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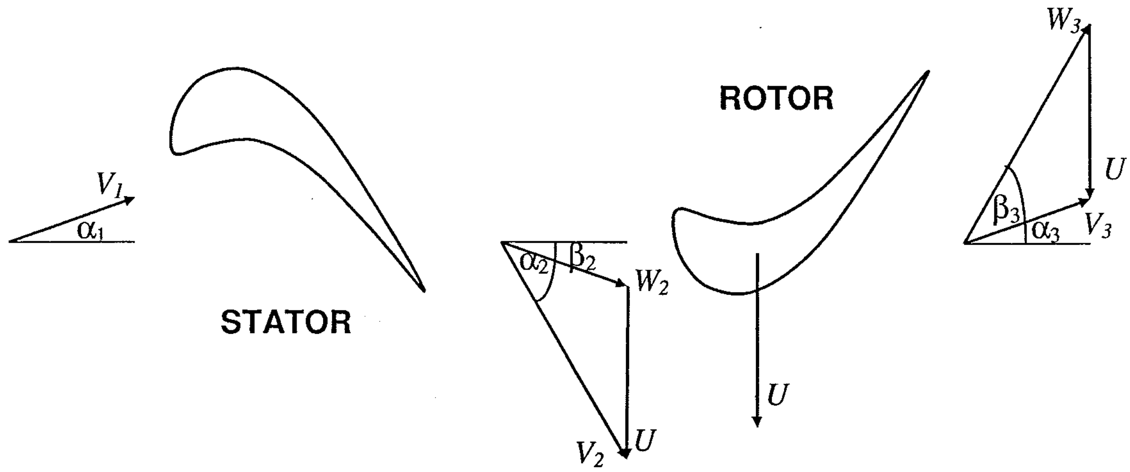
IIB Paper 4A3 Turbomachinery I Solutions 09 May 2010

1. a)
Velocity triangles (symmetrical, since $\Lambda = 0.5$):



The axial velocity is constant for a repeating stage, therefore,

$$\psi = \frac{\Delta h_0}{U^2} = \frac{U \Delta V_\theta}{U^2} = \frac{V_x}{U}(\tan\alpha_2 - \tan\alpha_1) = \phi(\tan\alpha_2 - \tan\alpha_1) \qquad (1)$$

Since the velocity triangles are symmetrical $\tan\beta_2 = -\tan\alpha_1$

From the definition of relative velocity, $\tan\beta_2 = \tan\alpha_2 - \frac{U}{V_x} = -\tan\alpha_1$

This can be substituted in (1) to give $\psi = \phi\left(2\tan\alpha_2 - \frac{1}{\phi}\right)$

Hence, $\tan\alpha_2 = \frac{\psi + 1}{2\phi}$ and $\frac{V_2}{U} = \frac{V_x}{U}\sqrt{1 + \tan^2\alpha_2} = \phi\sqrt{1 + \left(\frac{\psi+1}{2\phi}\right)^2}$

Rearranging gives, $\underline{\frac{V_2}{U} = \sqrt{\phi^2 + \left(\frac{\psi+1}{2}\right)^2}}$ [30%]

b)
For a repeating stage turbine, $V_2 = W_3$ (see vel triangles). Therefore,

$$\eta_{TT} = 1 - \frac{0.04}{\psi}\left[2\left(\frac{V_2}{U}\right)^2\right] = 1 - \frac{0.08}{\psi}\left[\phi^2 + \left(\frac{\psi+1}{2}\right)^2\right] = 1 - \frac{0.02}{\psi}\left[4\phi^2 + \psi^2 + 2\psi + 1\right]$$

$$\eta_{TT} = 1 - 0.02\left[\frac{4\phi^2 + 1}{\psi} + \psi + 2\right] \Rightarrow \frac{\partial \eta_{TT}}{\partial \psi} = -0.02\left[-\left(4\phi^2 + 1\right)\psi^{-2} + 1\right]$$

Peak efficiency design at $\frac{\partial \eta_{TT}}{\partial \psi} = 0$, when $\psi^2 = \left(4\phi^2 + 1\right)$

