ENGINEERING TRIPOS PART IIB

Thursday 7 May 2009 2.30 to 4

Module 4A14

SILENT AIRCRAFT INITIATIVE

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment
Data sheet for 4A14 (2 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Derive the relationship between aircraft weight W, lift force L, drag force D and engine thrust T for an aircraft established on a glide slope of angle γ. [10%]

(b) (i) What additional drag is required if an aircraft of mass 140,000 kg is to increase its glide slope angle from 3° to 6° ? [10%]

(ii) If this additional drag is to be produced by an airbrake consisting of a rectangular surface at 90° to the flow, determine the surface area A required if the drag coefficient $C_D = force\ /(\frac{1}{2}\rho_{air} U^2 A)$ is 1.0 and the approach velocity U is $60\,\mathrm{ms}^{-1}$. [10%]

(iii) Estimate the maximum Sound Pressure Level (SPL) on the ground outside the airport perimeter due to this air brake. [20%]

You may assume that

- the runway threshold is 1 km inside the airport perimeter;
- the main sound is dipole and due to an unsteady lift force on the airbrake at a Strouhal number $f\sqrt{A}\,/\,U = 2,$ where f is frequency of the force in Hz and the root mean square amplitude of the unsteady lift force is $0.01\left(\frac{1}{2}\rho_{air}U^2A\right)$;
- any reflection or scattering of the resulting sound field by the aircraft or the ground may be neglected.

(c) Discuss the benefits and disadvantages of [50%]

(i) a decrease in engine flight-idle thrust;

(ii) an increase in glide-slope angle;

(iii) a reduction in approach speed.

2 An acoustic liner for low frequency sound absorption consists of a perforated surface with open-area ratio of 10%. Each hole opens onto a cavity of volume $0.0004\ \mathrm{m}^3$ and together the hole and cavity behave like a Helmholtz resonator. The difference between the pressure perturbation just outside a hole and within its cavity is equal to

$$\rho_0 \ell \frac{du}{dt} + \alpha u$$

where ρ_0 is the mean density, ℓ is 0.6 times the hole diameter, u is the velocity of the air flowing through the hole, $\alpha = \rho_0 c_0 k$ with $k = 0.1$, and c_0 is the speed of sound. The ambient temperature is 600 K and pressure 1 bar.

(a) What hole size would you choose to absorb sound of frequency 250 Hz ? [40%]

(b) With this choice of hole size, determine the rate of absorption of sound energy at 250 Hz by a single hole in terms of p_{1rms}, the root mean square pressure perturbation at the hole. [40%]

(c) A plane sound wave of frequency 250 Hz travels along a duct of circular cross-section and diameter 0.8 m lined with this acoustic liner. Determine its attenuation in dB/unit length. [20%]

(TURN OVER

3 (a) The far-field acoustic density generated by a stationary compact distribution of quadrupoles is given by the equation

$$\rho'(\mathbf{x},t) = \frac{\hat{x}_i \hat{x}_j}{4\pi c_0^4 |\mathbf{x}|} \frac{d^2 S_{ij}(\tau)}{dt^2}$$

where $\hat{\mathbf{x}} = \mathbf{x}/|\mathbf{x}|$, $S_{ij}(\tau) = \int T_{ij} dV$ is the effective source strength, c_0 is the speed of sound, and time $\tau = t - |\mathbf{x}|/c_0$. Explain how this result can be used to derive Lighthill's eighth-power law for the acoustic intensity. [25%]

Show how this result is modified if the source distribution moves with a constant Mach number M. You should use without proof the fact that a source, which emits sound at a frequency ω when at rest, emits sound at a frequency $\omega/(1-M\cos\theta)$ when in motion, where θ is the angle between the observer and the direction of motion. [25%]

(b) The two-dimensional half space $y > 0$ is bounded by a straight wall on $y = 0$, and the fluid above the wall is at rest. The wall is then caused to vibrate in such a way that the pressure on the wall is $P_0 \exp[ikV(t - x/V)]$, with k and V constants. By solving the wave equation, find the acoustic pressure and the acoustic velocity in $y > 0$, in the case $V > c_0$, where c_0 is the speed of sound. [40%]

Without further detailed calculation, explain what happens when $V < c_0$. [10%]

4 (a) Suppose that in two dimensions (x, y) we have water in $x < 0$ and air in $x > 0$. A sound wave propagates in the water at a nonzero angle to the x axis, and is refracted by the air-water interface. Calculate the directions and amplitudes of the reflected and transmitted waves. [30%]

