ENGINEERING TRIPOS PART IIB

Monday 4 May 2009 9 to 10.30

Module 4F5

ADVANCED WIRELESS COMMUNICATIONS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Consider a pair of discrete random variables X,Y where $X \in \{a,b,c\}$ and $Y \in \{a,b,c\}$. Obtain the entropies $H(X), H(Y), H(X|Y), H(Y|X), H(X,Y)$ and the mutual information $I(X;Y)$ when their joint probabilities are: [25%]

$$P_{X,Y}(X=a,Y=a) = P_{X,Y}(X=b,Y=b) = P_{X,Y}(X=c,Y=c) = \frac{1}{6}$$
$$P_{X,Y}(X=a,Y=b) = P_{X,Y}(X=a,Y=c) = \frac{1}{12}$$
$$P_{X,Y}(X=b,Y=a) = P_{X,Y}(X=b,Y=c) = \frac{1}{12}$$
$$P_{X,Y}(X=c,Y=a) = P_{X,Y}(X=c,Y=b) = \frac{1}{12}$$

(b) Let Y_1 be the output of channel 1 to input X_1 and Y_2 be the output of channel 2 to input X_2 (see Fig. 1). Obtain the mutual information $I(X_1;Y_2)$ between the input of channel 1 and the output of channel 2 when the output of channel 1 to equally-likely input symbols is used as input for channel 2. [35%]

(c) Figure 2 shows the random coding error exponents $E_r(R)$, where R is the corresponding rate, of two different coding schemes (in solid and dashed lines, respectively). What can be said about their respective error probabilites and capacities? [15%]

(d) Explain why Gaussian signal constellations are not suitable for practical implementation. Show that the mutual information for QPSK modulation over the AWGN channel satisfies $I(X;Y) \leq 2$. [25%]

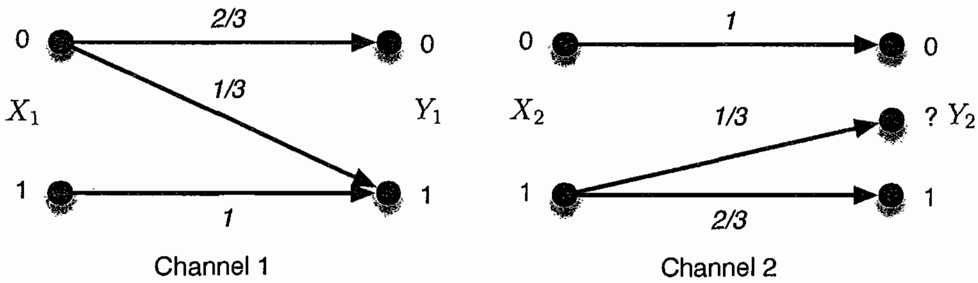

Fig. 1

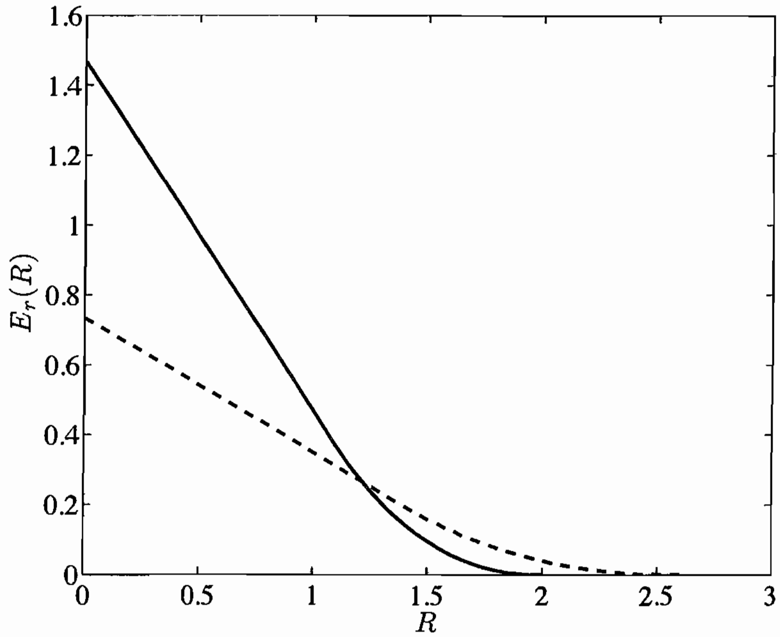

Fig. 2

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2 (a) Consider the signal set shown in Fig. 3.

(i) Find an orthonormal basis for the signal space. What is the dimension of the signal set? [25%]

(ii) Write down the signal constellation points (vectors) and draw the signal constellation. [25%]

(iii) What is the minimum distance of the signal constellation? [10%]

(b) BPSK modulation is employed to transmit over an AWGN channel. The channel introduces a phase rotation of 45 degrees, which is known at the receiver. How does this phase rotation affect the signal constellation using the standard basis? Does it affect the error probability? [40%]

