ENGINEERING TRIPOS PART IIA
ENGINEERING TRIPOS PART IIB

Tuesday 5th May 2009 2.30 to 4.00

Module 4C4

DESIGN METHODS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Module 4C4 Data Book (7 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Child-resistant containers are commonly used to store oral medication (both tablets and liquids). The most popular containers currently issued by pharmacies require the patient to push and twist the cap in order to access the medication. However, this can be a difficult task for many older patients, particularly for those with arthritis. You have been commissioned by the UK Department of Health to design a new, more accessible, yet safe container.

(a) Abstract the task to at least four levels and prepare an appropriate solution-neutral problem statement. [10%]

(b) List six key requirements for your new container. [10%]

(c) Establish the overall function for the container. Identify up to four sub-functions and arrange these in a process function structure. [20%]

(d) Describe, with detail of key features, a concept for a new container. [40%]

(e) Describe how the new container might be proven to be fit for purpose. [20%]

2 A soft drinks manufacturer wishes to introduce a novel device into their existing packaging to improve the flavour and appearance of the drink. The device, currently still under development, will need to be inserted into the packaging immediately before filling and sealing the pack.

The manufacturer wishes to launch the new product for the seasonal market in ten months time. In addition, they propose to use their existing filling line to manufacture the product. However, whilst there is space to introduce new equipment, the existing line must be kept operational for one eight hour shift per day.

The current line processes packs in batches of 360 at a time (taken from a pallet) at a rate of one batch every three minutes. The new devices will be supplied to the manufacturer loosely packed in boxes containing 1000 devices. Adjustments will need to be made to the filling machine to accommodate a taller pack and to the check-weighing equipment to accommodate the increase in pack weight.

Discuss a range of project management approaches the developer of the new device and the soft drinks manufacturer might adopt to maximise their chances of delivering the new product to the market on time. [100%]

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3 The main components of a gas spring used in a medical device are an aluminium pressure vessel and a piston sealed by two O-rings, as shown in Fig. 1. During manufacture the pressure vessel is first filled with gas (Fig. 1a), and then the piston is pushed up into the pressure vessel and latched in position (Fig. 1b). Table 1 shows a failure modes and effects analysis (FMEA) of the gas spring.

Fast fracture of the pressure vessel will occur if the stress intensity factor K is equal to the critical stress intensity factor (facture toughness) K_{IC} of the aluminium. K is a function of the pressure p and the maximum size a of cracks in the pressure vessel wall. Both p and a may be assumed to be normally distributed random variables, where the given limits cover six standard deviations. To determine the probability of fast fracture you should assume that K is also normally distributed. Relevant data is given in Table 2.

The probability of an O-ring being faulty is 10^{-2}.

(a) Draw a fault tree for gas leakage from the gas spring. [30%]

(b) What is the probability of gas leakage from a gas spring? [40%]

(c) Comment on the accuracy of your calculation. [10%]

(d) Suggest how the design of the gas spring might be improved to reduce the probability of gas leakage, and comment on the likely effectiveness of the improvements. [20%]

(cont.

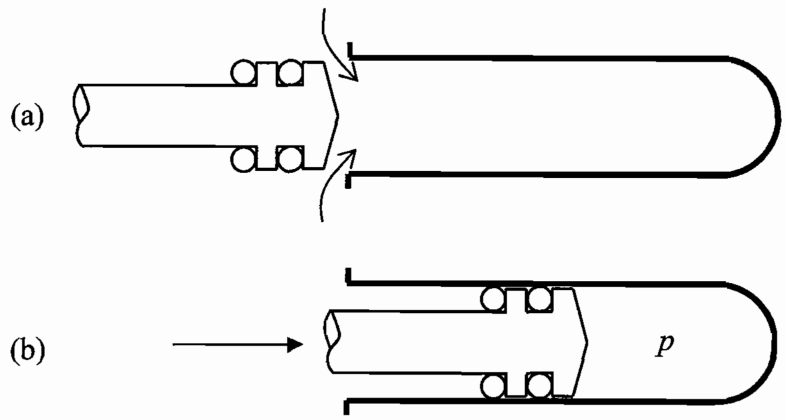

Fig. 1

Gas spring FMEA

Component	Function, states	Failure, modes	Failure causes	Effects on system
Pressure vessel	Holds gas	Fast fracture	Defect in wall too large	Gas leakage
Upper O-ring	Seals piston	O-ring leaks	Faulty O-ring	Lower O-ring takes pressure
Lower O-ring	Back-up for upper O-ring	O-ring leaks	Faulty O-ring	Gas leakage (if upper O-ring has also failed)

