PART IIB 4A11 TURBOMACHINERY II DR L XU
2010

4A11-2010 Cribs

1 a) (i)

Actual $F_\theta = \rho V_m s V_{\theta 1}$ $\quad (V_{\theta 2} = 0)$

$$= \rho s V_m^2 \tan\alpha_1$$

Idealised $F_\theta = \frac{1}{2}(p_{01} - p_1)c_m$

$$= \frac{1}{4}\rho V_1^2 c_m \quad \text{(incompressible)}$$

So the lift coefficient is:

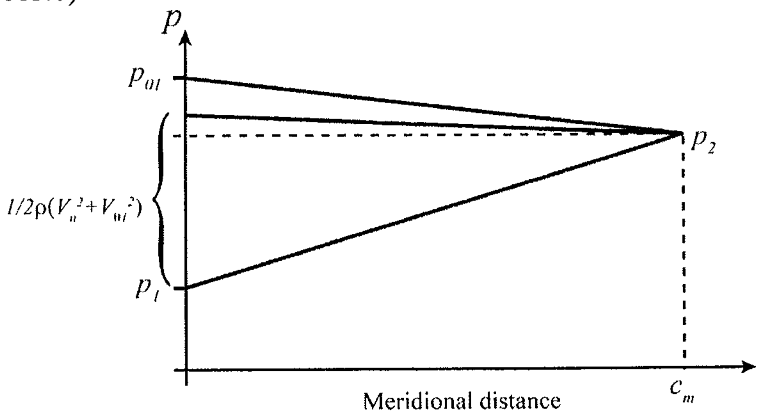
$$L = \frac{\rho s V_m^2 \tan\alpha_1}{1/4\,\rho V_1^2 c_m}$$

$$= 4\frac{s}{c_m}\bigg|_{\lambda=0} \frac{V_m^2 \tan\alpha_1}{V_m^2/\cos^2\alpha_1}$$

$$= 4\frac{s}{c_m}\bigg|_{\lambda=0} \tan\alpha_1 \cos^2\alpha_1$$

[30%]

1 a) (ii) The spanwise component of the inlet flow is not brought to rest. The maximum pressure that can be reached on the blade is not now p_{01} but $p_{01} - 1/2\rho V_s^2$ (for incompressible flow and Vs is the spanwise velocity component). So the "available" dynamic pressure is $1/2\rho(V_n^2 + V_\theta^2)$ where Vn is the inlet velocity component normal to the leading edge in the meridional view ($V_n = V_m \cos\lambda$)



[10%]

1 a) (iii)

Idealised $F_\theta = \frac{1}{4}\rho(V_n^2 + V_{\theta 1}^2)c_m$

$$= \frac{1}{4}\rho V_m^2(\cos^2\lambda + \tan^2\alpha_1)c_m$$

so the lift coefficient for the swept blade is:

$$L = 4\frac{s}{c_m}\bigg|_\lambda \frac{\tan\alpha_1}{\cos^2\lambda + \tan^2\alpha_1}$$

so, equating the lift coefficient for the unswept and swept cases, we obtain:

$$\frac{(s/c_m)_\lambda}{(s/c_m)_{\lambda=0}} = \cos^2\alpha_1\left(\cos^2\lambda + \tan^2\alpha_1\right)$$

[30%]

1 a) (iv) Assuming that the profile loss scales as the surface area of the blades, the loss will scale as the inverse of the result of part (iii) – i.e. the loss scales according to the chord-to-pitch ratio.

For inlet angle = 20 degrees and sweep angle = 30 degrees, we obtain:

$$\frac{\text{Blade area swept}}{\text{Blade area unswept}} = \frac{1}{\cos^2\alpha_1\left(\cos^2\lambda + \tan^2\alpha_1\right)} = 1.28$$

hence the loss increases by 28%.

[10%]

1 b). Due to the change of reference frame between rotating and stationary components, the boundary layer at the endwall of the machine will be skewed (at a different flow angle to the "freestream"). The amount of skew changes according to the operating point of the compressor. What is required is a compressor leading edge that is relatively insensitive to incidence. This can be achieved by reducing the loading at the leading edge by sweeping the endwall sections forward i.e. the chord at the endwall is increased to move the leading edges at the endwall further upstream. Sweep has this effect because the pressure gradient perpendicular to the endwall is small compared to the cross-passage pressure gradient. [20%]

Examiner's comments:

Well answered question. Apart from the one obtained the lowest mark the rest 4 candidates all did well. The physics were understood and algebraic derivations seemed less to be a problem.

