

ENGINEERING TRIPOS PART IIB

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Thursday 7 May 2009 2:30 to 4

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Module 4B5

NANOTECHNOLOGY

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS

Engineering data book

CUED approved calculator allowed

**You may not start to read the questions  
printed on the subsequent pages of this  
question paper until instructed that you  
may do so by the invigilator**

- 1 (a) In Quantum Mechanics, there are situations when one can represent electrons using plane waves, and others when wave-packets should be used. Under what circumstances are each of these representations appropriate? Discuss the differences between the properties of plane waves and wave-packets. [20%]
- (b) Describe the differences between electromagnetic and matter waves in Quantum Mechanics. [20%]
- (c) We have a transistor where the current flow is driven by a potential difference across the transistor of 2 V. As we shrink the transistor in all dimensions, a number of factors which have a detrimental effect on the transistor's operation come into play. Discuss the latter statement within the context of Quantum Mechanics, with particular reference to characteristic lengthscales. [40%]
- (d) Discuss ways in which these Quantum effects might be reduced in the transistor, and comment on their practicality. [20%]

2 (a) The band-gap of GaAs is 1.45 eV, and of  $\text{Ga}_{1-x}\text{Al}_x\text{As}$  is  $(1.45 + 1.247x)$  eV. For the case of 60% substitution of Al atoms for Ga atoms, sketch the potential as seen by an electron in a 10 nm wide quantum well formed using these materials, indicating which material is used, and where. [20%]

(b) Assuming that the electron effective mass in both materials is 0.067 times the free-electron mass, estimate the energy of the ground state in the well. How accurate do you expect your answer to be? Which assumptions were used in arriving at your estimate, and at which point would they start to break down? [50%]

(c) Sketch the wave-function and the probability density for an electron in the ground state and first excited state of this quantum well. Determine the value of the characteristic decay length of an electron outside the well, assuming the electron is in the ground state. Discuss how to determine the probability of finding the electron outside the well. [30%]

(TURN OVER

3 (a) Write down the Schrödinger equation for a quantum simple harmonic oscillator (QSHO) of stiffness  $k$ . By assuming a power-series solution to this equation, derive an expression for the energy levels of the oscillator, clearly describing the steps involved. [50%]

(b) For the case of a molecule where  $k = 4 \times 10^2 \text{ N.m}^{-1}$  and the mass of the molecule is  $10^{-26} \text{ kg}$ , determine the ground state vibration energy of this molecule in eV. [30%]

(c) Discuss the main differences between the Quantum and Classical descriptions of the simple harmonic oscillator. Under which circumstances do both descriptions converge? [20%]

- 4 (a) In conventional optical microscopy, what is the minimum feature size that can be resolved? What is the Rayleigh criterion? Briefly describe how this limit to resolution can be overcome. [20%]
- (b) Describe the basic principle of operation of Scanning-Probe Microscopes. [20%]
- (c) Sketch the configuration of an atomic force microscope (AFM), and describe the three most commonly used modes in which it can operate. [30%]
- (d) We would like to obtain a high-resolution ( $\sim 1$  nm) map of the electric potential across a working electronic device, which is made of metal. Discuss how to use an AFM for this purpose, and the measurement principle. [30%]

(TURN OVER

5 (a) Consider a potential step from 2 eV down to 1 eV, which is produced in a semiconductor heterostructure. For the ideal case at zero Kelvin, and where there are no defects present in the semiconductor materials, sketch the kinetic energy versus position of an electron at 3 eV that is incident from the left hand side. After the electron has passed the first interface what term can we use to describe it? [20%]

(b) Assuming now that band-bending can occur in the heterostructure, show how a 2-dimensional electron gas (2DEG) arises. Discuss the differences between 3-D and 2-D in quantum systems. [40%]

(c) What property of the 2-dimensional electron gas (2DEG) makes it so desirable from a device standpoint? How would you actually incorporate a 2DEG in a device to improve its operation? [30%]

(d) Briefly comment on the practical applications of semiconductor heterostructures in everyday devices

[10%]

**END OF PAPER**

4B5 2009 Numerical answers

Q2. (b) 56 meV

Q3.(b) 66 meV

May 2009

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