

ENGINEERING TRIPOS PART IIB

Monday 4 May 2009 9 to 10:30

Module 4D7

CONCRETE AND MASONRY STRUCTURES

Answer not more than three questions.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

*Attachments: (i) Concrete and Masonry Structures: Formula and Data Sheet (4 pages).
(ii) The Cumulative Normal Distribution Function (1 page).*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved

**You may not start to read the questions
printed on the subsequent pages of this
question paper until instructed that you
may do so by the Invigilator**

1 (a) On completion, the Burj Dubai will be the world's tallest building at over 800 m in height. The building is primarily a reinforced concrete frame structure.

Briefly discuss the Ultimate Limit States (ULS) and Serviceability Limit States (SLS) you would wish to consider when designing this structure. What considerations might you take into account when specifying the concrete mix for this building? Would you expect the mix to differ between different structural elements and different levels? If so, in what way? What key features could you include in your design to enhance the robustness and reduce the chance of disproportionate collapse? [30%]

(b) A reinforced concrete beam spanning 8 m between fully fixed supports is loaded at mid-span by a point load P with mean value 3 kN. The flexural strength of the beam in both hogging and sagging is everywhere assumed to be fully correlated and have a mean value of 6 kNm. Self-weight of the beam can be ignored. Assume the partial safety factors on material and load at ULS are $\gamma_m = 1.3$ and $\gamma_f = 1.2$ respectively.

Initially assume that load effect and resistance are normally distributed with both having coefficient of variation of 0.1.

(i) Determine the characteristic load effect and design load effect, and the characteristic bending strength and design bending strength for this beam. [10%]

(ii) Determine the reliability index, β , and hence the probability of failure for this beam. [10%]

(iii) Would this beam be considered safe in flexure? Justify your answer. [5%]

Further surveys indicate that the load effect would be better represented by a triangular probability density function rising linearly from zero at a load effect of 1 kNm to a maximum at the mean value $S_m = 3$ kNm and then falling linearly to zero at a load effect of 5 kNm. Similarly the strength probability density function rises linearly from zero at 4 kNm to a maximum at 6 kNm before falling linearly to zero at a strength of 8 kNm.

(iv) Write the convolution integral and hence find the probability of failure of this beam under the new loading and strength models. [45%]

- 2 (a) (i) Cement replacement materials offer the design engineer the scope to provide a variety of enhanced properties in a concrete mix design. List three cement replacement materials and explain the origin of each. [10%]
- (ii) There are a wide variety of specialist cements available in the construction industry. Name up to 4 different types of cement and detail the different properties of the concrete produced using each when compared to mix designs which only contain Ordinary Portland Cement (OPC). How are these changes achieved? [15%]
- (iii) Why is gypsum added in the final stages of production of OPC? [5%]
- (iv) Tricalcium aluminate (C_3A) can have detrimental effects on concrete. What are these and how can they be mitigated? [10%]

(b) A new reinforced concrete bridge is being considered for inclusion in a Public Finance Initiative (PFI) scheme and the contractor wishes to evaluate the net cost of including this structure in the project.

The estimated capital cost of constructing the bridge is £2.5M. The expected maintenance regime envisages renewing the waterproofing and applying silane treatment every 15 years at an estimated cost of £250k each time. Major patch repairs are expected every 25 years at an estimated cost of £100k each time. Provision has been made for cathodic protection (CP) to be installed at the start at an additional capital cost of £50k. Once installed the CP costs £1k per annum to operate.

The income stream from tolls on this bridge is estimated to generate a positive cash flow of £100k per annum over the entire life of the structure.

Assume a discount rate of 3% per annum, and apply continuous discounting where appropriate. The bridge design life is 75 years. All costs are in 2009 prices.

