ENGINEERING TRIPOS PART IIB

Wednesday 6 May 2009 2.30 to 4

Module 4C6

ADVANCED LINEAR VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

Version 7

1 (a) Sketch a typical arrangement for modal testing of an aerospace structure. Include in your sketch the instrumented impulse hammer, accelerometer, amplifiers, filters and the data acquisition system. Discuss the purpose and function of each of the principal components, and address in detail *two* of the following issues:

(i) the impulse hammer mass and tip stiffness;

(ii) the accelerometer mass, sensitivity and noise floor;

(iii) the charge amplifier gain and high-pass filter;

(iv) the purpose of a low-pass filter;

(v) sampling rate, clipping and quantization error.

Include where appropriate numerical examples to illustrate your answers. [50%]

(b) In a particular experiment an impulse is applied at one point j on the structure and an accelerometer is fixed to another point k. The sampling rate of the data acquisition system is 1000 Hz and there are 8192 data points per channel. Two modes are identified and the following data are obtained:

Mode number n	Frequency (Hz)	Q-factor	$u_j^{(n)} u_k^{(n)}$ (kg^{-1})
1	40	50	2
2	90	100	–5

Draw careful sketches of:

(i) the magnitude of the frequency-response function $H(j,k,\omega)$, which may be assumed to satisfy the formula

$$H(j,k,\omega) = \frac{y_k}{f_j} = \sum_{n=1}^{N} \frac{u_j^{(n)} u_k^{(n)}}{\omega_n^2 + 2i\omega\omega_n\zeta_n - \omega^2}\ ; \qquad [30\%]$$

(ii) the two modal circles. [20%]

2 (a) Describe briefly alternative approaches to the measurement of the damping factor η for different materials. How might you best measure the damping of

(i) an open-celled polymer foam;

(ii) window glass?

For each case comment on the difficulties likely to be encountered, and what steps you would take to get around them. [50%]

(b) A simple model for the bounce and pitch modes of a car is shown in Fig. 1. A uniform rigid beam of length $2L$, mass m and moment of inertia about its centre I is supported at its ends by two springs of stiffness k representing the suspension. Damping is modelled by allowing both spring stiffnesses to be complex with value $k(1+i\eta)$. The loss factor η may be assumed to be small.

In terms of the generalised coordinates z, θ shown in Fig. 1 calculate the mass and stiffness matrices. Write down the two vibration modes for the system in the absence of damping, and find the corresponding natural frequencies. [20%]

Use Rayleigh's principle to calculate approximate expressions for the complex frequencies of these two modes in the presence of the damping, briefly explaining the justification of the method. [30%]

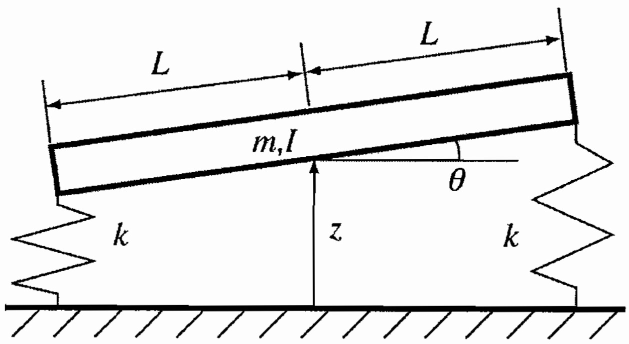

Fig. 1

3 (a) A uniform circular membrane with tension T and mass per unit area m can undergo small transverse vibration with displacement $w(r,\theta,t)$ in terms of plane polar coordinates. The membrane is fixed around the edge so that $w=0$ on $r=a$. The vibration is governed by the differential equation

$$T\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)=m\frac{\partial^2 w}{\partial t^2} .$$

Show how the method of separation of variables can be applied to obtain ordinary differential equations governing the r and θ motion. [35%]

(b) You may assume that the Bessel functions $J_n(z)$ satisfy the equation

$$z^2\frac{\partial^2 J_n}{\partial z^2}+z\frac{\partial J_n}{\partial z}+\left(z^2-n^2\right)J_n=0 .$$