For what incidence angle is all the incident energy transmitted to the air? [10%]

(b) A high-frequency sound wave is launched from the origin at an angle θ_0 to the x axis in a medium whose sound speed varies according to

$$c_0(x) = c_0(0)\, e^{\alpha x}$$

where $c_0(0)$ is positive and α is a constant. Use Snell's Law to show that the path satisfies

$$\frac{dy}{dx} = \frac{e^{\alpha x} \sin\theta_0}{\sqrt{1 - e^{2\alpha x} \sin^2\theta_0}}$$ [20%]

Hence, find an equation for the ray path. [20%]

Sketch typical ray paths in the cases $\alpha > 0$ and $\alpha < 0$. [20%]

Note that: $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$, where C is a constant.

END OF PAPER

USEFUL MATHEMATICAL FORMULAE

In spherical polar coordinates (r,θ,ϕ)

$$\nabla p' = \left(\frac{\partial p'}{\partial r}, \frac{1}{r}\frac{\partial p'}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial p'}{\partial \phi}\right)$$

For $\mathbf{v}' = (v'_r, v'_\theta, v'_\phi)$, $\quad \nabla\cdot\mathbf{v}' = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v'_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v'_\theta) + \frac{1}{r\sin\theta}\frac{\partial v'_\phi}{\partial\phi}$.

$$\nabla^2 p' = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p'}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial p'}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 p'}{\partial\phi^2}.$$

In cylindrical polar coordinates (r,θ,x_3)

$$\nabla p' = \left(\frac{\partial p'}{\partial r}, \frac{1}{r}\frac{\partial p'}{\partial \theta}, \frac{\partial p'}{\partial x_3}\right)$$

For $\mathbf{v}' = (v'_r, v'_\theta, v'_3)$, $\quad \nabla\cdot\mathbf{v}' = \frac{1}{r}\frac{\partial}{\partial r}(r v'_r) + \frac{1}{r}\frac{\partial v'_\theta}{\partial\theta} + \frac{\partial v'_3}{\partial x_3}$, $\quad \nabla^2 p' = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 p'}{\partial\theta^2} + \frac{\partial^2 p'}{\partial x_3^2}$.

Heaviside function $H(t-\tau) = 1 \quad t>\tau\ ; \ = 0 \quad t<\tau$

δ-functions

Kronecker delta $\delta_{ij} = 1$ if $i=j$; 0 if $i\neq j$

1D δ-function: $\delta(t) = 0$ for $t\neq 0$; $f(t)\delta(t-\tau) = f(\tau)\delta(t-\tau)$

$$\int_{-\infty}^{\infty}\delta(t)\,dt = 1 \text{ and } \int_{-\infty}^{\infty}\delta(t-\tau)f(t)\,dt = f(\tau) \text{ for any function } f(t).$$

3D δ-function: $\delta(\mathbf{x}) = \delta(x_1)\,\delta(x_2)\,\delta(x_3)$; $\delta(\mathbf{x}) = 0$ for $|\mathbf{x}|\neq 0$; $f(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) = f(\mathbf{y})\delta(\mathbf{x}-\mathbf{y})$

$\int_V \delta(\mathbf{x})\,dV = 1$ for any volume V that includes the origin

and

$\int_V \delta(\mathbf{x}-\mathbf{y})f(\mathbf{x})\,dV = f(\mathbf{x})$ for any function $f(\mathbf{x})$ and volume V that includes $\mathbf{x}$.

$$\nabla^2\left(\frac{1}{|\mathbf{x}|}\right) = -4\pi\delta(\mathbf{x}).$$

Autocorrelation

$F(\xi)$, the autocorrelation of $f(y) = \overline{f(y)f(y+\xi)}$

$$F(0) = \overline{f^2}$$

Integral lengthscale ℓ $\quad \ell\overline{f^2} = \int_{-\infty}^{\infty} F(\xi)\,d\xi$.