Using Whole Life Costing (WLC) principles calculate the net present value of this proposed bridge and advise whether on the basis of economic criteria the contractor should include it in the PFI scheme. [60%]

(TURN OVER

3 A reinforced concrete cross-section has a width of 200 mm and an effective depth of 400 mm. The top steel reinforcement consists of two 12 mm diameter bars and the bottom steel reinforcement consists of three 20 mm diameter bars, as shown in Fig. 1. The steel reinforcement has a Young's modulus of 210 GPa, and factored design yield stress of $f_{yd} = 400$ MPa. The concrete has a Young's modulus of 30 GPa and a factored design compressive cube strength of $f_{cd} = 25$ MPa. The concrete is assumed to fail in compression at a uniform stress of $0.6 f_{cd}$. The concrete tensile strength is 4 MPa. Assume the section is bending about its major axis.

(a) Evaluate the uncracked second moment of area of the section by transforming the steel to concrete. Find the bending moment at first cracking. [30%]

(b) Assuming the concrete carries no tension but is linear-elastic in compression, and that the reinforcing steel exhibits linear-elastic behaviour, find the cracked second moment of area. [25%]

(c) Assuming the compressive steel yields, find the ultimate moment capacity of the cross-section. [20%]

(d) Based on your answers in parts (a-c), plot a schematic moment-curvature diagram for the behaviour of the cross-section, identifying salient values and regions of interest. [25%]

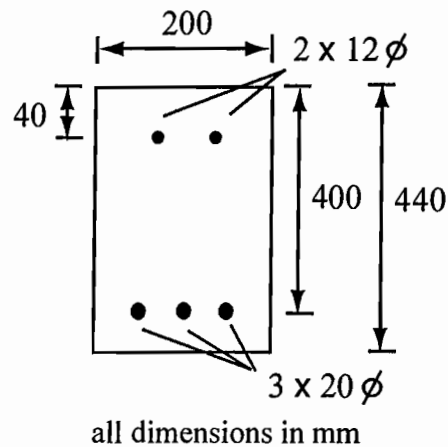


Fig. 1

4 (a) Describe the main features of a 'variable angle truss analogy' when applied to the design of the internal transverse reinforcing steel in a reinforced concrete beam subjected to:

- (i) vertical shear; or
- (ii) torsion.

For each case, include free body diagrams indicating how the force is assumed to be transferred using this analogy. [35%]

(b) Figure 2 shows a plane stone portal frame. The stones are square in cross-section with a density ρ . The frame is subjected to a horizontal load, P , as shown. Assuming that the stone portal cracks and hinges form in the locations A, B, C and D as indicated in the figure, find an upper bound collapse load associated with this mechanism. Assume there is no slip at any joints and that the stone has infinite compressive strength. [65%]