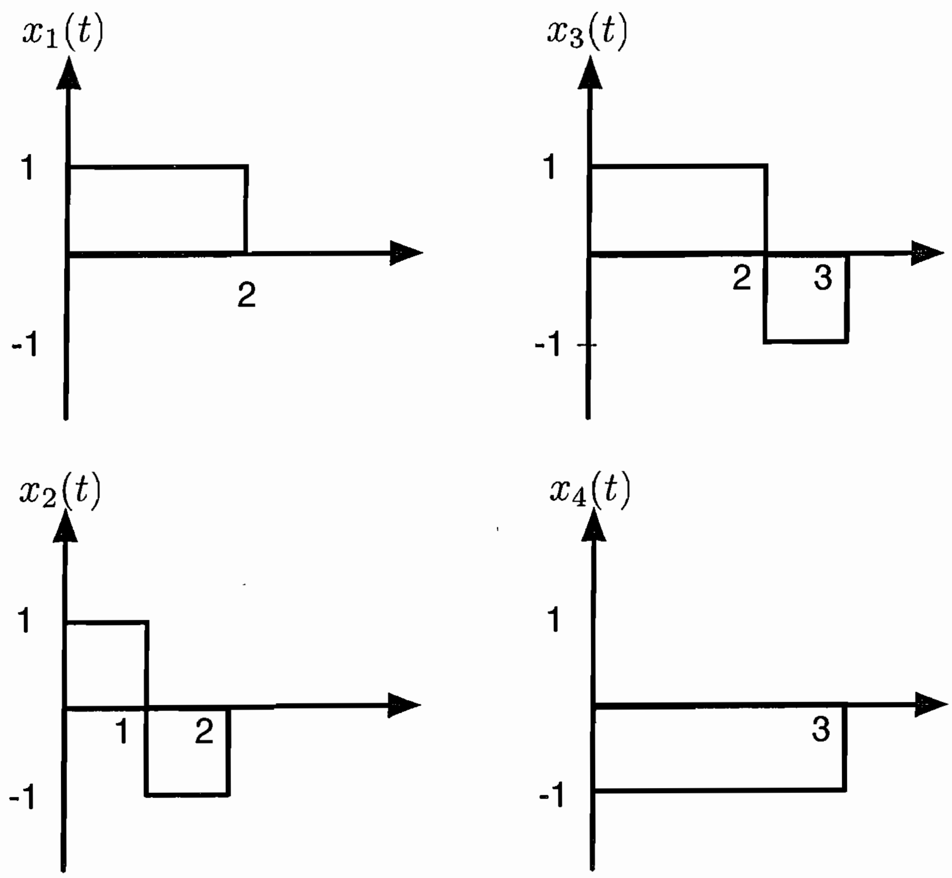

Fig. 3

3 Consider an $(8,4)$ linear binary code, with codewords of the form $(x_1, x_2, x_3, x_4, p_1, p_2, p_3, p_4)$ where x_i, $(i = 1,2,3,4)$, are the information bits and p_i, $(i = 1,2,3,4)$, are the parity check bits. The code is defined as

$$p_1 = x_2 + x_3 + x_4$$
$$p_2 = x_1 + x_2 + x_3$$
$$p_3 = x_1 + x_2 + x_4$$
$$p_4 = x_1 + x_3 + x_4$$

(where $+$ denotes modulo 2 addition).

(a) Find a generator matrix and a corresponding parity-check matrix. [25%]

(b) Find the minimum distance of the code. [20%]

(c) Draw the factor graph describing the code. Write down the variable and check-node degree distributions of the code, interpreted as a low-density parity-check code. [30%]

(d) The code is used for transmission over a binary erasure channel. Can the iterative decoder successfully decode if x_2, x_4 and p_3 are erased while all other bits are received correctly? [25%]

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4 Consider the convolutional code generated by the encoder shown in Fig. 4.

(a) What are the code generators in octal form? Draw a section of the trellis diagram specifying clearly the contents of the memory in each state and the input and the output corresponding to each transition. Find the free distance, d_{free}, of the code. [25%]

(b) What is the *diversity* achieved by the code when transmitted using BPSK modulation over a fast Rayleigh fading channel? Compare with the diversity of uncoded BPSK modulation. [25%]

(c) What is the diversity achieved by the code when transmitted using bit-interleaved coded modulation with 16-QAM over a fast Rayleigh fading channel? Justify your answer. [25%]

(d) Draw the block diagram of a parallel turbo-code of rate $R = 1/3$ using the recursive encoder corresponding to the feedforward encoder shown in Fig. 4. [25%]

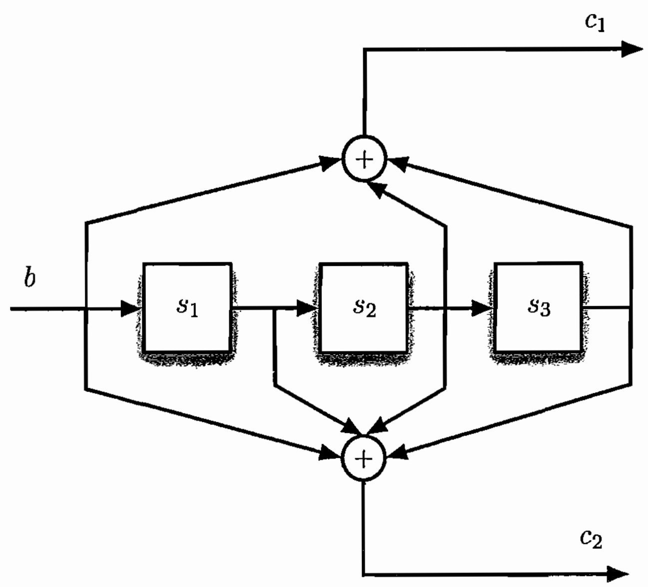

Fig. 4

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