ENGINEERING TRIPOS PART IIB

Monday 27 April 2009 9 to 10.30

Module 4D6

DYNAMICS IN CIVIL ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments: 4D6 Data sheets (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A cantilever beam with flexural stiffness EI of 8×10^6 Nm^2 and a self weight of 200 kgm^{-1} is shown in Fig. 1. This beam is expected to undergo flexural vibrations with a mode shape as follows:

$$\bar{u}_n = 1 - \cos \frac{n \pi x}{12}$$

where n indicates the mode of vibration. Determine the first and second mode natural frequencies for this beam. [30%]

(b) Sketch the first and second mode shapes of flexural vibration for this beam and comment on their suitability. [10%]

(c) An impact load $F(t)$ is applied at the tip of the beam as shown in Fig. 1. The variation of the impact load with time is shown in Fig. 2. Calculate the peak displacement of the tip of the beam using the SRSS method. [30%]

(d) Define the term 'ductility factor' for an inelastic structure. By comparing deformation energies in an elastic structure and an elasto-plastic structure, show that the force in the elastic system F_{el} is related to the force at first yield F_y in the elasto-plastic system by:

$$F_y = \frac{F_{el}}{\sqrt{2\mu - 1}}.$$

[30%]

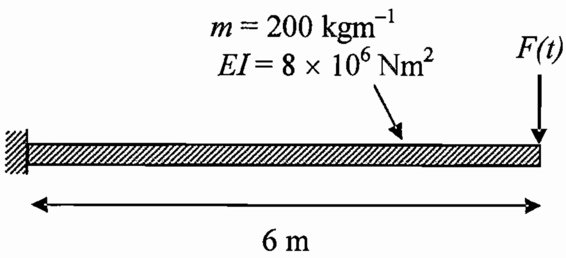


Fig. 1

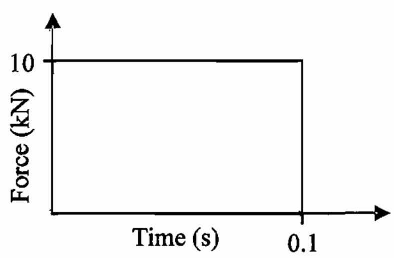


Fig. 2

2 Figure 3 shows a sway frame model of a building constructed on a slope. The building can be modelled as consisting of rigid beams and light flexible columns. All columns have flexural rigidity EI = 10 MNm^2 and the two floors each have a mass of 1000 kg.

(a) Find the mode shapes and corresponding natural frequencies of the frame. [40%]

(b) The building experiences earthquake loading which can be idealised as a triangular pulse of ground acceleration, as shown in Fig. 4. Estimate the maximum lateral displacement of the top storey relative to the ground for each mode and hence estimate the combined response. [60%]

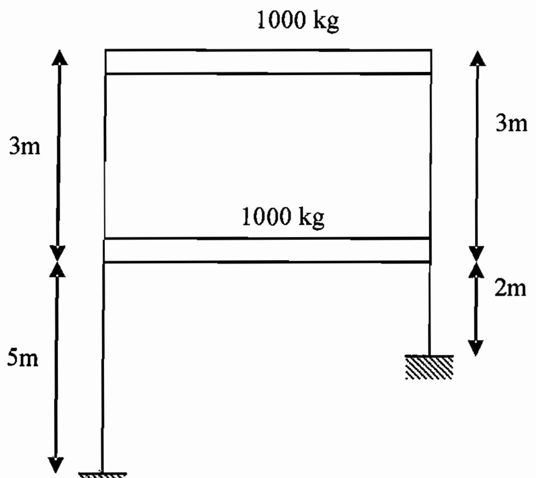


Fig. 3

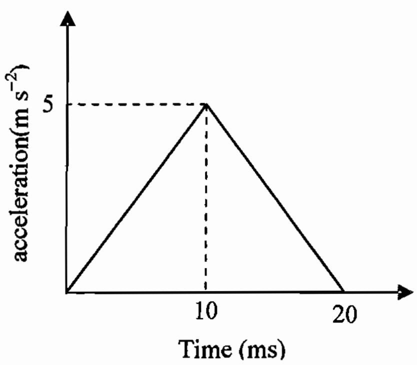


Fig. 4

(TURN OVER

3 (a) Explain why 'soil liquefaction' can occur in loose, saturated, sandy soils but not in dense, saturated, sandy soils that are subjected to earthquake loading. What type of damage can soil liquefaction cause in civil engineering structures? [15%]

