ENGINEERING TRIPOS PART IIB

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Monday 27 April 2009 9 to 10.30

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Module 4C15

MEMS DESIGN

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*There are no attachments to this paper.*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| Single-sided script paper | Engineering Data Book<br>CUED approved calculator allowed |

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

1 (a) Explain briefly the idealisations implicit in a Hertzian analysis of a normally loaded contact between a sphere and a plane. [15%]

(b) The tip of a platinum probe in an Atomic Force Microscope has an effective radius of $140\,\text{nm}$. It is loaded against a plane mica surface by a force $P$ equal to $100\,\text{nN}$.

(i) In the absence of any surface forces, estimate the peak Hertz pressure and the radius of the contact patch. Sketch the way in which the pressure is distributed across the contact patch. [15%]

(ii) The influence of surface effects can be assessed by the numerical value of the Maugis parameter $\lambda$ defined as

$$\lambda = \sigma_0 \left\{ \frac{9R}{2\pi w E^{*2}} \right\}^{1/3}$$

in which $R$ is the reduced radius, $E^*$ the contact modulus, $\sigma_0$ the theoretical junction strength and $w$ is the work of adhesion. For the case of a contact between platinum and mica these latter two quantities take the numerical values $2.05\,\text{GPa}$ and $0.4\,\text{Jm}^{-2}$ respectively. Evaluate $\lambda$. [10%]

(iii) What qualitative effect will the presence of adhesive van der Waal forces have on the contact patch if the normal force is maintained at a value of $100\,\text{nN}$? Under these circumstances, the contact stress distribution can be taken to be equal to the sum of a Hertzian pressure $p(r)$ acting over a patch of radius $a$ together with a distribution $p_a(r)$ which accounts for the phenomenon of adhesion and which extends to a radius $c = a + d$. Provided $d/a \leq 0.2$, a possible approximation for $p_a(r)$ is

$$\left.\begin{aligned} p_a(r) &= -\frac{\sigma_0}{\pi} \arccos\left\{ \frac{a^2 - r^2 - 2ad}{a^2 - r^2 + 2ad} \right\} & r &\leq a \\ p_a(r) &= -\sigma_0 & a &< r \leq c \end{aligned}\right\}$$

Sketch the form of this distribution. [25%]

(iv) Under these circumstances it can be shown that the value of $a$ satisfies the equation

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$$a^3 \frac{4E^*}{3R} = P + 2\sqrt{2\pi E^* w}\; a^{3/2} \; .$$

Evaluate the increase in the radius of the contact patch of the platinum probe which is due to the action of adhesion with the mica surface. [25%]

(v) If the value of the dimension $d$ is given the relation

$$d = \frac{3}{4\lambda^2}\left\{\frac{3\pi w R^2}{4E^*}\right\}^{1/3}$$

then how appropriate are the expressions for $p_a(r)$. [10%]

Mica: $E = 56.5\,\mathrm{GPa}$ $\nu = 0.098$ Platinum: $E = 177\,\mathrm{GPa}$ $\nu = 0.39$

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*Hertzian point contact under load* $P$

Reduced radius $R$ given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

Contact modulus $E^*$ by $\frac{1}{E^*} = \frac{1-\nu_1{}^2}{E_1} + \frac{1-\nu_2{}^2}{E_2}$

Radius of contact circle $a = \left\{\frac{3PR}{4E^*}\right\}^{1/3}$

Maximum contact pressure $p_0 = \frac{3P}{2\pi a^2} = \left\{\frac{6PE^{*2}}{\pi^3 R^2}\right\}^{1/3}$

Mean contact pressure $\bar{p} = \frac{2}{3} p_0$

Approach of distant points $\delta = \frac{a^2}{R} = \left\{\frac{9P^2}{16RE^{*2}}\right\}^{1/3}$

Maximum shear stress is of magnitude $0.31 p_0$ and at depth $0.48a$.

2 A microfluidic device is adapted for the separation of DNA molecules of different sizes in a buffer solution. A schematic top view of the arrangement is shown in Fig. 1 in which $V_1$ to $V_4$ are the associated potentials.

(a) What is electro-osmosis? How is it utilised to pump ionic solutions in glass microchannels? [20%]

(b) The Navier-Stokes equation for electro-osmotic driven fluid flow can be solved to obtain an expression describing 'plug flow' which describes a nearly uniform fluid flow profile through the thickness of the channel under the assumption that the Debye length is much smaller than the channel width. The velocity of the fluid plug can be expressed as

$$U = -\frac{\sigma_w E_x L_D}{\eta}$$

where $U$ is the plug velocity, $\sigma_w$ is the glass wall charge, $E_x$ is applied electric field, $L_D$ is the Debye length and $\eta$ is the solution viscosity.