For this design with $\phi = 0.5$, $\psi = \sqrt{4(0.5)^2 + 1} = \underline{\sqrt{2}}$ [15%]

$$\alpha_2 = tan^{-1}\left(\frac{\psi + 1}{2\phi}\right) = tan^{-1}\left(1 + \sqrt{2}\right) = \underline{67.5^o}$$

$$\alpha_1 = tan^{-1}\left(\frac{1}{\phi} - \frac{\psi + 1}{2\phi}\right) = tan^{-1}\left(\frac{1 - \psi}{2\phi}\right) = tan^{-1}\left(1 - \sqrt{2}\right) = \underline{-22.5^o}$$

$$\eta_{TT} = 1 - 0.02\left[\frac{4\phi^2 + 1}{\psi} + \psi + 2\right] = 1 - 0.02\left[\frac{1 + 1}{\sqrt{2}} + \sqrt{2} + 2\right] = \underline{0.903}$$

$$\eta_{TS} = \eta_{TT} - \frac{0.5V_3^2}{\Delta h_0} = 0.903 - \frac{0.5\phi^2}{cos^2(\alpha_3).\psi} = 0.903 - \frac{0.5 \times 0.5^2}{cos^2\left(22.5^0\right)\sqrt{2}} = \underline{0.800}$$

[20%]

c)

$$Power = \dot{m} \times N_{stage} \times \psi \times U^2 \qquad \Rightarrow \bar{r}^2 = \frac{Power}{\dot{m} \times N_{stage} \times \psi \times \Omega^2}$$

$$\bar{r} = \sqrt{\frac{3.5e6}{25 \times 4 \times \sqrt{2} \times (100\pi)^2}} = \underline{0.501m}$$

$$V_1 = \frac{\phi \bar{r} \Omega}{cos\,\alpha_1} = \frac{0.5 \times 0.501 \times 100\pi}{cos\left(22.5^0\right)} = \underline{85.2m/s}$$

$$\dot{m} = \rho A_x V_x = \rho \times 2\pi \bar{r} h \times \phi \bar{r} \Omega$$

The blade height, $h = \frac{\dot{m}}{2\pi\phi\bar{r}^2\Omega\rho} = \frac{25}{2\pi \times 0.5 \times 0.501^2 \times 100\pi \times 1.65} = 0.0612$

$\Rightarrow$ <u>h = 61 mm</u> [20%]

<u>Advantages of 50% reaction design:</u>

1. Symmetrical velocity triangles gives similar rotor and stator blade shapes and thus makes the aero and mechanical design easier.
2. Lower overall losses as there are similar turning and relative velocity levels in both rotor and stator (see efficiency correlation).

[15%]

Question 1: 50% reaction turbine design

This was a popular question but the candidates found it difficult. In part (a) students didn't take the time to carefully draw correct velocity triangles and this led to difficulty in deriving the proof required and generated errors in the following parts of the question. The candidates that didn't see the velocity triangles were symmetric had a lot of problems and produced a lot of inaccurate algebra. Part (b) required differentiation to find the stage loading for peak efficiency. This was performed well, although some candidates failed to see that the rotor exit relative velocity was equal to the stator exit absolute velocity. Most candidates couldn't get the total-to-static efficiency from the total-to-total efficiency for the design, but calculating the flow angles was found easier. Part (c) required some fairly straightforward design calculations and was generally well answered, although there were frequent numerical slip-ups or errors carried over that were taken into consideration in the marking. The discussion of the advantages of 50% reaction in part (d) was variable with some very good as well as some very wrong statements.

2. a)

The Diffusion Factor as defined represents the diffusion of the flow over the suction surface of a compressor cascade. It is therefore a measure of the loading on the blades and the likelihood of separation. The first term (1-V2/V1) represents the bulk deceleration of the flow. The second term represents the flow turning through the cascade. The DF can be used in prelim design since it only involves inlet/exit velocities and the pitch/chord ratio. Therefore, detailed blade shape is not required. It can be used to set the pitch/chord ratio and thus the number of blades to get reasonable diffusion at design (DF~0.45), and used to see when the blades may stall (DF~0.6)

[20%]

b)

$$Y_P = \frac{1 - p_{02rel}/p_{01,rel}}{1 - p_1/p_{01,rel}} \Rightarrow \frac{p_{02,rel}}{p_{01,rel}} = 1 - Y_P\left(1 - p_1/p_{01,rel}\right)$$

$$\therefore \frac{p_{02}}{p_{01}} = 1 - 0.038 \times (1 - 0.7528) = 0.9906 \qquad \text{[Using Tables]}$$