(b) The pad foundation for a column carrying an axial load of 270 kN has the dimensions of 3 m × 3 m × 0.5 m. The foundation is to be embedded to a depth of 2 m below ground surface. The site has a loose, sandy layer with a saturated unit weight of 18.5 kN m^{-3} and a void ratio of 0.85. The Poisson's ratio of sand may be taken as 0.3. The water table at this site is at the ground surface. By considering a reference plane 2 m below the underside of the pad foundation, calculate the horizontal and rotational stiffness offered by the foundation. [30%]

(c) A simplified structural model for the column in Part (b) above assumes that the loading from the building is concentrated as a lumped mass of 27,000 kg at a height of 2 m above ground surface. Estimate the natural frequency of the soil-column system for horizontal and rocking vibrations. [25%]

(d) During a strong earthquake event the natural frequency of the soil-column system for rocking vibration is expected to decrease by 75% of the value obtained in Part (c) above. Calculate the natural frequency for horizontal vibrations during this earthquake. Comment on the vulnerability of the soil-column system to excessive horizontal and rocking vibrations during this strong earthquake. [30%]

4 (a) In the wind engineering approach to the dynamic response of a flexible structure to the randomly-varying gustiness of the wind, the design value X_{des} of the displacement may be expressed as

$$X_{des} = \overline{X} + g_f \sigma_X$$

where $\overline{X}$ is the static displacement due to the time-average wind forces, g_f is a peak factor and σ_X is the standard deviation of the time-varying displacement.

(i) Assuming that the structure moves as a rigid body on elastic supports, explain how σ_X may be estimated, starting from the power spectrum S_{UU} of the incident wind velocity U. [30%]

(ii) Describe shortcomings and assumptions of the overall approach. [10%]

(iii) Explain how and why the method differs from the Response Spectrum approach used in earthquake engineering. [10%]

(iv) Describe briefly what further steps would be required to extend the method to more generally flexible structures. [10%]

(b) Describe some of the strategies that might be adopted to design blast-resistant buildings. [40%]

END OF PAPER

Module 4D6: Dynamics in Civil Engineering

Data Sheets

Approximate SDOF model for a beam

for an assumed vibration mode $\bar{u}(x)$, the equivalent parameters are

$$M_{eq} = \int_0^L m\,\bar{u}^2 dx \qquad K_{eq} = \int_0^L EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx \qquad F_{eq} = \int_0^L f\,\bar{u}dx + \sum_i F_i\,\bar{u}_i$$

Frequency of mode $u(x,t) = U\sin\omega t\,\bar{u}(x)$ $\quad f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M_{eq}}} \qquad \omega = 2\pi f$

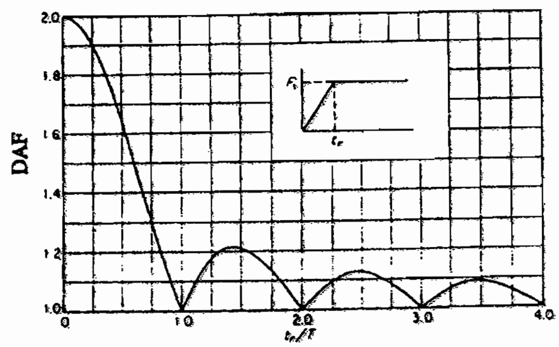
Modal analysis of simply-supported uniform beams

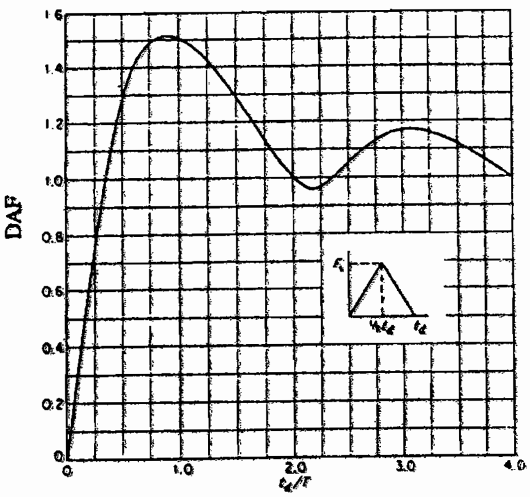
$$u_i(x) = \sin\frac{i\pi x}{L} \qquad M_{i\,eq} = \frac{mL}{2} \qquad K_{i\,eq} = \frac{(i\pi)^4 EI}{2L^3}$$