(i) If the channel width and height are both $100\,\mu\text{m}$ estimate the volumetric flow rate for an ionic solution with Debye length of 1 nm, a wall charge of $0.1\,\text{C}\,\text{m}^{-2}$, an applied electric field of $10^4\,\text{V}\,\text{m}^{-1}$ and solution viscosity equal to $1.5\times10^{-3}\,\text{kg}\,\text{m}^{-1}\text{s}^{-1}$. Are the assumptions for plug flow satisfied? [10%]

(ii) Calculate the time taken for a plug of fluid to travel a distance of 10 mm . [10%]

(iii) Explain by a series of diagrams how this separation may be conducted by switching potentials across ports 1 and 2 to ports 3 and 4 as shown in Fig. 1. [20%]

(c) Explain the principle of electrophoresis. [20%]

(d) Two differentially charged DNA molecules are to be separated by electrophoresis in the separation column. Their electrophoretic mobilities vary by $10^{-9}\,\text{m}^2\text{V}^{-1}\text{s}^{-1}$. Calculate their relative separation distance when the bulk solution has traveled a distance of 1 mm. [20%]

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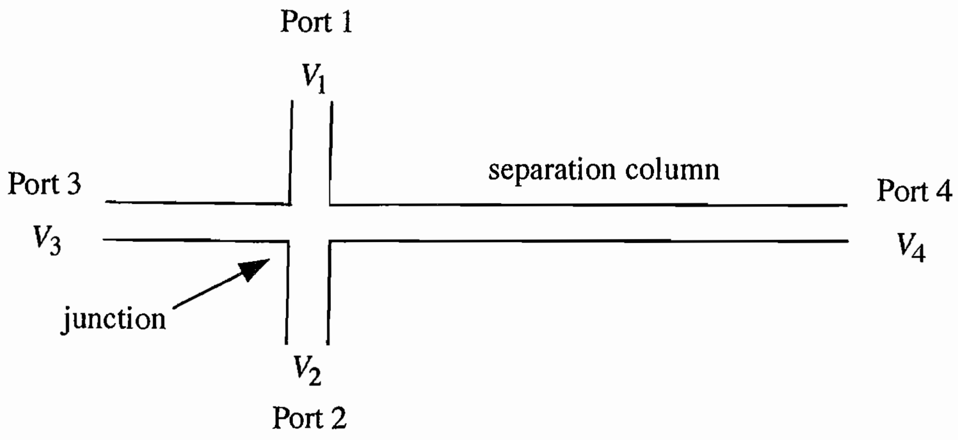

Fig. 1

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3 (a) Explain the usefulness of test structures for MEMS design. What is the M-test? [30%]

(b) An electro-mechanical switch is shown in cross-section in Fig. 2. It comprises a mechanical element restrained by springs and actuated vertically using electrostatic forces. Mechanical limit stops are fabricated as 'dimples' to prevent failure by stiction.

(i) Sketch a process flow to fabricate this device. [20%]

(ii) By treating the actuator as a voltage controlled parallel-plate electrode system, derive an expression for the pull-in voltage of the actuator from first principles. Define carefully any variables you use. [20%]

(iii) Explain qualitatively how the pull-in voltage is influenced by film stress. [10%]

(c) Electro-mechanical switches can be combined to construct simple building blocks representative of mechanical logic. Compare the advantages and disadvantages of logic gates built using electromechanical switches as opposed to wholly electronic devices i.e. transistors. [20%]

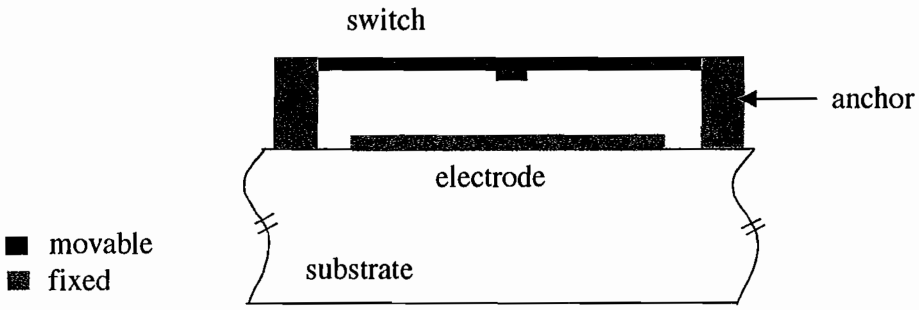

Fig. 2

4 (a) Explain the principle of operation of a micromachined vibratory rate gyroscope. What is *mode matching* and how is it utilised to enhance the sensitivity of micromachined vibratory rate gyroscopes? [20%]

(b) The Analog Devices micromachined vibratory rate gyroscope uses two vibratory masses that are driven out of phase using electrostatic comb drives. From first principles, derive an expression for the electrostatic actuation force generated by a comb drive actuator as a function of applied voltage and geometric parameters. What are the advantages of a comb drive actuator as compared to a parallel plate actuator? Why are two vibratory masses used? [20%]

(c) The Analog Devices gyroscope also utilizes a comb drive structure to sense the position of the vibrating mass. Derive an expression for the capacitive current obtained using a comb drive structure as a function of geometry and applied DC voltage. [20%]

(d) Parallel plate electrodes are utilized to sense the position of the gyroscope proof mass in the sense mode. Obtain a relation between the fractional change in capacitance as a function of the displacement of the proof mass. [10%]

(e) A micromachined vibratory gyroscope is constructed in a surface micromachining process in a polysilicon structural layer of thickness $4\,\mu\text{m}$ with a minimum gap equal to $1\,\mu\text{m}$.

(i) Using the expression for actuator force derived in part (b), derive a relation that minimizes the product of the applied AC and DC voltages to generate vibratory motion in the proof mass at the resonance frequency. You may assume that your system is lightly damped. [15%]

(ii) The gyroscope proof mass of magnitude $1\,\mu\text{g}$ is actuated to a displacement of $5\,\mu\text{m}$ at a frequency of 10 kHz. Estimate the magnitude of the Coriolis force to be measured for an applied rotation rate of $1\,\text{rad}\,\text{s}^{-1}$. [15%]

**END OF PAPER**