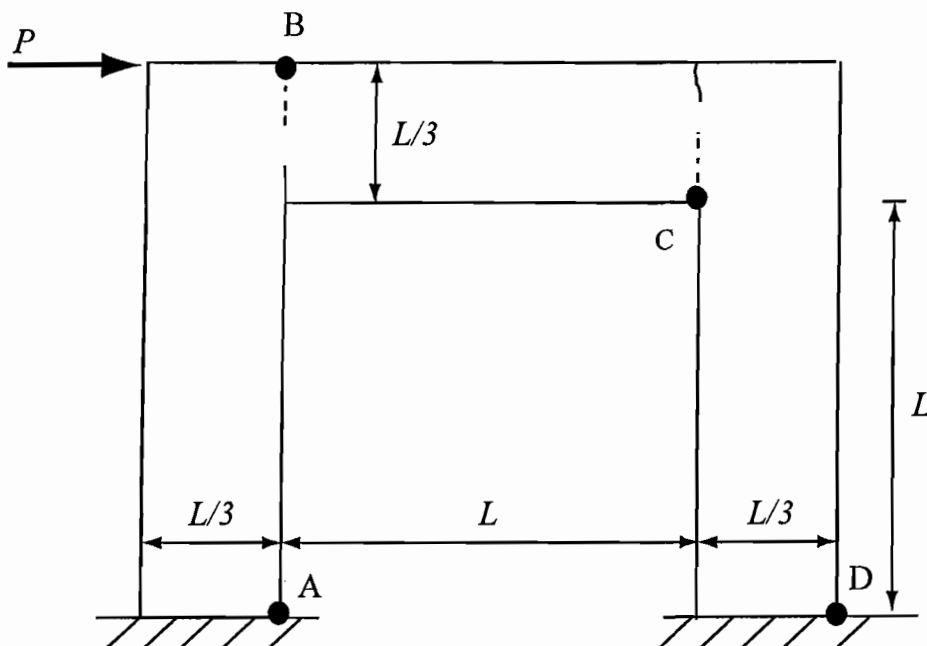


Fig. 2

END OF PAPER

Module 4D7 : Concrete and masonry structures**Formula and Data Sheet**

The purpose of this sheet is to list certain relevant formulae (mostly from Eurocode 2) that are so complex that students may not remember them in full detail. Symbols used in the formulae have their usual meanings, and only minimal definitions are given here. The sheet also gives some typical numerical data.

Material variability, partial safety factors and probability of failure

The word 'characteristic' usually refers to a 1 in 20 standard. At SLS, usually $\gamma_m = 1.0$ on all material strengths, $\gamma_f = 1.0$ on all loads.

At ULS, usually γ_m is 1.15 for steel, 1.5 for concrete; and γ_f is 1.4 for permanent loads, 1.6 for live loads (possibly reduced for combinations of rarely-occurring loads).

The difference between two normally-distributed variables is itself normally distributed, with mean equal to the difference of means, and variance the sum of the squares of the standard deviations.

Convolution integral

$$P_f = \int_{-\infty}^{+\infty} f_S(x) F_R(x) dx$$

Cement paste

The density of cement particles is approx. 3.15 times that of water. On hydration, the solid products have volume approx. 1.54 times that of the hydrated cement, with a fixed gel porosity approx. 0.6 times the hydrated cement volume. This gives capillary porosity about

$$\left[3.15 \frac{W}{C} - 1.14h \right] / \left[1 + 3.15 \frac{W}{C} \right]$$

for hydration degree h :

and gel/space ratio (gel volume / gel + capillaries) $2.14h / [h + 3.15 W/C + a]$.

Mechanical properties of concrete

Cracking strain typically 150×10^{-6} , strain at peak stress in uniaxial compression typically 0.002. Lateral confinement typically adds about 4 times the confining stress to the unconfined uniaxial strength, as well as improving ductility. In plane stress, the peak strength under biaxial compression is typically 20% greater than the uniaxial strength.

Durability considerations

Present value of some future good: $S_i / (1 + r)^i$ for stepped,

or $S_i / \exp(rct_i)$ for continuous discounting

where $(1 + r) = \exp(r_c)$

Water penetration : cumulative volume uniaxial inflow / unit area is sorptivity times square root of time. On sharp-wet-front theory penetration depth is $\{2k(H + h_c)/\Delta n\}^{1/2} t^{1/2}$.

Uniaxial diffusion into homogeneous material : $\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$

solution $c = c_o(1 - \text{erf}(z)), z = x/2\sqrt{Dt}$

Table of erf(z) :

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	
erf(z)	0	0.11	0.22	0.33	0.43	0.52	0.60	0.68	
z	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	∞
erf(z)	0.74	0.80	0.84	0.88	0.91	0.93	0.95	0.97	1.00

Passivation for pH > 12 and Cl⁻ < 0.4% by weight cement.

Corrosion unlikely for corrosion current < 0.2 $\mu\text{A}/\text{cm}^2$, resistivity > 100 k Ω cm, half-cell potential > -200 mV (but probable for < -350 mV).

SLS : cracking

Steel minimum area $A_{s,\min} \sigma_s = k_c k_{f_{ct,eff}} A_{ct}$

in tension zone, to produce multiple cracks.

Then, limitation of crack width to about 0.3 mm under quasi-permanent loads depending on exposure.

Maximum (characteristic) width $w_k = s_{r,\max}(\varepsilon_{sm} - \varepsilon_{cm})$

Where crack spacing $s_{r,\max} = 3.4c + 0.425k_1k_2\phi / \rho_{p,eff}$

with k_1 0.8 for high bond, 1.6 for plain bars;
 k_2 1.0 for tension, 0.5 for bending.