Applying continuity across the cascade,

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hs \cos\alpha_1\, p_{01}} = Q(M_{1rel}) = \frac{\dot{m}\sqrt{c_p T_{02}}}{hs \cos\alpha_2\, p_{02}} \times \frac{\cos\alpha_2}{\cos\alpha_1} \times \frac{p_{02}}{p_{01}}$$

$$\therefore \cos\alpha_2 = \frac{Q(M_1)}{Q(M_2)} \times \cos\alpha_1 \times \frac{p_{01}}{p_{02}} \qquad \text{(true since } \dot{m}/hs \text{ is constant)}$$

$$\cos\alpha_2 = \frac{Q(0.65)}{Q(0.44)} \times \cos 55^0 \times \frac{1}{0.9906} = \frac{1.128}{0.8691} \times 0.5736 \times \frac{1}{0.9906} = 0.7515$$

[Using Tables]

$$\Rightarrow \underline{\alpha_2 = 41.3^0} \quad \text{(angles are +ve)}$$

This is 4.3 degrees greater than the metal angle due to deviation. Deviation arises due to (i) inviscid effects – the flow across the passage does not follow the trailing edges as it is diffusing (ii) viscous effects – boundary layer blockage increases effective exit metal angle.

[25%]

c)
$$DF = 0.45 = 1 - \frac{V_2}{V_1} + \frac{|V_{\theta 2} - V_{\theta 1}|}{2V_1}\frac{s}{c}$$

$$\frac{s}{c} = \left(0.45 - 1 + \frac{V_2}{V_1}\right)\frac{2V_1}{|V_{\theta 2} - V_{\theta 1}|} = \left(-0.55 + \frac{V_2/\sqrt{c_p T_0}}{V_1/\sqrt{c_p T_0}}\right)\frac{2V_1/\sqrt{c_p T_0}}{\left(sin\,\alpha_1 . V_1/\sqrt{c_p T_0} - sin\,\alpha_2 . V_2/\sqrt{c_p T_0}\right)}$$

From tables,

$$\frac{s}{c} = \left(-0.55 + \frac{0.273}{0.3948}\right)\frac{2 \times 0.3948}{\left(sin 55^0 . 0.3948 - sin 41.3^0 . 0.273\right)} = \underline{0.780}$$ [20%]

d) With 5 degrees incidence for the same inlet Mach number, the mass flow through the cascade will change

$$\therefore Q(M_{2,new}) = Q(M_1) \times \frac{cos\,\alpha_{1,new}}{cos\,\alpha_2} \times \frac{p_{01}}{p_{02}} = 1.128 \times \frac{cos\,60^0}{cos\,41.3^0} \times \frac{1}{0.9906} = 0.7579$$

$\Rightarrow M_{2,new} = 0.371$, $V_{2,new}/\sqrt{c_p T_0} = 0.2317$ [From Tables]

$$DF_{new} = \left(1 - \frac{V_{2,new}/\sqrt{c_p T_0}}{V_1/\sqrt{c_p T_0}}\right) + \frac{s}{c}\frac{\left(sin\,\alpha_{1,new} . V_1/\sqrt{c_p T_0} - sin\,\alpha_2 . V_{2,new}/\sqrt{c_p T_0}\right)}{2V_1/\sqrt{c_p T_0}}$$

$$DF_{new} = \left(1 - \frac{0.2317}{0.3948}\right) + 0.78\frac{\left(sin 60^0 . 0.3948 - sin 41.3^0 . 0.2317\right)}{2 \times 0.3948} = \underline{0.60}$$

[20%]

e) Applying mass conservation between inlet and minimum area,

$$\frac{\dot{m}\sqrt{c_p T_{01}}}{hs\,cos\,\alpha_1\,p_{01}} = Q(M_1) = \frac{\dot{m}\sqrt{c_p T_{01}}}{ho\,p_o^*} \times \frac{o}{s\,cos\,\alpha_1} \times \frac{p_o^*}{p_{01}}$$

With the throat choked, $Q(1) = 1.281$ and there is no loss upstream,

$$\Rightarrow \alpha_{1,choke} = cos^{-1}\left(\frac{Q(1)}{Q(M_1)}\frac{o}{s}\right) = cos^{-1}\left(\frac{1.281}{1.128} \times 0.6\right) = 47.04^0$$

Therefore, the incidence at choking = 47.04 – 55 $\underline{= -7.95^0}$

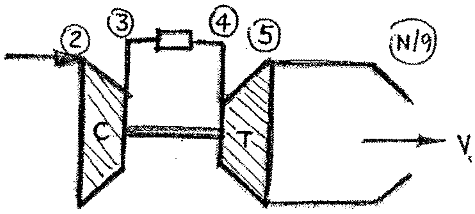
[15%]