2. a). This is the result of radial equilibrium due to the presence of the swirl. The swirl generates a radially inwards pressure gradient which reduces the static pressure at the hub. For a compressor stage, lower pressure at the hub at the rotor exit means less pressure rise through the rotor, thus lower reaction. For a turbine stage, lower pressure at the nozzle exit means higher expansion through the nozzle and less so through the rotor, again results in lower reaction.

Low reaction causes high diffusion in compressor stator, in particular at the hub corner it is likely to trigger large scale corner separation. For turbines the rotor will have to produce large flow turnings with little pressure drop, indicating high diffusion on the suction surface which again is prone to cause high loss and/or separation. [15%]

b). (i) V_r small means only three forces/accelerations in radial direction: radial pressure gradient $\frac{dp}{dr}$, centripetal acceleration $\rho\frac{V_\theta^2}{r}$, and the body force F_r.

Control volume: $\Delta(Vol) = r\cdot\delta\theta\cdot\delta r\cdot\delta x$

Radial pressure force: $F_{r,\Delta p} = -\frac{dp}{dr}\cdot r\cdot\delta\theta\cdot\delta r\cdot\delta x$

Centripetal force: $F_{cen} = \rho\frac{V_\theta^2}{r}\cdot r\cdot\delta\theta\cdot\delta r\cdot\delta x$

Blade body force: $F_{r,b} = F_r\cdot r\cdot\delta\theta\cdot\delta r\cdot\delta x$

$$\Sigma F\big|_r = 0 \quad\Rightarrow [\rho\frac{V_\theta^2}{r} + F_{r,b} + (-\frac{dp}{dr})]\cdot r\cdot\delta\theta\cdot\delta r\cdot\delta x = 0$$

$$\rho\frac{V_\theta^2}{r} = \frac{dp}{dr} - F_{r,b}$$ [15%]

Moment of momentum balance for $\Delta(Vol)$ on the rotor axis:

$$rF_\theta\cdot r\cdot\delta\theta\cdot\delta r\cdot\delta x = \frac{d}{dx}(\dot{m}rV_\theta)\delta x = \dot{m}\frac{d}{dx}(rV_\theta)\delta x$$

$$\because \dot{m} = \rho V_x r\cdot\delta\theta\cdot\delta r \quad\Rightarrow rF_\theta\cdot r\cdot\delta\theta\cdot\delta r\cdot\delta x = \rho V_x r\cdot\delta\theta\cdot\delta r\frac{d}{dx}(rV_\theta)\delta x$$

$$\therefore rF_\theta = \rho V_x\frac{d}{dx}(rV_\theta) \Rightarrow F_\theta = \frac{\rho V_x}{r}\frac{d}{dx}(rV_\theta)$$

[15%]

But $F_{r,b} = |\mathbf{F}_b|\sin\delta;\ F_{\theta,b} = |\mathbf{F}_b|\cos\delta;\ \Rightarrow F_{r,b} = F_{\theta,b}\tan\delta;$ [10%]

Substitute into radial equilibrium:

$$\rho\frac{V_\theta^2}{r} = \frac{dp}{dr} - F_{r,b} = \frac{dp}{dr} - F_{\theta,b}\tan\delta = \frac{dp}{dr} - \rho\frac{V_x}{r}\tan\delta\frac{d}{dx}(rV_\theta)$$

$$\therefore p_o = constant;\ T_o = constant; \Rightarrow \frac{dp}{dr} = -\rho V_x\frac{dV_x}{dr} - \rho V_\theta\frac{dV_\theta}{dr} - \rho V_r\frac{dV_r}{dr}$$

