ENGINEERING TRIPOS PART IIB

Monday 4 May 2009 2.30 to 4

Module 4C16

ADVANCED MACHINE DESIGN

*Answer **two** questions from Section A and **one** question from Section B.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Data sheet (3 pages).

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book
	CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 (a) Two parallel plates are separated by a thin, incompressible fluid film of thickness h and viscosity η as indicated in Fig. 1(a). The upper plate is at rest and the lower moves with speed U in the x-direction. The plates are sufficiently wide for there to be no fluid flow in the y-direction. The distribution of fluid velocity $u(z)$ is given by

$$\frac{u(z)}{U} = \frac{1}{2\eta U}\frac{\mathrm{d}p}{\mathrm{d}x}z(z-h)+\left(1-\frac{z}{h}\right)$$

where $\frac{\mathrm{d}p}{\mathrm{d}x}$ is the pressure gradient within the fluid in the x-direction. Explain briefly the physical significance of each of the two terms on the right-hand side of this expression.

[10%]

(b) The stationary pad of an infinitely wide, fixed inclination, plain bearing has the form of a step of height d with parallel inlet and outlet regions each of length $B/2$ as shown in Fig. 1(b). The speed of the lower plane surface is U, the bearing is supplied with lubricant of viscosity η and the outlet film thickness is h_0.

(i) By considering continuity of flow, or otherwise, show that the load supported per unit width P' is given by the expression

$$P' = \frac{3\eta B^2 U}{2}\frac{d}{(h_0+d)^3+h_0{}^3}\,.$$

[50%]

(ii) If $d/h_0 = 0.68$, derive an expression for u/U as a function of z/h_0 and hence sketch the way in which fluid velocity varies with z in the outlet section of the bearing, i.e. where $h = h_0$. [40%]

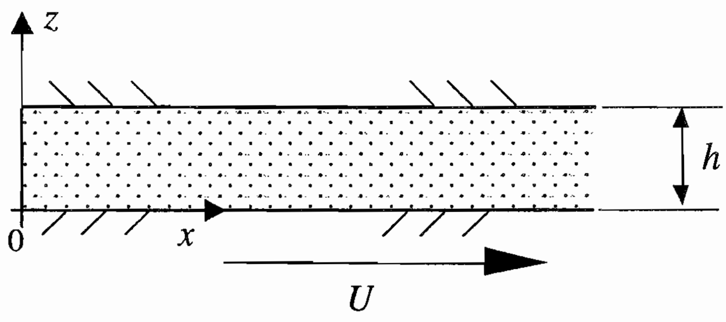

Fig. 1(a)

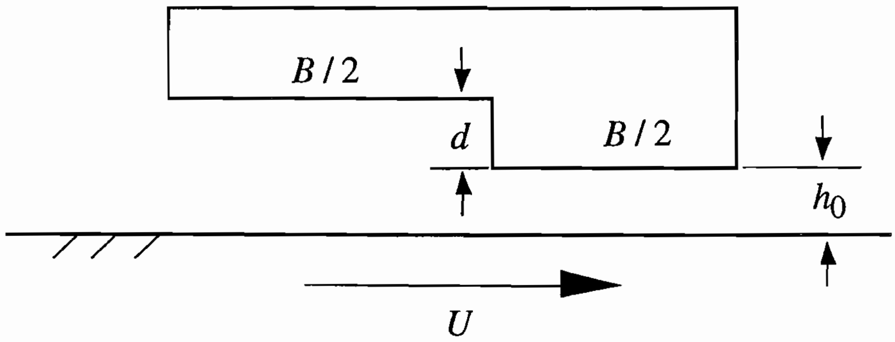

Fig. 1(b)

(TURN OVER

2 Figure 2, not drawn to scale, shows a cam-operated mechanism to open a trap door releasing sweets from a vending machine. The cam has equal tip and base circle radii r, with a tip circle centre offset $3r/2$. The base and tip circles are connected by straight flanks. The trap door pivots about point A, which is at the same height as the centre O of the cam base circle and a distance $3r/\sqrt{2}$ away from O. A spring, not shown, keeps the trap door in contact with the cam. The top edge of the trap door has a circular arc of radius $r/2$, whose centre is a distance $3r/2$ from A. In the closed position as shown by the solid outline of the door, the trap door makes contact with the cam on the base circle and the door is at an angle of 45° to the horizontal. The door is opened by rotating the cam anti-clockwise, with the position of the cam defined by the angle ϕ between the horizontal and the line connecting the tip and base circles of the cam. In the fully open position the trap door is vertical, as shown by the dashed outline of the door. Note that the cam does not undergo complete rotation.