Question 2: Compressible flow compressor cascade

This was the most popular question and most candidates answered it well. In part (a) many explained what the diffusion factor (DF) represented but didn't describe how it was used in design. Part (b) was completed accurately by almost all candidates, although some had odd ideas about the source of deviation in axial compressors and even got confused with the source of slip in radial machines. Part (c) required the application of the compressible flow tables and DF to find the blade spacing and there were some minor slip-ups with the trigonometry. Part (d) required re-calculation of the exit Mach number from the cascade using compressible flow and continuity to derive a new DF. This was answered surprisingly well as it is a tricky calculation. Part (e) considered at what inlet angle the cascade would choke. Most candidates could do this, but several failed to turn the result into a (negative) incidence.

3(a) For the compressor and turbine:

$$\frac{T_{03}}{T_{02}} = \left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma\eta_p}} \quad \text{(Ia)}, \quad \frac{T_{05}}{T_{04}} = \left(\frac{P_{05}}{P_{04}}\right)^{\eta_p\frac{\gamma-1}{\gamma}} \quad \text{(Ib)}$$



For choked flow

$$\frac{\dot{m}\sqrt{CpT_{0IN}}}{P_{0IN}A_{IN}} = \Pi_m = constant \qquad \text{(II)}$$

CHOKING CONSTRAINT

$\Pi_{m4} = \Pi_{m9}$ = constant (choked) (III)

Combining Eqns (II & III) with $p_{09}=p_{05}$, $T_{09}=T_{05}$ (negligible losses & heat transfer)

$$\frac{\sqrt{T_{05}}}{P_{05}A_9} = \frac{\sqrt{T_{04}}}{P_{04}A_4} \qquad \text{(IV)}$$

Eqns (Ib) and (IV) yields

$$\frac{T_{05}}{T_{04}} = \left(\frac{A_4}{A_9}\right)^{nt}, \quad nt = \left(\frac{\gamma}{(\gamma-1)\eta_p} - \frac{1}{2}\right)^{-1} \qquad \text{(VI)}$$

Note nt = constant for γ and η_p constant. So turbine PR and TR fixed by geometry

ENERGY CONSERVATION ($W_C = W_T$)

$$c_p(T_{03} - T_{02}) = c_p(T_{04} - T_{05})$$

$$T_{04} - T_{05} = T_{04}\left(1 - \frac{T_{05}}{T_{04}}\right)$$

k_H

$$(T_{03} - T_{02}) = C_0 T_{04} \qquad \text{(VII)}$$

$$C_0 = 1 - \left(\frac{A_4}{A_9}\right)^{nt}$$

Using the polytropic efficiency Eqn (Ia) with Eqn (VII)

$$\frac{P_{03}}{P_{02}} = \left(1 + \frac{T_{03} - T_{02}}{T_{02}}\right)^{\eta_p\frac{\gamma}{\gamma-1}} = \left(1 + C_0\frac{T_{04}}{T_{02}}\right)^{C_1} \qquad \text{(A)}$$

Where $C_1 = \eta_p \frac{\gamma}{\gamma-1}$

MASS CONSERVATION ($\dot{m}_{air,4} = \dot{m}_{air,2}$)
Assuming negligible pressure drop ($p_{04} = p_{03}$)

$$\frac{\dot{m}\sqrt{C_p T_{02}}}{P_{02}A_2} = \frac{\dot{m}\sqrt{C_p T_{04}}}{P_{04}A_4} \times \sqrt{\frac{T_{02}}{T_{04}}} \times \frac{A_4}{A_2} \times \frac{P_{03}}{P_{02}} \qquad \text{(B)}$$

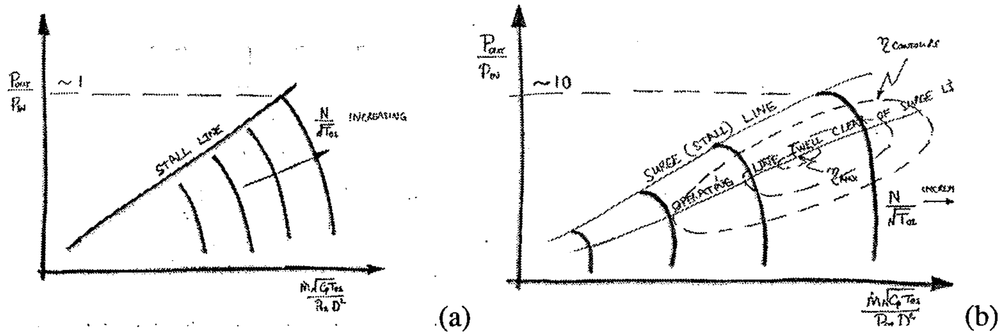
Combining equations (A) and (B)