The last term in RHS small and can be neglected, leading to

$$\rho\frac{V_\theta^2}{r} = -\rho V_x\frac{dV_x}{dr} - \rho V_\theta\frac{dV_\theta}{dr} - \rho\frac{V_x}{r}\tan\delta\frac{d}{dx}(rV_\theta)$$

$$\therefore p_o = constant;\ T_o = constant; \Rightarrow \frac{dp}{dr} = -\rho V_x\frac{dV_x}{dr} - \rho V_\theta\frac{dV_\theta}{dr} - \rho V_r\frac{dV_r}{dr}$$

$$\therefore \frac{V_\theta^2}{r} + \frac{V_\theta dV_\theta}{dr} = \frac{V_\theta}{r}\frac{d(rV_\theta)}{dr} = 0 \quad \because (rV_\theta\ \textit{function of}\ x\ \textit{only})$$

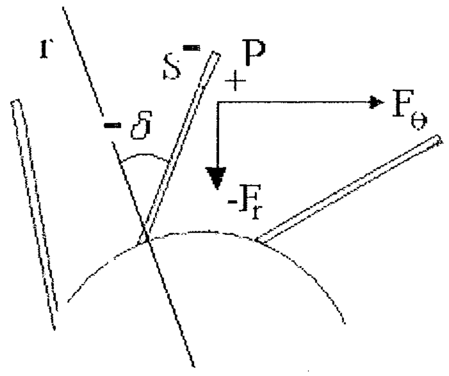
$$\therefore V_x\frac{dV_x}{dr} = \frac{V_x}{r}\tan\delta\frac{d}{dx}(rV_\theta) \Rightarrow dV_x = \tan\delta\frac{d}{dx}(rV_\theta)(-\frac{dr}{r})$$

$$\Rightarrow V_{x,c} - V_{x,h} = \ln\frac{r_h}{r_c}\frac{d}{dx}(rV_\theta)\tan\delta$$

[30%]

2. c. $\rho\frac{V_\theta^2}{r}=\frac{dp}{dr}-F_{r,b}$ states that radial equilibrium is off-set by $-F_{r,b}$. For the same V_θ^2, if $F_{r,b}$ is pointing inwards, the value of $\frac{dp}{dr}$ will be reduced. This is achieved by leaning the blade with the pressure surface facing downwards as shown in the sketch. This will reduce the hub reaction as the reduction of the hub static pressure is relieved. This is particularly

effective with high aspect ratio blades when the integrated effect of the pressure difference along a long blade span is significant. [15%]

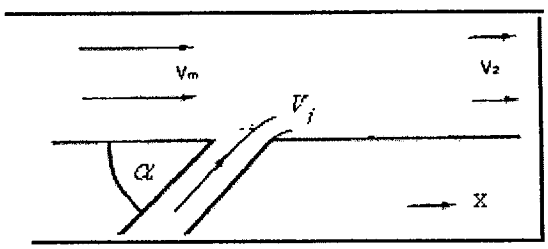


Examiner's comments:

Relatively the most popular question and the lowest averaged mark. It is a relatively long question and the main part, the simple radial equilibrium with radial blade force, was well answered, but most of the candidates were unable to derive the relationship between the circumferential blade force and blade circulation thus not able to complete the question. The physics of using the blade lean to control the stage reaction, however, was well understood and answered fully.

3. a). The aerodynamic losses in the tip region of axial turbine stages are mainly due to the entropy rise caused by the interaction of the leakage flow and the main stream flow. The loss level is, to the first approximation, proportional to the leakage mass flow rate. The mechanisms for driving the leakage mass flow are different for shrouded and unshrouded rotor tips. For the shrouded tip, the driving force is the pressure difference across the rotor tip (from inlet to the exit) but for the unshrouded rotor, the driving force is the pressure difference across the blade tip section, from pressure side to the suction side. For a low reaction stage, the pressure difference across the rotor row is small but high blade loading means that the pressure difference across the tip is high. This means with the shrouded tip, there is no pressure surface to suction surface leakage due to the shroud and the driving force for the leakage flow through the shroud from the leading edge to the trailing edge is small. Thus shrouded tip is preferred for a low reaction stage. For higher reaction stage, the trend of the pressure difference is reversed so that the unshrouded tip is preferred. The stage reaction controls the driving force for the leakage flow therefore is an important design parameter to consider. [25%]

