ENGINEERING TRIPOS PART IIB

Monday 20 April 2009 9 to 10.30

Module 4F7

DIGITAL FILTERS AND SPECTRUM ESTIMATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) The four basic classes of adaptive filtering applications are Identification, Inverse modelling, Prediction and Interference cancellation. Describe any three classes of applications with the aid of diagrams. For convenience, the following notation may be used: u =input signal to the adaptive filter, y =output of the adaptive filter, d =desired response, $e = d - y =$ estimation error. [35%]

(b) Suppose the input to the adaptive filter has autocorrelation (at lag k)

$$r_u(k) = \alpha^{|k|}, \qquad |\alpha| < 1.$$

Find the eigenvalues of the autocorrelation matrix

$$\mathbf{R} = E\left\{[u(n) \quad u(n-1)]^T [u(n) \quad u(n-1)]\right\}.$$

[20%]

(c) The convergence of a M-tap LMS filter depends on the eigenvalues of the $M \times M$ autocorrelation matrix $\mathbf{R}$ of the input signal. The eigenvalues in turn depend on M. Let $S_u(e^{j\omega})$ be the power spectrum of the input signal. A well-known result states that the maximum and minimum eigenvalues approach the maximum and minimum values of the power spectrum respectively as $M \to \infty$, i.e. $\lambda_{\max} \to \max_\omega S_u(e^{j\omega})$, $\lambda_{\min} \to \min_\omega S_u(e^{j\omega})$.

For the input signal in part (b), find the asymptotic values of the maximum and minimum eigenvalues of the matrix $\mathbf{R}$ as $M \to \infty$. [45%]

2 Adaptive filters are commonly used for prediction. The aim is to form a linear predictor of the real valued signal $\{x(n)\}$ from noisy measurements of the signal:

$$u(n) = x(n) + v(n)$$

where $v(n)$ is zero mean white noise that is uncorrelated with $x(n)$. The variance of $v(n)$ is σ_v^2.

(a) State the M-tap LMS algorithm, with all quantities carefully defined, for designing a linear predictor for $\{x(n)\}$. (You may use the following standard notation: write the LMS algorithm as $\mathbf{h}(n+1) = \mathbf{h}(n) + \cdots$ where $\mathbf{h}(n)$ is the vector of filter coefficients.) [30%]

(b) State the range of values of step-size for which the LMS converges in mean. State this limit point, namely

$$\lim_{n \to \infty} E\{\mathbf{h}(n)\},$$

in terms of the autocorrelation matrix of $\{x(n)\}$. [30%]

(c) The fact that the limit point is a function of σ_v^2 is undesirable and regarded as an introduced bias in the result. The γ-LMS has been proposed as an adaptive filtering algorithm to combat the effect of measurement noise. The γ-LMS algorithm is:

$$\mathbf{h}(n+1) = \gamma \mathbf{h}(n) + \cdots$$

where the missing terms, indicated by "$\cdots$", are precisely the same as the standard LMS algorithm you have detailed in part (a). The only difference between γ-LMS and the standard LMS is that the scalar term γ multiplies the previous weight vector $\mathbf{h}(n)$ in the γ-LMS. How would you select the step-size and γ to remove the bias? (You may use the *Independence Assumption* if necessary: $E\left\{\mathbf{u}(n-1)\mathbf{u}(n-1)^{\mathrm{T}}\mathbf{h}(n)\right\} \approx E\left\{\mathbf{u}(n-1)\mathbf{u}(n-1)^{\mathrm{T}}\right\} E\{\mathbf{h}(n)\}$) [40%]

3 (a) What are the conditions that a random process $\{X_n\}$ has to satisfy for it to be Wide Sense Stationary (WSS) ? Define the *Power Spectrum* or *Spectral Density* for a WSS random process. [20%]

(b) Assume N data points $\{x_0, x_1, \ldots, x_{N-1}\}$ are available from the WSS random process. Define the *Correlogram* and *Periodogram* estimates of the power spectrum. State the well known simplification for the periodogram estimate which avoids direct computation of the autocorrelation function. What favourable property is revealed by this simplification? [30%]

(c) Obtain the expected value of the periodogram estimate. Is this estimator unbiased? [20%]

(d) Compute the expected value of the periodogram estimate when the input sequence $\{X_n\}$ is zero-mean white noise with variance σ^2. [10%]

(e) Define the Blackman-Tukey estimate and obtain the expected value of this estimator. Explain how it might improve the power spectrum estimate and its disadvantage. [20%]

4 (a) Describe the parametric approach to *power spectrum estimation*. Include the ARMA model in your discussion. [35%]

(b) Describe in detail the *least-squares method*, the *autocorrelation method* and the *covariance method* for fitting an AR model with P parameters to the data sequence $\{x_0, x_1, \ldots, x_{N-1}\}$. [35%]

(c) Using the autocorrelation method, estimate the parameters of a second-order AR process using the data sequence $\{x_0, x_1, \ldots, x_4\} = \{1, -2, 3, -4, 5\}$. [30%]

END OF PAPER