$$\frac{\dot{m}\sqrt{C_p T_{02}}}{P_{02}A_2} = C_2 \times \sqrt{\frac{T_{02}}{T_{04}}} \times C_3 \times \left(1 + C_0 \frac{T_{04}}{T_{02}}\right)^{C_1} \qquad \text{(C)}$$

where $C_2 = \dfrac{\dot{m}\sqrt{C_p T_{04}}}{P_{04}A_4}$, $C_3 = \dfrac{A_4}{A_2}$

[40%]

b) The single stage has a greater flow range than the multistage. Eqns (A) and (C)

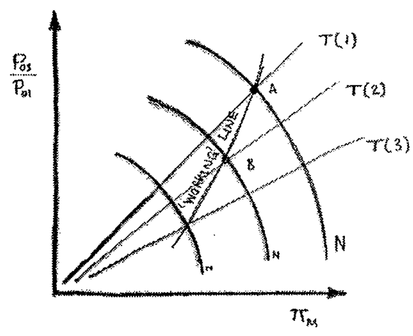


Single and multistage compression maps

will give the working line. Eqn. (A) will give the 'y' axis values and (C) the 'x'. Varying T_{04} is equivalent to varying the engine fuel flow i.e. the engine control variable.

[20%]

c) Eqn. (B), for a fixed T_{04}/T_{02} gives straight lines radiating from the origin



$$\frac{P_{03}}{P_{02}} = CONST \frac{\dot{m}\sqrt{T_{02}}}{P_{02}A_2}$$

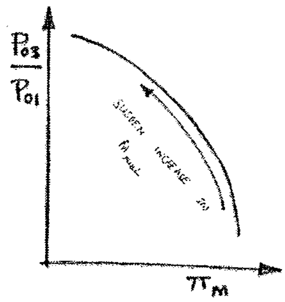
with a slope T(*) α T_{04}/T_{02}. Two points (A) and (B) on a 'working line' are shown with T(A) > T(B). T(1) has a higher (T_{04}/T_{02}) than T(2) and T(3). Point (B) has a lower T_{04}/T_{02} and N than (A). Points (A) and (B) assume the same η_p. Taking into account that $\eta_{pB} < \eta_{pA}$ Eqn (A) shows

$$\frac{P_{03}}{P_{02}} = f\left(\frac{T_{04}}{T_{02}}\right)^{C\eta_p}$$

indicating point (B) should now have a smaller pressure ratio. Moving down the T(2) line (and hence across the map) to a lower pressure ratio is a move of the working line towards the surge line. The working line eqns show that a reduction in the nozzle area will reduce k and thus move the working line away from the surge line.

[15%]

d) A sudden fuel increase will accelerate the engine. However, acceleration (of the heavy metal components) is quite slow on the scales characterising compressor and turbine operation (aerothermal time scales). Therefore, during this type of transient the compressor can be considered to operate on the same characteristics plot as for steady operation. The fuel increase, increases T_{04}/T_{02} which increases the pressure in the combustor. This acts like an increase in back pressure for an isolated compressor test.



[15%]

e) Even the crudest surge model (Greitzer) involves the solution of coupled non-linear differential equations. Surge is preceded by stall and the computation of this again needs solution of coupled non-linear differential equation sets. Hence, surge/stall are non-linear instability problems. The working line equation evaluation just needs basic conservation properties satisfied the multiply choked components dramatically simplifying the physics of the modelled process.

[10%]

C A Hall
09 May 2010

Question 3: Turbojet off-design operation

This was the least popular question but those who answered it seemed to know the topic well and were able to accurately recite many details for the discussion parts. Part (a) required a mathematical analysis for the working line of a turbojet and most applied the correct principles and assumptions, although some failed to evaluate all the constants completely. Part (b) required sketches of compressor characteristics and several answers to this part were incomplete. Part (c) required a careful explanation and many candidates could reproduce some correct facts but clearly didn't understand how the engine working line was modified by the engine parameters. Many candidates omitted part (d) or gave incorrect answers. For part (e), most just stated that the engine became unsteady at surge, but didn't explain further why it is problematic to determine mathematically.