(a) Sketch equivalent mechanisms for:

(i) just after the trap door begins to open; [10%]

(ii) when the trap door is in the fully open position. [10%]

(b) Find the cam orientation angle ϕ in the fully open position. [20%]

(c) Consider the point when the trap door just begins to open. Assuming that the cam is rotating at a constant angular velocity ω at this point find the angular velocity and angular acceleration of the trap door. [60%]

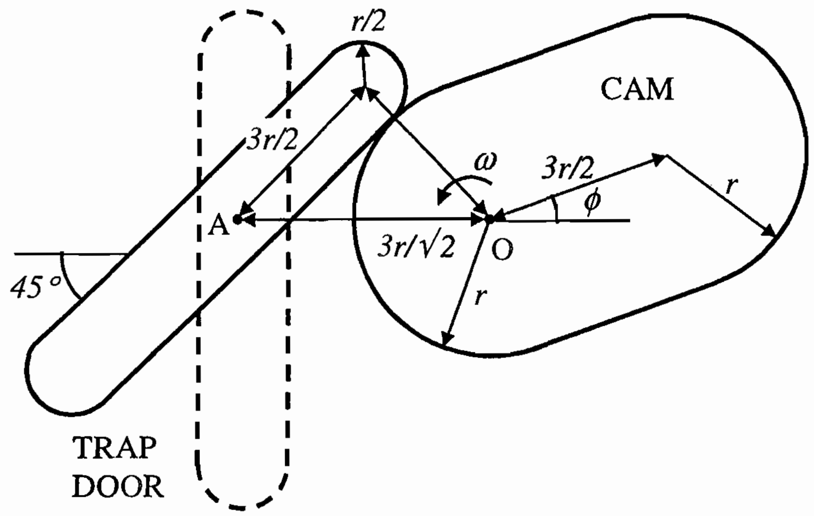

Fig. 2

3 The force F required to move an urban delivery vehicle can be idealised as $F = ma + r$, where m is the vehicle mass, a is the acceleration and r is a constant resistance force that opposes motion. The operating cycle of the vehicle is shown in Fig. 3 and consists of constant acceleration for time T to speed $v=U$, constant speed U for time wT, then constant deceleration for time T to zero speed $v=0$.

(a) Sketch a graph of the power required to drive the vehicle as a function of time. [15%]

(b) Show that the ratio of peak power to mean power is

$$\frac{P_{\text{peak}}}{P_{\text{mean}}} = \left(1 + \frac{mU}{rT}\right)\left(\frac{2+w}{1+w}\right)$$

and use this expression to suggest the conditions for which a hybrid drive would be most beneficial. [40%]

(c) The vehicle is equipped with a hybrid drive consisting of a constant power source (supplying the mean power P_{mean}) and an energy storage device. There is full recovery of the vehicle's kinetic energy and there are no losses in the storage device.

(i) Sketch the energy in the storage device as a function of time. [10%]

(ii) Derive an expression for the minimum energy required in the storage device at the beginning of the operating cycle to ensure that the energy store is not depleted. [35%]