SOURCES

Point sources

monopole of strength $Q(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t) = \frac{Q(t-|\mathbf{x}|/c_0)}{4\pi|\mathbf{x}|}.$$

dipole of strength $\mathbf{F}(t)$ at the origin generates a pressure field

$$p'(\mathbf{x},t) = -\frac{\partial}{\partial x_i}\left[\frac{F_i(t-|\mathbf{x}|/c_0)}{4\pi|\mathbf{x}|}\right] = \frac{x_i}{4\pi}\left[\frac{1}{|\mathbf{x}|^3}F_i(t-|\mathbf{x}|/c_0) + \frac{1}{|\mathbf{x}|^2 c_0}\frac{\partial F_i}{\partial t}(t-|\mathbf{x}|/c_0)\right]$$

Distributed sources

Monopole, strength $q(\mathbf{x},t)$, wave equation $\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = q$, pressure field $p'(\mathbf{x},t) = \int \frac{q(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c_0)}{4\pi|\mathbf{x}-\mathbf{y}|}dV$

Dipole, strength $\mathbf{f}(\mathbf{x},t)$, wave equation $\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = -\nabla\cdot\mathbf{f}$, $\quad p'(\mathbf{x},t) = -\frac{\partial}{\partial x_i}\int \frac{f_i(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c_0)}{4\pi|\mathbf{x}-\mathbf{y}|}dV$

Quadrupole, strength $T_{ij}(\mathbf{x},t)$, equation $\left(\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p' = \frac{\partial T_{ij}}{\partial x_i\partial x_j}$, $\quad p'(\mathbf{x},t) = \frac{\partial^2}{\partial x_i\partial x_j}\int \frac{T_{ij}(\mathbf{y},t-|\mathbf{x}-\mathbf{y}|/c_0)}{4\pi|\mathbf{x}-\mathbf{y}|}dV$

Far-field form $|\mathbf{x}| >> |\mathbf{y}|$, $\mathbf{y}$ near origin

$$|\mathbf{x}-\mathbf{y}| \sim |\mathbf{x}| - \frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|} + 0(|\mathbf{x}|^{-2}),\ \frac{1}{|\mathbf{x}-\mathbf{y}|} \sim \frac{1}{|\mathbf{x}|} + 0(|\mathbf{x}|^{-2}),\ \frac{\partial}{\partial x_i} \sim -\frac{x_i}{|\mathbf{x}|c_0}\frac{\partial}{\partial t} + 0(|\mathbf{x}|^{-1}).$$

Physical sources

Lighthill's theory shows that jet noise is generated by quadrupoles of strength $T_{ij} = \rho v_i v_j + \left(p' - c_0^2\rho'\right)\delta_{ij} - \tau_{ij}$.

The Ffowcs-Williams Hawkings equation shows that foreign bodies in linear motion generate far-field sound

$$p'(\mathbf{x},t) = \frac{1}{4\pi|\mathbf{x}|}\frac{\partial}{\partial t}\int_S \rho_0 d\mathbf{S}\bullet\mathbf{v}\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{c_0} + \frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c_0}\right) + \frac{x_i}{4\pi|\mathbf{x}|^2 c_0}\frac{\partial}{\partial t}\int_S dS_i\, p\left(\mathbf{y}, t - \frac{|\mathbf{x}|}{c_0} + \frac{\mathbf{x}\cdot\mathbf{y}}{|\mathbf{x}|c_0}\right)$$

SOUND POWER

Sound power from a source, $P = \int_S \bar{\mathbf{I}}\bullet d\mathbf{S} = \int_{S_\infty} \frac{\overline{p'^2}}{\rho_0 c_0}dS$ for a statistically stationary source.

For a spherically symmetrical sound field $P = \frac{\overline{p'^2}}{\rho_0 c_0}4\pi r^2$ where p' is the pressure at radius r.

For a sound field, which is a function of spherical polar coordinates r, θ only, and independent of ϕ,

$$P = 2\pi r^2 \int_0^\pi \frac{\overline{p'^2}}{\rho_0 c_0}\sin\theta\, d\theta$$

USEFUL DATA AND DEFINITIONS

Physical Properties

Speed of sound in an ideal gas $\sqrt{\gamma RT}$, where T is temperature in Kelvins

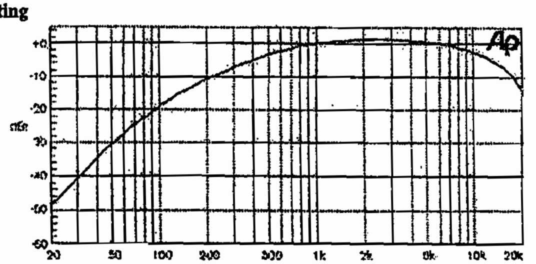
Units of sound measurement

SPL (sound pressure level) $= 20\log_{10}\left(\dfrac{p'_{rms}}{2.10^{-5}\,\text{Nm}^{-2}}\right)$ dB