Explain how this allows the radial variation of modes of the membrane to be expressed in terms of Bessel functions, and hence how the natural frequencies of the membrane can be calculated. Using the graphs of the first few Bessel functions shown in Fig. 2, obtain an approximate formula for the lowest natural frequency. [40%]

(c) As $z\to\infty$ the Bessel functions tend to a limiting form:

$$J_n(z)\to\sqrt{\frac{2}{\pi z}}\cos\left(z-\frac{(2n+1)\pi}{4}\right).$$

Deduce an expression for the corresponding limiting form of the natural frequencies of the membrane for a fixed value of n. [25%]

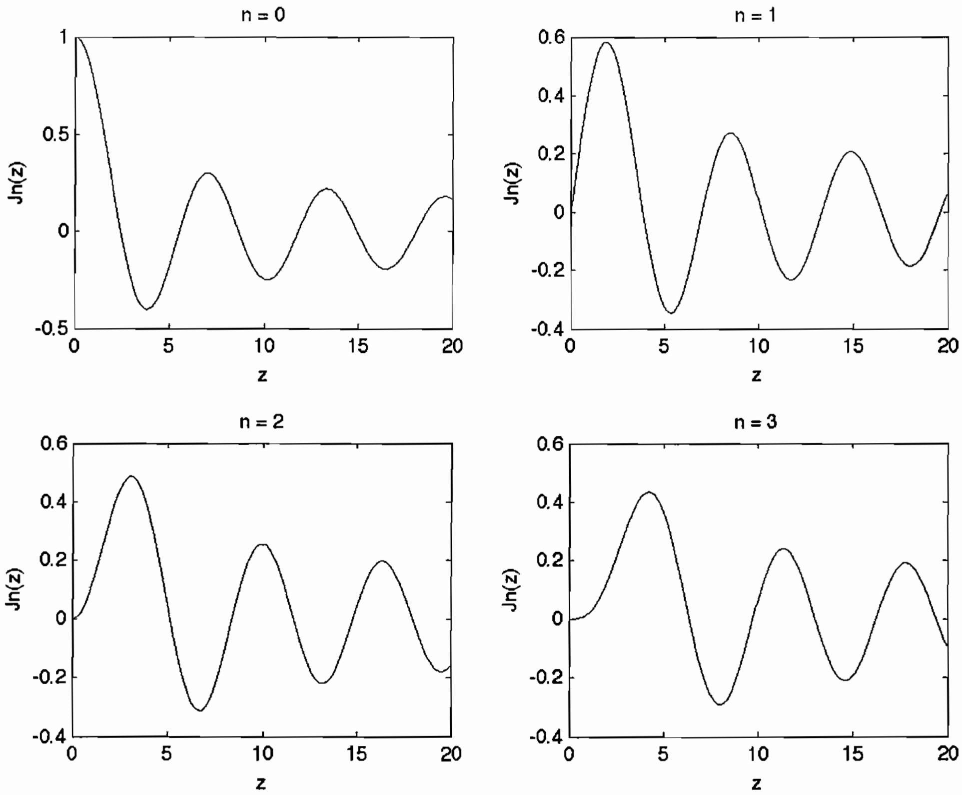

Fig. 2

(TURN OVER

4 (a) What, if anything, can the interlacing theorem say about the following alterations to vibrating systems? Detailed calculations are not required.

(i) A point mass is added to a structure. [20%]

(ii) A point mass is moved from one point to another on a structure. [20%]

(iii) Two cantilever beams lie along the same straight line "head to head", with the free ends adjacent and the clamped ends outermost, as sketched in Fig. 3. Now the tips of the two beams are welded rigidly together. [20%]

(b) A discrete vibrating system with N degrees of freedom is described by a mass matrix M and a stiffness matrix K. One of the generalised coordinates is then constrained to have zero displacement. What is the effect on the matrices M and K? What happens to the natural frequencies of the system? Suggest a way in which this result could be used as part of a computer algorithm to find the natural frequencies of the original system. [40%]

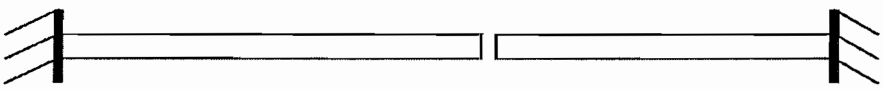

Fig. 3

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