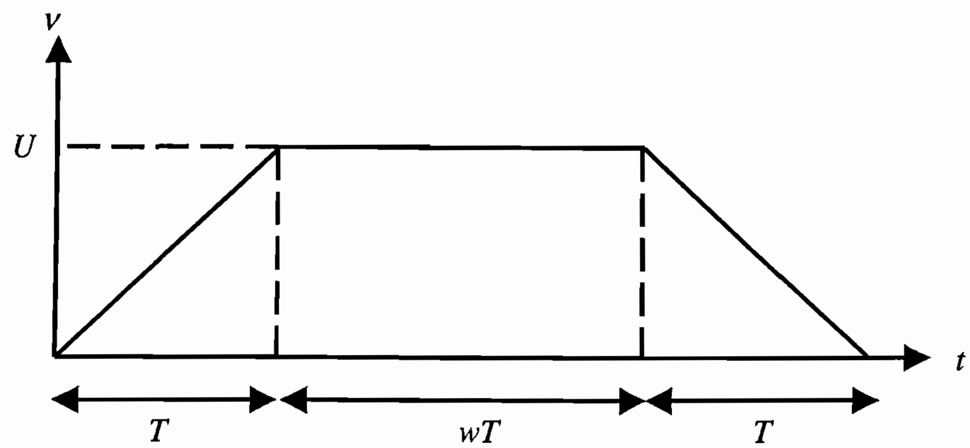

Fig. 3

(TURN OVER

SECTION B

4 You are a member of a team designing a large mechanical assembly which contains a number of high speed thrust and journal bearings: typical bearing radii R are 500 mm. On the basis of the paper 'Tribological Scaling' R I Taylor, (2004) Proc. Instn. Mech. Engrs., 221, Part J, Engng Tribology, 413-425, your colleagues suggest that potential tribological designs can be validated with small-scale tests in which dynamical similarity with the full-scale design is maintained.

(a) Do you feel as confident as they do of this approach? What difficulties might be encountered in relying on tests at, say, one tenth full size? [50%]

(b) What additional investigations would you recommend? It can be assumed that you have access to samples of the correct materials. [50%]

For thrust bearings $W*$ is defined as $W* = \dfrac{\bar{p}}{\eta U}\dfrac{h_0^2}{B}$ while for journals $W* = \dfrac{\bar{p}}{\eta\omega}\left(\dfrac{c}{R}\right)^2$ where $\bar{p}$ is the nominal pressure on the contact, B is a typical bearing linear dimension, R is journal radius, U and ω are sliding and rotation speeds, c and h_0 are radial clearance and minimum film thickness respectively and η is lubricant viscosity.

5 The paper 'Modelling and analysis of a high-speed circuit breaker mechanism with a spring-actuated cam', KY Ahn and SH Kim (2001), Proc Inst Mech Engrs, **215**, Part C, 663-672 describes the design of a cam mechanism for a circuit breaker. Use their model and results as a basis for discussion of the following points related to cam design.

(a) Ahn and Kim pay particular attention to modelling of friction associated with the camshaft rotation. Discuss why friction should affect the lift curve, and how the model for such a frictional element would depend on the details of the mechanism. [35%]

(b) What factors need to be taken into account to optimise the lift curve for practical high-speed cams? [35%]

(c) Cam mechanisms commonly contain springs. What factors govern the choice of spring element? [30%]

6 The paper 'Power management strategy for a parallel hybrid electric truck' C-C Lin, H Peng, JW Grizzle, J-M Kang, IEEE Transactions on Control Systems Technology, vol. 11, no. 6, Nov. 2003, p839-849, describes the development of a power management strategy for a parallel hybrid truck. With appropriate references to this work, discuss the following points related to the design of hybrid drives.

(a) What are the main objectives of the power management strategy of a parallel hybrid vehicle? What are the problems associated with implementing an ideal strategy, and what approaches can be taken to overcome them? [50%]

(b) The vehicle studied in the paper used a lead-acid battery for energy storage. What other storage devices might be considered? What criteria should be used in selecting an energy storage device? In terms of these criteria and the truck application, how does a lead-acid battery compare to other storage devices? [50%]

END OF PAPER

ENGINEERING TRIPOS Part IIB

Module 4C16 Data Sheet

HYDRODYNAMIC LUBRICATION

Viscosity: temperature and pressure effects

$\eta = \eta_0$ at $p = 0$ and $T = T_0$

Vogel formula $$\eta = \eta_0 \exp\left\{\frac{b}{T + T_c}\right\}$$

Barus equation $$\eta = \eta_0 \exp\{\alpha p\}$$

Roelands equation $$\eta = \eta_0 \exp\left\{\ln\left(\frac{\eta_0}{\eta_r}\right)\left[\left(1 + \frac{p}{p_r}\right)^\beta \left(\frac{T_0 + T_r}{T + T_r}\right) - 1\right]\right\}$$

p_r, T_r and η_r are reference values

Viscous pressure flow

Rate of flow q_x per unit width of fluid of viscosity η down a channel of height h due to pressure gradient $\frac{\mathrm{d}p}{\mathrm{d}x}$

$$q_x = -\frac{h^3}{12\eta}\frac{\mathrm{d}p}{\mathrm{d}x}$$

Reynolds' Equation for a steady configuration

1–D flow: $$\frac{\mathrm{d}p}{\mathrm{d}x} = 12\eta\overline{U}\left\{\frac{h - h^*}{h^3}\right\}$$

$\overline{U}$ is the entraining velocity so that $|\overline{U}h^*|$ is flow per unit width through the contact.