IL (intensity level) $= 10\log_{10}\left(\dfrac{\text{intensity}}{10^{-12}\,\text{watts m}^{-2}}\right)$ dB

PWL (power level) $= 10\log_{10}\left(\dfrac{\text{sound power output}}{10^{-12}\,\text{watts}}\right)$ dB

A-weighting



SPL mode A-Weighting Curve

Definitions

Surface impedance, Z_s, relates the pressure perturbation applied to a surface, p', to its normal velocity v_n; $p' = Z_s v_n$.

Characteristic impedance of a fluid $\rho_0 c_0$

Nondimensional surface impedance of a surface $Z_s/\rho_0 c_0$

Transmission loss $= 10\log_{10}\left(\dfrac{\text{incident sound power}}{\text{transmitted sound power}}\right)$

Absorption coefficient of a sound absorber $= \dfrac{\text{sound power absorbed}}{\text{incident sound power}}$

Wavelength, λ, for sound waves with angular frequency ω, $\lambda = 2\pi c_0/\omega$

Wave-number $k_0 = \omega/c_0 = 2\pi/\lambda$

Phase speed $= \omega/k$

Group velocity $= \dfrac{\partial\omega}{\partial k}$

Helmholtz number (or compactness ratio) $= k_0 D$, where D is a typical dimension of the source.

Strouhal number $= \omega D/(2\pi U)$ for sound of frequency ω, produced in a flow with speed U, length scale D.

BASIC EQUATIONS FOR LINEAR ACOUSTICS

Conservation of mass $\dfrac{\partial\rho'}{\partial t} + \rho_0\,\nabla\cdot\mathbf{v}' = 0$

Conservation of momentum $\rho_0\dfrac{\partial\mathbf{v}'}{\partial t} + \nabla p' = 0$

Isentropic $c_0^2 = \left.\dfrac{dp}{d\rho}\right|_s$

These equations combine to give the wave equation $\dfrac{1}{c_0^2}\dfrac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0$

Energy density $e = \frac{1}{2}\rho_0 v'^2 + \frac{1}{2}p'^2/\rho_0 c_0^2$

Intensity $\mathbf{I} = p'\mathbf{v}'$

$\text{div}\bar{\mathbf{I}} = 0$ for statistically stationary (in time) sound fields.

Velocity potential $\phi'(\mathbf{x},t)$ satisfies the wave equation and $p' = -\rho_0\dfrac{\partial\phi'}{\partial t}$, $\mathbf{v} = \nabla\phi'$.

SIMPLE WAVE FIELDS

1D or plane wave

The general solution of the 1D wave equation is $p'(x,t) = f(t - x/c_0) + g(t + x/c_0)$, where f and g are arbitrary functions. In a plane wave propagating to the right $p' = \rho_0 c_0 u'$; in a plane wave propagating to the left $p' = -\rho_0 c_0 u'$, u' being the particle velocity.

Spherically symmetric sound fields

The general spherically symmetric solution of the 3D wave equation is $\phi'(r,t) = \dfrac{f(t - r/c_0)}{r} + \dfrac{g(t + r/c_0)}{r}$

where r is the distance from the source; f and g are arbitrary functions.

cos θ dependence

The general solution of the 3D wave equation with $\cos\theta$ dependence is

$$p'(\mathbf{x},t) = \frac{\partial}{\partial x}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right] = \cos\theta\,\frac{\partial}{\partial r}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right].$$

In a cylindrical duct, radius a_0

$p'(\mathbf{x},t) = e^{i(\omega t + n\theta)} J_n(z_{mn} r/a_0)(Ae^{-ikx_3} + Be^{ikx_3})$, where z_{mn} is the mth zero of $J'_n(z)$ and $k = (k_0^2 - z_{mn}^2/a_0^2)^{1/2}$.

SCATTERING

For an incident plane wave of amplitude p_I, propagating at an angle θ_0, the diffracted pressure a distance r from a sharp edge is

$$p_I\left(\frac{2}{\pi k_0 r}\right)^{1/2}\frac{\sin(\theta_0/2)\sin(\theta/2)}{\cos\theta_0 + \cos\theta}\exp(-ik_0 r - i\pi/4)$$