Table 1

Design data for gas spring

Property	Formula, value or range
Stress intensity factor	$K = \sigma\sqrt{\pi a}$
Critical stress intensity factor for aluminium	$K_{IC} = 22$ MPa m$^{1/2}$
Fast fracture condition	$K = K_{IC}$
Hoop stress in pressure vessel wall	$\sigma = pr/t$
Pressure vessel radius	$r = 10$ mm
Pressure vessel wall thickness	$t = 1$ mm
Filled pressure	$p = 50 \pm 9$ MPa
Maximum crack size	$a = 0.3 \pm 0.3$ mm

Table 2

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4 A nuclear engineer has been asked to design a flask to contain a certain volume V of some highly radioactive nuclear waste. In order to shield workers adequately the walls of the flask must be of a certain thickness t. The engineer has decided to make the flask cylindrical. The interior cavity in which the waste will be stored will be of radius R and height H, as indicated schematically in Fig. 2.

In order to provide effective shielding the material from which the flask will be made is very dense. The engineer therefore decides to minimize the volume of material used in the construction of the flask. As t is fixed, the design variables are R and H.

(a) Formulate this task as a constrained optimization problem with one equality constraint. [10%]

(b) By using the equality constraint to eliminate H from your expression for the objective function, show that the task can be formulated as an unconstrained univariate minimization problem with an objective function

$$f(R) = 2\pi(R+t)^2 t + V\left[\left(1+\frac{t}{R}\right)^2 - 1\right]$$ [10%]

(c) Estimate, using a Golden Section line search, the value of R that minimizes f for the case where $V = 1000\pi \text{ cm}^3$ and $t = 10 \text{ cm}$. A suitable initial interval for R is between 5 and 10 cm, and the search can be halted when the interval has been reduced four times. [40%]

(d) (i) By using appropriate optimality criteria find an analytical expression in terms of V and t for the value of R that minimizes f.

(ii) Hence find the optimal flask design for the case detailed in (c), and comment on the performance of the Golden Section line search.

(iii) How does the optimal flask design for a given value of V change with the wall thickness t? [40%]

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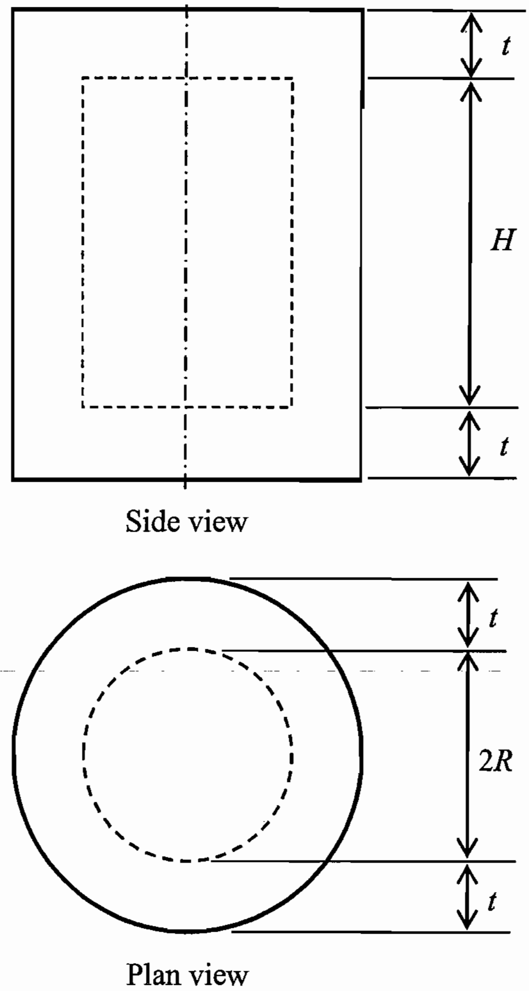

Fig. 2

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1995

(Revised 2001)

(Revised 2002)

(Revised 2003)

(Revised 2006)

(Revised 2007)

MODULE 4C4

DATA BOOK

1.0 OPTIMIZATION

DATA SHEET

1.1 Series

Taylor Series

For a function of one variable:

$$f(x_k+\delta)=f(x_k)+\delta f'(x_k)+\frac{1}{2}\delta^2 f''(x_k)+\ldots \quad \text{where} \quad x_{k+1}=x_k+\delta$$

