PART II B 2010 DR S BARRETT

SOLUTIONS TO 4A8 (ENVIRONMENTAL FLUID MECHANICS) 2010

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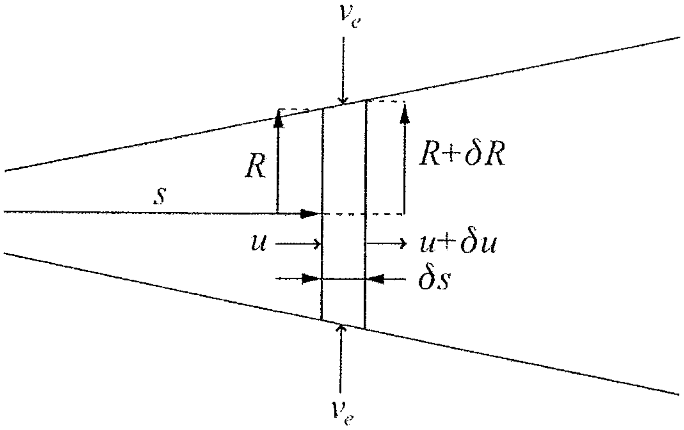
1 (a) (i) An entrainment velocity is the effective rate at which ambient fluid is mixed into a jet or plume. For a simple jet/plume it can be approximated by the MTT relation:

$$v_e = \alpha u,$$

where v_e is the entrainment velocity, $\alpha \sim 0.1$ is a constant and u is the velocity of the plume relative to the ambient fluid.

(ii) Start with a clear diagram:



Conservation of mass: change in mass flux = mass of fluid entrained

$$\Rightarrow \delta(\pi R^2 u \rho) = 2R v_e \rho \delta s$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}s}(R^2 u) = 2\alpha u R$$

Conservation of momentum: change in momentum flux = net force on CV

$$\Rightarrow \delta(\pi R^2 u^2 \rho) = 0$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}s}(R^2 u^2) = 0$$

(iii) Take $R = As^a$ and $u = Bs^b$. Substituting into conservation of mass:

$$\frac{\mathrm{d}}{\mathrm{d}s}(A^2 s^{2a} B s^b) = 2\alpha B s^b A s^a$$

$$\Rightarrow A(2a+b)s^{2a+b-1} = 2\alpha s^{a+b}$$

This applies at any s so $2a+b-1=a+b$, i.e. $a=1$. Similarly substituting into conservation of momentum we get

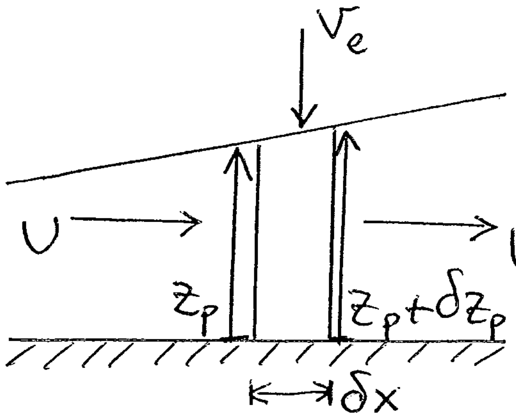
$$(2a+2b)s^{2a+2b-1}=0 \Rightarrow 2a+2b=0$$

So $a=-b=-1$.

(b) (i)

$$\begin{aligned} 0.1\times\chi(z=0) &= \chi(z=z_p) \\ \Rightarrow 0.1 &= \exp\left(-\frac{Uz}{\kappa u_* x}\right) \\ \Rightarrow -2.3 &= -\frac{Uz}{\kappa u_* x} \\ z_p &\simeq 2.3\kappa u_* x/U \end{aligned}$$

(ii) Diagram: Applying conservation of mass:



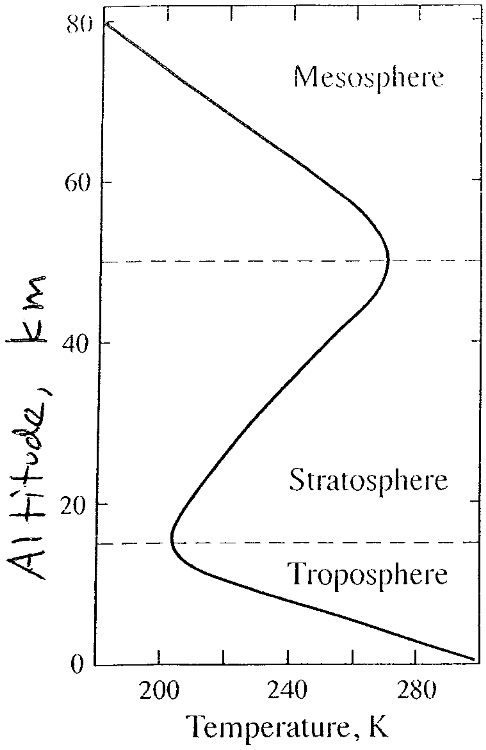
$$\begin{aligned} Uz_p+v_e\delta x &= U(z_p+\delta z_p) \\ \Rightarrow v_e\delta x &= U\delta z_p \\ \Rightarrow v_e &= U\frac{\partial z_p}{\partial x} \\ \Rightarrow v_e &= 2.3\kappa u_* \end{aligned}$$

Note that this means $v_e \simeq u_*$.