SLS : deflection

Interpolated curvature

$$\alpha = \zeta \alpha_{II} + (1 - \zeta) \alpha_I$$

where $\zeta = 1 - \beta(\sigma_{sr} / \sigma_s)^2$

β is 1.0 for short-term, 0.5 for sustained load,
 σ_{sr} is steel stress, for cracked section, but using loads which first cause cracking at the section considered.
 σ_s is current steel stress, calculated for cracked section.

ULS : moment and axial force

It is usual to assume failure at a cross-section to occur when the extreme-fibre compressive strain in the concrete reaches a limiting value, often $\varepsilon_{cm} = 0.0035$. The yield strain of steel ε_y of course depends on strength, as roughly f_y/E . Initial calculations often use uniform stress of $0.6 f_{cd}$ on the compression zone at failure. With these assumptions, for a singly-reinforced under-reinforced rectangular beam

$$M_u = A_s f_y d (1 - 0.5 x/d) / \gamma_s;$$

where

$$x/d = \frac{\gamma_c A_s f_y}{\gamma_s 0.6 f_{cu} b d};$$

over-reinforcement for $x/d > 0.5$.

For Tee beams, effective flange width b in compression is of order

$$b_w + l_o / 5 \leq b_{actual},$$

where l_o is span between zero-moment points.

For long columns, extra deflection prior to material failure is of order

$$e_2 = \frac{l_o^2}{\pi^2} \kappa_m$$

where κ_m is curvature at mid-height at failure and l_o is effective length.

Eurocode multiplies by further factors K_r and K_ϕ ,

$$\text{where } K_r = \left(\frac{n_u - n}{n_u - n_{bal}} \right) \leq 1$$

Shear in reinforced concrete

For *unreinforced* webs at ULS, shear strength in Code is

$$V_{Rd,c} = \left[\frac{0.18}{\gamma_c} k (100 \rho_l f_{ck})^{1/3} + 0.15 \sigma_{cp} \right] b_w d$$
$$\geq (v_{\min} + 0.15 \sigma_{cp}) b_w d$$

where: $k = 1 + \sqrt{200/d} \leq 2.0$ a factor that varies with effective depth, d (with d in mm),

ρ_l is the reinforcement ratio of anchored steel = $A_s/b_w d$ but $\rho_l \leq 0.02$.

$$v_{\min} = 0.035 k^{3/2} f_{ck}^{1/2}$$

For *reinforced* webs at ULS, shear strength in Code is

- Concrete resistance

$$V_{Rd,max} = f_{c,max} (b_w 0.9d) / (\cot \theta + \tan \theta)$$

where:

$$f_{c,max} = 0.6(1 - f_{ck}/250) f_{cd}$$

- Shear stirrup resistance

$$V_{Rd,s} = A_{sw} f_y (0.9d) (\cot \theta) / (s \gamma_s)$$

Torsion at ULS

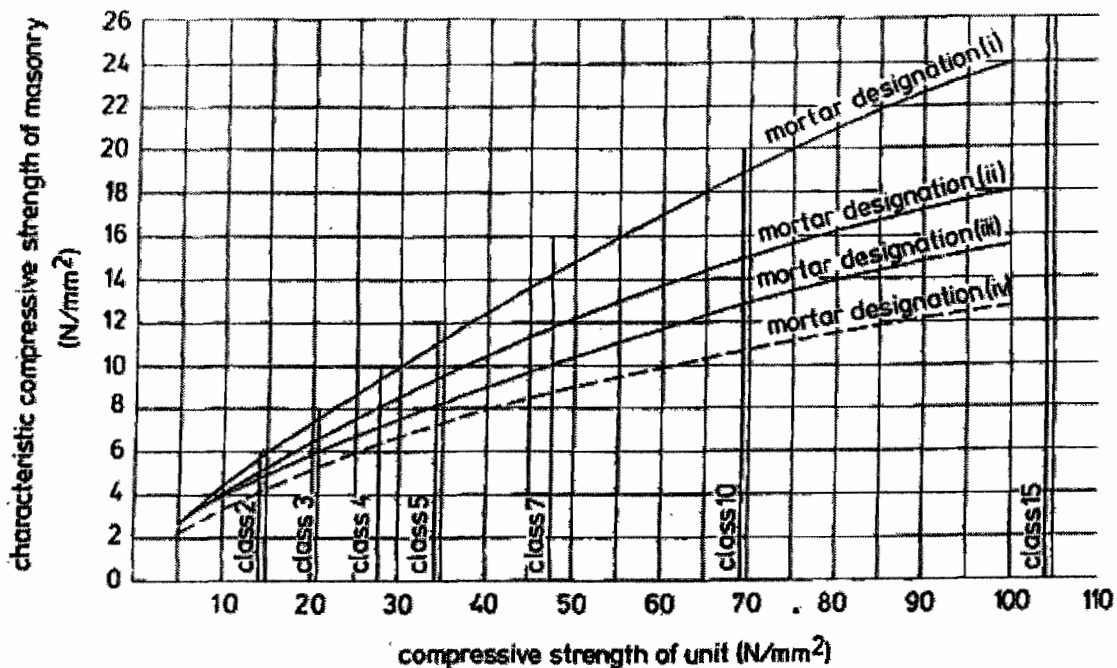
Based on truss analogy with variable strut angle, for a thin-walled box section; shear flow