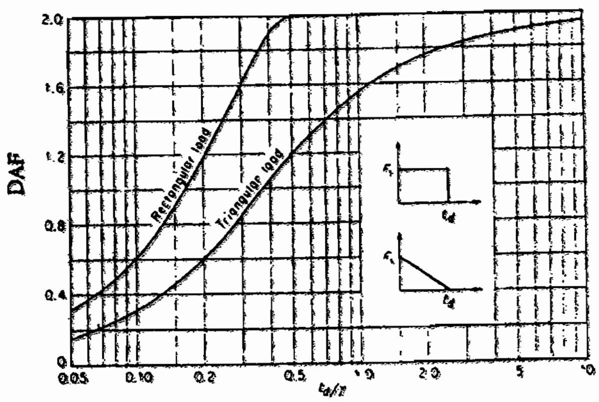
Ground motion participation factor

$$\Gamma = \frac{\int m\bar{u}dx}{\int m\bar{u}^2 dx}$$

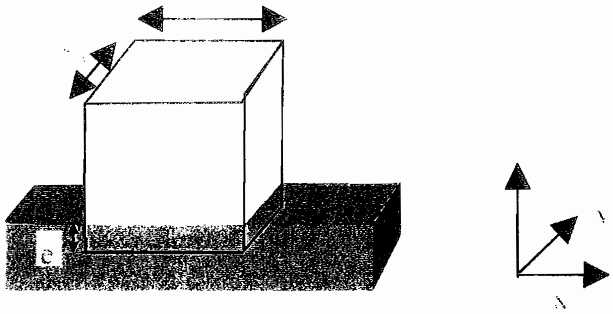
Dynamic amplification factors







Approximate relations for evaluating soil stiffness for an embedded prismatic structure of dimensions $2l$ and $2b$, embedded to a depth e are:



$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 2.4\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{hy} = \frac{Gb}{2-\nu}\left[6.8\left(\frac{l}{b}\right)^{0.65} + 0.8\frac{l}{b} + 1.6\right]\left[1 + \left(0.33 + \frac{1.34}{1+\frac{l}{b}}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{v} = \frac{Gb}{2-\nu}\left[3.1\left(\frac{l}{b}\right)^{0.75} + 1.6\right]\left[1 + \left(0.25 + \frac{0.25b}{l}\right)\left(\frac{e}{b}\right)^{0.8}\right]$$

$$K_{rx} = \frac{Gb^3}{1-\nu}\left[3.2\frac{l}{b} + 0.8\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\frac{l}{b}}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(\frac{l}{b}\right)^{2.4} + 0.27\right]\left[\left(1 + \frac{e}{b} + \frac{1.6}{0.35+\left(\frac{l}{b}\right)^4}\left(\frac{e}{b}\right)^2\right)\right]$$

$$K_{tor} = Gb^3\left[4.25\left(\frac{l}{b}\right)^{2.45} + 4.06\right]\left[\left(1 + \left(1.3 + 1.32\frac{b}{l}\right)\left(\frac{e}{b}\right)^{0.9}\right)\right]$$

Unit weight of soil:

$$\gamma = \frac{(G_s + eS_r)\gamma_w}{1+e}$$

where e is the void ratio, S_r is the degree of saturation, G_s is the specific gravity of soil particles.

For dry soil this reduces to

$$\gamma_d = \frac{G_s \gamma_w}{1 + e}$$

Effective mean confining stress

$$p' = \sigma'_v \frac{(1 + 2K_o)}{3}$$

where σ'_v is the effective vertical stress, K_o is the coefficient of earth pressure at rest given in terms of Poisson's ratio ν as

$$K_o = \frac{\nu}{1 - \nu}$$

Effective stress Principle:

$$p' = p - u$$

Shear modulus of sandy soils can be calculated using the approximate relation:

$$G_{max} = 100 \frac{(3 - e)^2}{(1 + e)} (p')^{0.5}$$

where p′ is the effective mean confining pressure in **MPa,** e is the void ratio and Gmax is the small strain shear modulus in **MPa**

Shear modulus correction for strain may be carried out using the following expressions;

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

where

$$\gamma_h = \frac{\gamma}{\gamma_r} \left[1 + a \cdot e^{-b\left(\frac{\gamma}{\gamma_r}\right)} \right]$$

'a' and 'b' are constants depending on soil type; for sandy soil deposits we can take

$$a = -0.2 \ln N$$

$$b = 0.16$$

where N is the number of cycles in the earthquake, γ is the shear strain mobilised during the earthquake and γ_r is reference shear strain given by

$$\gamma_r = \frac{\tau_{\max}}{G_{\max}}$$

where

$$\tau_{\max} = \left[\left(\frac{1 + K_o}{2}\sigma_v' \sin\phi'\right)^2 - \left(\frac{1 - K_o}{2}\sigma_v'\right)^2\right]^{0.5}$$

Shear Modulus is also related to the shear wave velocity v_s as follows;

$$v_s = \sqrt{\frac{G}{\rho}}$$

where G is the shear modulus and ρ is the mass density of the soil.

Natural frequency of a horizontal soil layer f_n is;

$$f_n = \frac{v_s}{4H}$$

where v_s is shear wave velocity and H is the thickness of the soil layer.

SPGM
January, 2006