Q1 Entrainment

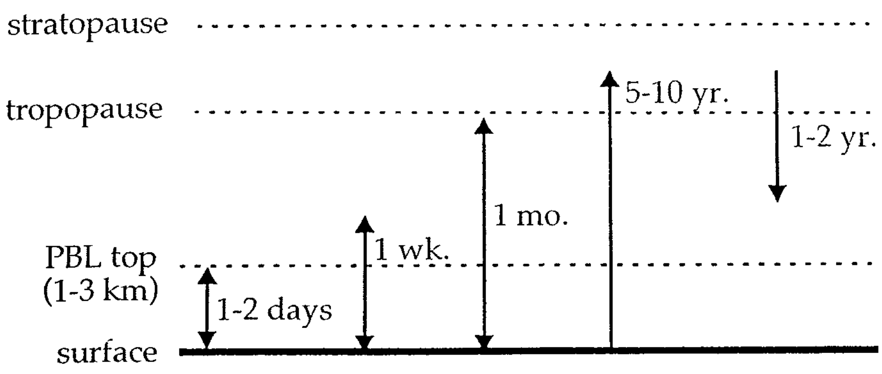
All candidates demonstrated an understanding of entrainment in a textbook setting and could derive momentum jet equations. The key differentiator was in the application of the concept of entrainment to a passive plume.

2 (a) Temperature vs. altitude (km):



The stratosphere is heated due to photolysis of ozone in the ozone layer. The troposphere is heated convectively at the surface and its temperature approximately follows an adiabatic lapse rate until the tropopause. The stratosphere is very stable while the troposphere can be unstable or stable.

Vertical mixing times:



(b) (i) Starting from $T\mathrm{d}s = \mathrm{d}h - v\mathrm{d}p$, where $s = \mathrm{const.}$,

$$\Rightarrow \mathrm{d}p = \mathrm{d}h/v \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}z} = \frac{c_p}{v}\frac{\mathrm{d}T}{\mathrm{d}z}$$

Use the hydrostatic equation:

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\rho g$$

Note $\rho = 1/v$ and that $\mathrm{DALR} = -\mathrm{d}T/\mathrm{d}z$, thus:

$$\mathrm{DALR} = \frac{g}{c_p}$$

(ii) This time include latent heat, i.e.

$$\mathrm{d}h = v\mathrm{d}p + \mathrm{d}q$$

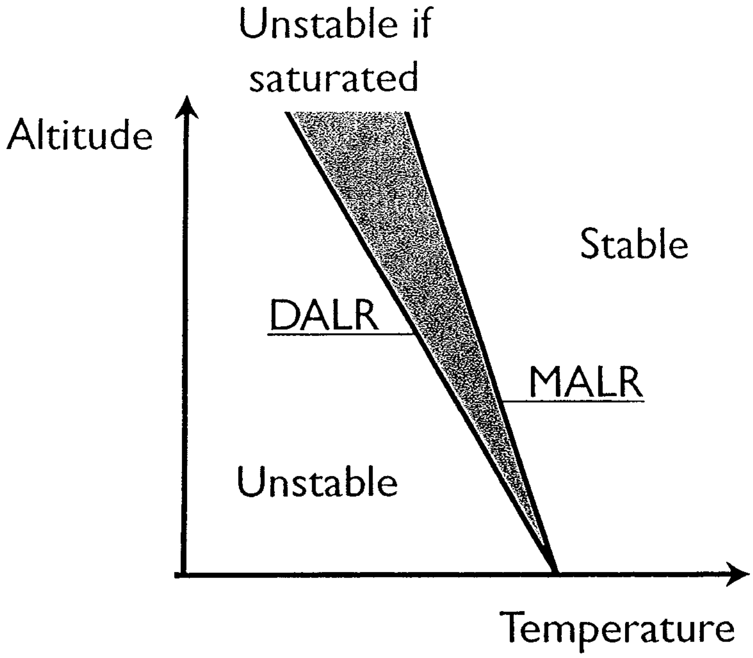
$$\Rightarrow c_p\frac{\mathrm{d}T}{\mathrm{d}z} = v\frac{\mathrm{d}p}{\mathrm{d}z} - \Delta H_v\frac{\mathrm{d}w_v}{\mathrm{d}z}$$

$$\Rightarrow \mathrm{MALR} = -\frac{\mathrm{d}T}{\mathrm{d}z} = \frac{g}{c_p + \Delta H_v\frac{\mathrm{d}w_v}{\mathrm{d}T}}$$