For a function of several variables:

$$\mathrm{f}(\underline{\mathrm{x}}_k+\underline{\delta\,\mathrm{x}})=\mathrm{f}(\underline{\mathrm{x}}_k)+\{\nabla\mathrm{f}(\underline{\mathrm{x}}_k)\}^t\underline{\delta\,\mathrm{x}}+\frac{1}{2}\underline{\delta\,\mathrm{x}}^t\mathbf{H}(\underline{x}_k)\underline{\delta\,\mathrm{x}}+\ldots \quad \text{where} \quad \underline{\mathrm{x}}_{k+1}=\underline{\mathrm{x}}_k+\underline{\delta\,\mathrm{x}}$$

where $\{\nabla f(\underline{x}_k)\}^t$ is the Grad of the function at $\underline{x}_k$:

$$\begin{bmatrix} \dfrac{\partial f(\underline{x}_k)}{\partial x_1} & \dfrac{\partial f(\underline{x}_k)}{\partial x_2} & \cdots & \dfrac{\partial f(\underline{x}_k)}{\partial x_n} \end{bmatrix}$$

and $\mathbf{H}(\underline{x}_k)$ is the Hessian of the function at $(\underline{x}_k)$:

$$\begin{bmatrix} \dfrac{\partial^2 f(\underline{x}_k)}{\partial x_1^2} & \dfrac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_2} & \cdots & \dfrac{\partial^2 f(\underline{x}_k)}{\partial x_1 \partial x_n} \\ \dfrac{\partial^2 f(\underline{x}_k)}{\partial x_2 \partial x_1} & & & \\ \vdots & & & \\ \dfrac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_1} & \dfrac{\partial^2 f(\underline{x}_k)}{\partial x_n \partial x_2} & \cdots & \dfrac{\partial^2 f(\underline{x}_k)}{\partial x_n^2} \end{bmatrix}$$

Note: 1. $\nabla f(\underline{x}_k)$ is defined as a column vector.
2. The Hessian is symmetric.
3. If $f(\underline{x})$ is a quadratic function the elements of the Hessian are constants and the series has only three terms.

1.2 Line searches

Golden Section Ratio = $\frac{\sqrt{5}-1}{2} \approx 0.6180$

Newton's Method (1D)

When derivatives are available: $x_{k+1} = x_k - \{f'(x_k)\}/\{f''(x_k)\}$

When derivatives are unavailable:

$$x_4 = \frac{1}{2}\frac{\left(x_2^2 - x_3^2\right)f(x_1) + \left(x_3^2 - x_1^2\right)f(x_2) + \left(x_1^2 - x_2^2\right)f(x_3)}{\left(x_2 - x_3\right)f(x_1) + \left(x_3 - x_1\right)f(x_2) + \left(x_1 - x_2\right)f(x_3)}$$

1.3 Multidimensional searches

Conjugate Gradient Method

To find the minimum of the function

$f(\underline{x}) = f(\underline{x}_0) + \nabla f(\underline{x}_0)^t\, \partial\underline{x} + \frac{1}{2}\partial\underline{x}^t \mathbf{H} \partial\underline{x}$, where $\partial\underline{x} = \underline{x} - \underline{x}_0$ and $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$

where $\alpha_k = \frac{-\underline{s}_k^t \nabla f(\underline{x}_k)}{\underline{s}_k^t \mathrm{H} \underline{s}_k}$ (which minimises $f(\underline{x})$ along the defined line)

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \frac{\nabla f(\underline{x}_{k+1})^t \mathbf{H} \underline{s}_k}{\underline{s}_k^t \mathbf{H} \underline{s}_k}$

For a quadratic function, the method converges at $\underline{x}_n$.

Fletcher-Reeves Method

To find the minimum of the function f($\underline{x}$) where $\underline{x}$ has n dimensions:

First move is in direction $\underline{s}_0$ from $\underline{x}_0$ where:

$$\underline{s}_0 = -\nabla f(\underline{x}_0)$$

Then $\underline{x}_{k+1} = \underline{x}_k + \alpha_k \underline{s}_k$ such that f($\underline{x}$) is minimised along the defined line.

Then $\underline{s}_{k+1} = -\nabla f(\underline{x}_{k+1}) + \beta_k \underline{s}_k$

where $\beta_k = \dfrac{(\nabla f(\underline{x}_{k+1}))^2}{(\nabla f(\underline{x}_k))^2}$

For quadratic functions, the method will converge at $\underline{x}_n$. For higher order functions, the method should be restarted when $\underline{x}_n$ is reached.

