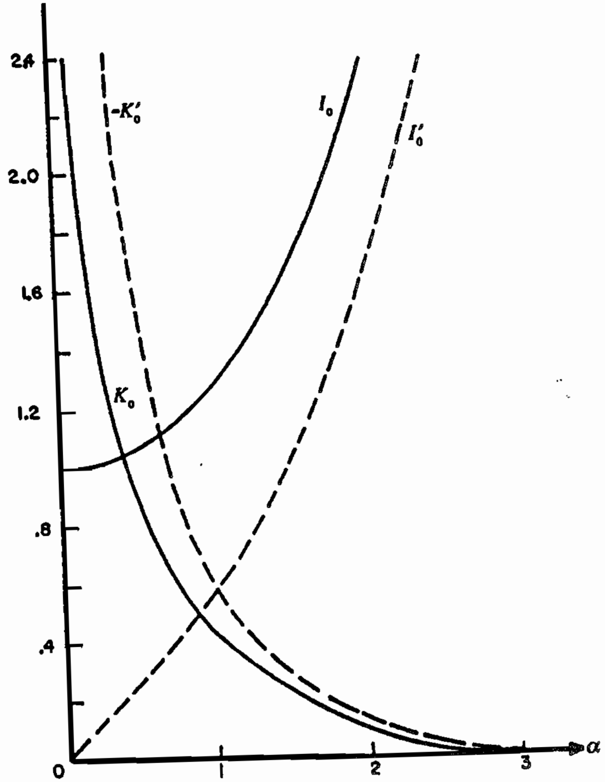
Modified Bessel equation

$I_0(kr)$ and $K_0(kr)$ are two independent solutions of

$$\frac{d^2 f}{dr^2}+\frac{1}{r}\frac{df}{dr}-k^2 f=0.$$

$I_0(kr)$ is finite at $r=0$ and tends to infinity as $r\to\infty$,

$K_0(kr)$ is infinite at $r=0$ and tends to zero as $r\to\infty$.



$I_0(\alpha)$, $K_0(\alpha)$, $I_0'(\alpha)$, $K_0'(\alpha)$
where $'$ denotes a derivative

APD September 1998