$$q = f_{yd} \sqrt{\frac{A_w \cdot \sum A_t}{s \cdot u}}$$

where

$$\sigma_c < v \cdot f_{cd}$$

Masonry walls in compression



interpolation for classes of loadbearing bricks not shown on the graph may be used for average crushing strengths intermediate between those given on the graph, as described in clause 10 of BS 3921: 1985 and clause 7 of BS 187: 1976.

Figure 5.6(a) Characteristic compressive strength, f_k , of brick masonry (see Table 5.4)

Note. Mortar designations in the figure above range from (i) a strong mix of cement and comparatively little sand with 28 day site compressive cube strength of around 11 MPa, through (ii) and (iii) with strengths around 4.5 and 2.5 MPa respectively, to (iv) soft mortars e.g. of cement, lime and plentiful sand or cement, plasticizer and plentiful sand, with strength around 1.0 MPa.

CRM/JML Feb 2009

THE CUMULATIVE NORMAL DISTRIBUTION FUNCTION

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u e^{-\frac{x^2}{2}} dx \quad \text{FOR } 0.00 \leq u \leq 4.99.$$

u	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9014
1.3	.9032	.9049	.9065	.9082	.9098	.9114	.9130	.9146	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9250	.9264	.9278	.9292	.9305	.9318
1.5	.9331	.9344	.9357	.9369	.9382	.9394	.9406	.9417	.9429	.9440
1.6	.9452	.9463	.9473	.9484	.9495	.9505	.9515	.9525	.9535	.9544
1.7	.9554	.9563	.9572	.9581	.9590	.9599	.9608	.9616	.9624	.9632
1.8	.9640	.9648	.9656	.9663	.9671	.9678	.9685	.9692	.9699	.9706
1.9	.9712	.9719	.9725	.9732	.9738	.9744	.9750	.9755	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9807	.9812	.9816
2.1	.9821	.9825	.9830	.9834	.9838	.9842	.9846	.9850	.9853	.9857
2.2	.9861	.9864	.9867	.9871	.9874	.9877	.9880	.9884	.9887	.9890
2.3	.9892	.9895	.9898	.9901	.9904	.9907	.9910	.9913	.9916	.9919
2.4	.9921	.9924	.9927	.9929	.9932	.9935	.9937	.9940	.9942	.9945
2.5	.9947	.9949	.9951	.9953	.9955	.9957	.9959	.9961	.9963	.9965
2.6	.9967	.9969	.9971	.9973	.9975	.9977	.9979	.9981	.9983	.9985
2.7	.9987	.9989	.9991	.9992	.9994	.9995	.9997	.9998	.9999	.9999
2.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
2.9	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.0	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.1	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.2	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.3	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.4	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.5	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.6	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.9	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.0	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.1	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.2	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.3	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.4	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.5	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.6	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
4.9	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

Example: $\Phi(3.57) = .98215 = 0.9998215.$