Taking $\mathrm{d}w_v/\mathrm{d}T = 2.7 \times 10^{-4}\ \mathrm{K}^{-1}$ and $\Delta H_v = 2.5\ \mathrm{MJkg}^{-1}$:

$$\frac{\mathrm{MALR}}{\mathrm{DALR}} = 0.6.$$

(iii) Yes. The region of instability is larger:



Q2 Thermal Structure of the Atmosphere

Attempted by all candidates. Most candidates demonstrated knowledge of the thermal structure of the atmosphere and could derive the dry adiabatic lapse rate, with some having difficulty extending this to the moist adiabatic lapse rate. Most knew the effect of clouds on stability, although the quality of the explanations varied.

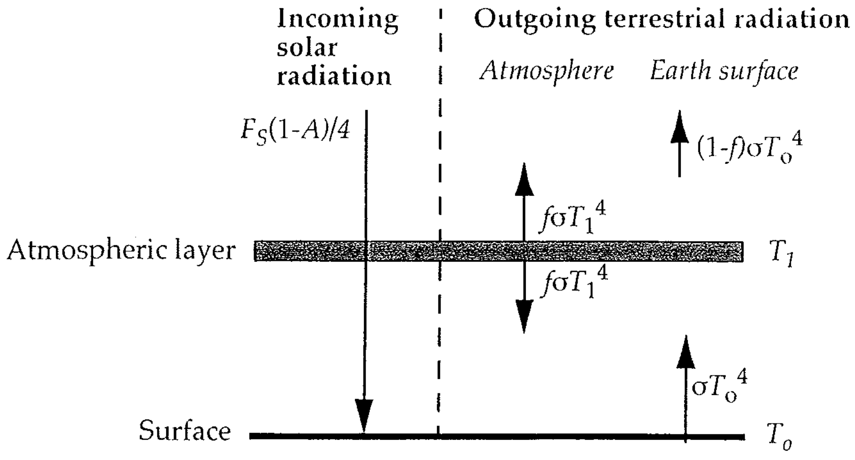
3 (a) Energy incident on earth = $F_S \pi R_E^2$, of which $(1-A)$ is absorbed. This is spread over (on average) $4\pi R_E^2$. Thus the mean solar radiation flux absorbed by the earth (per unit area) is $F_S(1-A)/4$.

(b) Assuming the Earth is a black body:

$$\sigma T_E^4 = F_S(1-A)/4 \Rightarrow T_E = 255 \text{ K}.$$

This is too cold mainly because the atmosphere absorbs some of the outgoing thermal radiation from the Earth, which is not accounted for.

(c) Diagram of fluxes:



Energy balance of earth + atmosphere system:

$$F_S(1-A)/4 = (1-f)\sigma T_0^4 + f\sigma T_1^4$$

Energy balance of atmosphere:

$$f\sigma T_0^4 = 2f\sigma T_1^4$$
$$\Rightarrow T_0 = 2^{1/4} T_1$$
$$\Rightarrow F_S(1-A)/4 = (1-f)\sigma T_0^4 + (f/2)\sigma T_0^4$$
$$\Rightarrow T_0 = \left[\frac{F_S(1-A)}{4\sigma\left(1-\frac{f}{2}\right)}\right]^{1/4}$$

Assuming $f = 0.77$, : $T_0 = 288\,K$ and $T_1 = 241\,K$. This is more realistic and the surface is colder than the atmosphere. This is a good estimate for both the surface temperature and the atmosphere's temperature at the atmospheric scale height, $H \simeq 7$ km.