3. b). For the mixing loss of a small jet with the main flow at constant pressure, assuming that p_o and T_o for both streams are the same, applying axial momentum equation, with mass continuity and energy equation:



$$T\Delta s = \Delta h = \Delta h_o - \frac{1}{2}\Delta V^2;$$

$$\Delta h \cdot \dot{m}_m = \frac{1}{2}\dot{m}_m (V_1^2 - V_2^2)$$

$$(momentum\ in\ flow\ direction):\ \dot{m}_m V_m + \dot{m}_j V_j \cos\alpha \cong (\dot{m}_m + \dot{m}_j)V_2$$

$$substitute: \frac{1}{2}\dot{m}_m (V_1^2 - V_2^2) = \dot{m}_j (V_2^2 - V_1 V_j \cos\alpha) + H.O.T(\frac{\dot{m}_j}{\dot{m}_m})$$

For small $\frac{\dot{m}_j}{\dot{m}_m}$, when $V_2 \approx V_1 \approx V$

$$\Delta h \cdot \dot{m}_m = \dot{m}_j (V_2^2 - VV_j \cos\alpha) \quad \Rightarrow \quad \Delta h = \frac{\dot{m}_j}{\dot{m}_m} V^2 (1 - {}^{V_j}\!/_{V} \cos\alpha)$$

[20%]

3. c). In the rotor relative frame of reference, the stagnation temperature of the leakage flow is the same as that of the main flow. As the kinetic energy of each velocity component adds up such as, $\frac{1}{2}V^2 = \frac{1}{2}V_x^2 + \frac{1}{2}V_\theta^2 + \frac{1}{2}V_r^2$ etc, the mixing loss in 3.b). above can be expressed as the kinetic energy loss in x and θ directions respectively. Assuming the r component of the velocity is very small:

$$\Delta h = \frac{\dot{m}_l}{\dot{m}_m}[V_\theta^2 (1 - \frac{V_{\theta l}}{V_{\theta 2}}) + V_x^2 (1 - \frac{V_{xl}}{V_{x2}})]$$

Compared to $V_{\theta,2}^2$ term, $V_{x,2}^2$ is small and assume V_{xl} is close to V_{x1} so $V_{x,2}^2(1 - \frac{V_{xl}}{V_{x2}})$ term is much smaller than $V_{\theta,2}^2(1 - \frac{V_{\theta l}}{V_{\theta 2}})$ term and can be neglected, so the shrouded tip mixing loss can be approximated as:

$$\Delta h = \frac{\dot{m}_l}{\dot{m}_m} V_\theta^2 (1 - \frac{V_{\theta l}}{V_{\theta 2}}) = \frac{\dot{m}_l}{\dot{m}_m}(1 - \frac{V_{x1}\tan\alpha_1}{V_{x2}\tan\alpha_2}) V_2^2 \sin^2\alpha_2$$

Further assume the change of the axial velocity across the blade row is small: $V_{x1} \approx V_{x2}$

$$\Delta h = \frac{\dot{m}_l}{\dot{m}_m}(1 - \frac{\tan\alpha_1}{\tan\alpha_2}) V_2^2 \sin^2\alpha_2;$$

Loss coefficient: $\varsigma = \frac{T\Delta s}{0.5 \cdot V_2^2} \cong \frac{\Delta h}{0.5 \cdot V_2^2} = 2\frac{\dot{m}_l}{\dot{m}_m}(1 - \frac{\tan\alpha_1}{\tan\alpha_2})\sin^2\alpha_2$

[30%]

3. d). At the ideal seal condition $\dot{m}_l$=0 so that the Δh due to the leakage flow mixing is zero. However the fluid in cavities up- and downstream of the blade tip shroud will be driven by the pressure field of the blade to create some mixing losses in the cavities as well as in the blade passages due to vortex interactions, which is driven out of the cavities by the blade pressure field. The shroud surface friction also contributes some entropy generation, which is associated with the shroud configurations. Minimizing the dimension of the cavity wells as well as smoothing the surface of the shroud in circumferential direction can reduce these losses.

[25%]

Examiner's comments:

This was a generally well answered question. The physics of the loss generation was well understood and the first part, qualitative comparison of the different types of the rotor tip configurations was answered correctly. Most candidates were able to derive the mixing loss due to a small jet into a main stream flow but less so when asked to extend this to the shrouded tip cases. Again the qualitative question relating the "offset" loss was well answered, showing the candidates were more comfortable with physical explanations than with algebraic or quantitative analyses.