1.4 Constrained Minimisation

Penalty and Barrier functions

The most common Penalty function is:

$$q(\mu, \underline{x}) = f(\underline{x}) + \frac{1}{\mu}\sum_{i=1}^{p} (\max[0, g_i(\underline{x})])^2$$

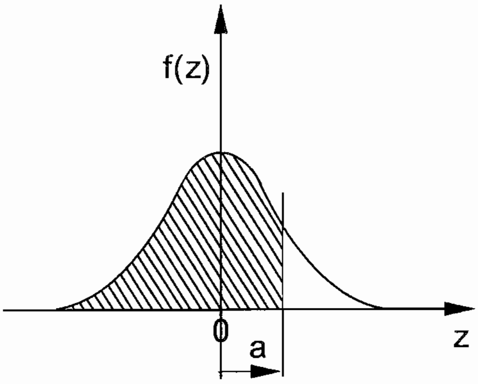
where $f(\underline{x})$ is subject to the constraints $g_1(\underline{x}) \le 0, \ldots, g_p(\underline{x}) \le 0$

A typical Barrier function for the same problem is:

$$q(\mu, \underline{x}) = f(\underline{x}) - \mu \sum_{i=1}^{p} g_i(\underline{x})^{-1}$$

2.0 STATISTICS DATA SHEET

2.1 Standardised normal probability density function



$$P(z<a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a} e^{-\frac{z^2}{2}}\,dz$$

$$z=\frac{x-\mu}{\sigma}$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9349	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9723	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

TABULATED VALUES

2.2 Moments of a randomly distributed variable

Expectation

$$E[g(x)] = \int_{-\infty}^{\infty} g(x) f(x) dx \qquad \text{where} \qquad P(a < x < b) = \int_a^b f_x(x) dx$$

Central and non-central moments

Moment	Definition	Name	Normal Distribution
1^{st} non-central	$E[x] = \mu_x$	Mean	μ
1^{st} central	$E[x - \mu_x] = 0$		0
2^{nd} central	$E\left[(x - \mu_x)^2\right] = \sigma_x{}^2$	Variance	σ^2
3^{rd} central	$E\left[(x - \mu_x)^3\right]$	Skew	0
4^{th} central	$E\left[(x - \mu_x)^4\right]$	Kurtosis	$3\sigma^4$

Due to its symmetry the *odd* central moments of a *normal distribution* are all zero. The *even* central moments of a *normal distribution* are given by:

$$\{\sigma^2,\ 3\sigma^4,\ 3\times 5\sigma^6,\ 3\times 5\times 7\sigma^8,\ 3\times 5\times 7\times 9\sigma^{10}, ...\}$$

$$= \{\sigma^2,\ 3\sigma^4,\ 15\sigma^6,\ 105\sigma^8,\ 945\sigma^{10}, ...\}$$

Relating central and non-central moments

$$E\left[(x - \mu_x)^n\right] = E\left[\sum_{i=0}^{n} \binom{n}{i} x^i (-\mu_x)^{n-i}\right] = \sum_{i=0}^{n} \binom{n}{i} (-\mu_x)^{n-i} E\left[x^i\right]$$

$$E\left[x^n\right] = E\left[((x - \mu_x) + \mu_x)^n\right] = \sum_{i=0}^{n} \binom{n}{i} E\left[(x - \mu_x)^i\right] \mu_x{}^{n-i}$$

where $\quad \binom{n}{i} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

2.3 Combining distributed variables

For the function $y = g(x_1, x_2, \ldots x_n)$

where x_1, x_2 etc. are independent and defined by their respective distributions:

Exact formulae for one and two variables

	y	μ_y	σ_y^2
1	$x + a$	$\mu_x + a$	σ_x^2
2	ax	$a\mu_x$	$a^2\sigma_x^2$
3	$a_1x_1 + a_2x_2$	$a_1\mu_1 + a_2\mu_2$	$a_1^2\sigma_1^2 + a_2^2\sigma_2^2$
4	x_1x_2	$\mu_1\mu_2$	$\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2 + \sigma_1^2\sigma_2^2$
5 (Normal distributions only)	x_1/x_2	μ_1/μ_2	$\frac{1}{\mu_2^2}\left(\frac{\mu_1^2\sigma_2^2 + \mu_2^2\sigma_1^2}{\mu_2^2 + \sigma_2^2}\right)$

Where: μ = mean; σ = standard deviation; a = constant.

Approximate formulae

$$\mu_y \approx g(\mu_1, \mu_2, \ldots) + \frac{1}{2}\left\{\left[\frac{\partial^2 g}{\partial x_1^2}\right]_\mu \sigma_1^2 + \left[\frac{\partial^2 g}{\partial x_2^2}\right]_\mu \sigma_2^2 + \ldots\right\} + \ldots$$

$$\sigma_y^2 \approx \left[\frac{\partial g}{\partial x_1}\right]_\mu^2 \sigma_1^2 + \left[\frac{\partial g}{\partial x_2}\right]_\mu^2 \sigma_2^2 + \ldots$$