2–D flow: $$\frac{\partial}{\partial x}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial y}\right\} = 12\overline{U}\frac{\partial h}{\partial x}$$

ELASTIC CONTACT STRESS FORMULAE

Suffixes 1, 2 refer to the two bodies in contact.

Effective curvature $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

where R_1, R_2 are the radii of curvature of the two bodies (convex positive).

Contact modulus $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

where E_1, E_2 and ν_1, ν_2 are Young's moduli and Poisson's ratios.

	Line contact (width $2b$; load W' per unit length)	Circular contact (diameter $2a$; load W)
Semi contact width or contact radius	$b = 2\left\{\frac{W'R}{\pi E^*}\right\}^{1/2}$	$a = \left\{\frac{3WR}{4E^*}\right\}^{1/3}$
Maximum contact pressure ("Hertz stress")	$p_0 = \left\{\frac{W'E^*}{\pi R}\right\}^{1/2}$	$p_0 = \frac{1}{\pi}\left\{\frac{6WE^{*2}}{R^2}\right\}^{1/3}$
Approach of centres	$\delta = \frac{2W'}{\pi}\left[\frac{1-\nu_1^2}{E_1}\left\{\ln\left(\frac{4R_1}{b}\right)-\frac{1}{2}\right\} + \frac{1-\nu_2^2}{E_2}\left\{\ln\left(\frac{4R_2}{b}\right)-\frac{1}{2}\right\}\right]$	$\delta = \frac{a^2}{R} = \frac{1}{2}\left\{\frac{9}{2}\frac{W^2}{E^{*2}R}\right\}^{1/3}$
Mean contact pressure	$\bar{p} = \frac{W'}{2b} = \frac{\pi}{4}p_0$	$\bar{p} = \frac{W}{\pi a^2} = \frac{2}{3}p_0$
Mean shear stress	$\tau_{max} = 0.30p_0$ at $x = 0, z = 0.79b$	$\tau_{max} = 0.31p_0$ at $r = 0, z = 0.48a$ for $\nu = 0.3$
Maximum tensile stress	zero	$\frac{1}{3}(1-2\nu)p_0$ at $r = a, z = 0$

Mildly elliptical contacts

If the gap at zero load is $h = \frac{1}{2}Ax^2 + \frac{1}{2}By^2$, and $0.2 < A/B < 5$

Ratio of semi-axes $b/a \cong (A/B)^{2/3}$

To calculate the contact **area** or Hertz **stress** use the circular contact equations with $R = (AB)^{-1/2}$ or better $R_e = [AB(A+B)/2]^{-1/3}$.

For **approach** use circular contact equation with $R = (AB)^{-1/2}$ (**not** R_e)

ELASTOHYDRODYNAMIC LUBRICATION

Formulae for line contact film thickness

$\bar{U}$ is the entraining velocity, R is the effective radius of curvature and E^* is the contact modulus (see elastic contact stress formulae).

Rigid isoviscous (Kapitza)

$$h_c = 4.9 \frac{\bar{U}\eta_0 RL}{W}$$

Ertel-Grubin

$$\frac{\bar{h}}{R} = 1.37 \left(\frac{\eta_0 \alpha (2\bar{U})}{R} \right)^{3/4} \left(\frac{E^* RL}{W} \right)^{1/8}$$

Dowson and Higginson

$$\frac{\bar{h}}{R} = 1.6 \left(2\alpha E^*\right)^{0.54} \left(\frac{\bar{U}\eta_0}{2E^* R} \right)^{0.7} \left(\frac{W/L}{2E^* R} \right)^{-0.13}$$

EPICYCLIC SPEED RULE

$$\omega_s = (1+R)\omega_c - R\omega_a \quad \text{where} \quad R = \frac{A}{S}$$

MPFS, DJC, JAW
January 2009

Engineering Tripos Part IIB 2009

Module 4C16 Advanced Machine Design

Answers

1. b) ii) $\frac{u}{U} = -0.355\frac{z}{h_0}\left(\frac{z}{h_0}-1\right)+1-\frac{z}{h_0}$

2. b) $\phi = \pi - \cos^{-1}\left(\sqrt{2}-1\right) \approx 2.00\text{rad}$

c) angular velocity = 0

angular acceleration = ω^2 anticlockwise

3. c) ii) $E_{start} = \frac{1}{2}mU^2 + rUT\left(\frac{1+w}{2+w}-\frac{1}{2}\right)$