(d) The base situation is:

$$\frac{F_S(1-A)}{4} = \sigma\left(1-\frac{f}{2}\right)T_0^4 \qquad (1)$$

Now RF $= \Delta F$ is defined as the reduction in outgoing radiation that would be caused by a change in GHG concentrations that resulted in a change Δf in absorption:

$$\Delta F = \sigma\left(1-\frac{f}{2}\right)T_0^4 - \sigma\left(1-\frac{f+\Delta f}{2}\right)T_0^4 = \frac{\Delta f}{2}\sigma T_0^4 \qquad (2)$$

(3) If the perturbation is maintained for some time, eventually T_0 will adjust to a new equilibrium $T_0+\Delta T_0$. This is given by:

$$\frac{F_S(1-A)}{4} = \sigma\left(1-\frac{f+\Delta f}{2}\right)(T_0+\Delta T_0)^4 \qquad (3)$$

Assuming $\Delta T_0/T_0 \ll 1$:

$$(T_0+\Delta T_0)^4 \simeq T_0^4 + 4T_0^3\Delta T_0 \qquad (4)$$

Now (1) = (3), unless the sun gets hotter or the Earth's albedo changes — but we're assuming they don't. Substituting (4) into (3), equating the RHS of (1) and (3), and neglecting small terms, we get

$$\Delta T_0 = \frac{T_0\Delta f}{8(1-f/2)}$$

Finally, replacing Δf with ΔF from (2):

$$\Delta T_0 = \frac{\Delta F}{4\left(1-\frac{1}{2}f\right)\sigma T_0^3}$$

So the climate sensitivity parameter is:

$$\lambda = \frac{1}{4\left(1-\frac{1}{2}f\right)\sigma T_0^3}$$

Q3 Radiative Forcing

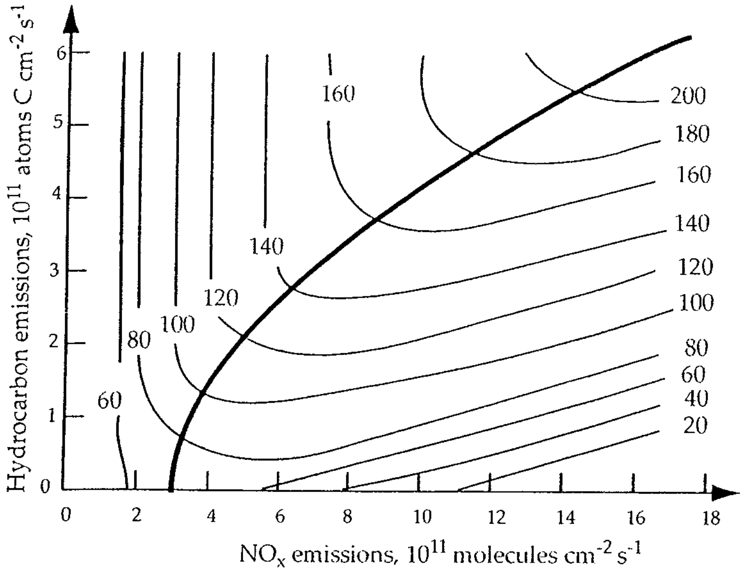
Almost all candidates could derive the mean solar radiation flux absorbed by the Earth and estimate the Earth's temperature assuming it was a black body. Most were able to draw a convincing diagram demonstrating they understood how to model absorption by the Earth's atmosphere. Few candidates produced a correct and convincing derivation of the climate sensitivity parameter, but most had familiarity with the material.

4 (a) (i) Lower primary PM emissions will improve air quality near the routes. Removing sulfur from fuels will also reduce 'primary' sulfate emissions and corresponding concentrations.

(ii) Again lower primary PM. Lower secondary sulfate. May result in higher nitrates if reduction in sulfates results in increased free ammonia to form ammonium nitrate.

(iii) Reduced CO_2 beneficial globally and in the long-term, so little regional or short-term effect. Increased methane will increase short-term and northern-hemispheric warming. May increase ozone too. Reduction in black carbon emissions may reduce local warming.

(b) Ozone isopleth (numbers not needed):



In high–NOx locations, cutting NOx emissions can increase ozone because the HOx + NOx reaction (which yields nitric acid) terminates the ozone production chain reaction. The authority should cut HC emissions, or ideally both HC and NOx emissions.

(c) For NO_2:

$$\frac{d[NO_2]}{dt} = -j[NO_2] + k_3[O_3][NO]$$

Now set

$$\frac{d[NO_2]}{dt} = 0 \Rightarrow [O_3]_{ss} = \frac{j[NO_2]}{k_3[NO]}$$

(d) OH 'cleans' the atmosphere of anthropogenic pollutants including CO, VOCs and SO2 (by converting it to sulfate, which gets removed from the atmosphere). At the same time, OH is produced from ozone, of which some is attributable to anthropogenic NOx and VOC emissions.

Q4 Qualitative Application

Attempted by weaker candidates on average. Variable depth of understanding/ability to apply concepts in part (a), but all could make the more basic points and a couple showed a significant depth of understanding linking disparate concepts. Most candidates could answer the qualitative question on ozone formation in polluted air and derive the steady-state ozone concentration in clean air. Almost all candidates could make a correct basic conceptual point about the role of OH relative to anthropogenic pollutants, while a few demonstrated a more in-depth knowledge.