

Question 1

This is a straightforward application of the theory discussed in class and notes.

(a) [20%]

$$\dot{m}c_p \frac{dT}{dx} - \lambda \frac{d^2T}{dx^2} - wQ = 0,$$

In the preheat zone, we assume that there are *no chemical reactions* as the temperature is relatively low. Therefore, we can neglect the third term in the equation, and write:

$$\dot{m}c_p \frac{dT}{dx} - \lambda \frac{d^2T}{dx^2} = 0$$

which can be integrated once to yield:

$$\frac{\dot{m}c_p}{\lambda}(T - T_R) = \left(\frac{dT}{dx}\right)_{x=0^-}$$

The boundary conditions are on the reactant side, $x \rightarrow -\infty$, $T = T_R$ to the boundary of the reaction zone, $x = 0^-$, QED.

(b) [20%]

Starting from the same equation, in the reaction zone we assume that the *convective term is negligible*, as there is little change in the temperature, so there is a balance between diffusive and reactive terms:

$$\begin{aligned} -\lambda \frac{d^2T}{dx^2} - wQ &= 0 \\ -\lambda \frac{1}{2} \frac{d}{dx} \left(\frac{dT}{dx}\right)^2 &= wQ \left(\frac{dT}{dx}\right) \\ \left(\frac{dT}{dx}\right)_{x=0^+}^2 &= 2 \frac{Q}{\lambda} \int_T^{T_P} w dT \end{aligned}$$

, where we integrate from the flame front at $x = 0^+$ to the product region, where $\left(\frac{dT}{dx}\right) = 0$.

(c) [20%] The gradients from two equations above can be matched at the flame front $x = 0$:

$$\frac{\dot{m}c_p}{\lambda}(T_0 - T_R) = \frac{\rho_R u_L c_p}{\lambda}(T_0 - T_R) = \left(2 \frac{Q}{\lambda} \int_{T_0}^{T_P} w dT\right)^{1/2}$$

We now recall that:

$$\dot{m}c_p(T_P - T_R) = \dot{m}Q$$

and we make the approximation that at the interface of the reaction zone, the temperature is not very different from the products, so that $(T_0 - T_R) \approx (T_P - T_R)$

Substituting, we have:

$$\frac{\rho_R u_L c_p}{\lambda} (T_P - T_R) = \left(2 \frac{Q}{\lambda} \int_{T_0}^{T_P} w dT \right)^{1/2}$$

$$u_L = \frac{\lambda}{\rho_R c_p (T_P - T_R)} \left(2 \frac{c_p (T_P - T_R)}{\lambda} \int_{T_0}^{T_P} w dT \right)^{1/2}$$

which simplifies to the desired expression

$$u_L = \left(\frac{2\lambda}{\rho_R^2 c_p (T_P - T_R)} \int_{T_0}^{T_P} w dT \right)^{\frac{1}{2}}$$

Notes: Several students had trouble with transforming the integral in space to an integral in temperature. A second problem appeared in recognizing the relationship between temperature rise and Q

(d) [20%] Here we make the assumption of *high activation energy*, where E/RT is a large number in the reaction region. This leads to two consequences: (i) the reaction only starts at very high temperatures, so that $T(0) \approx T_P$, (ii) the exponential term can be expanded in series for the small parameter $\epsilon = \frac{T_P - T}{T_P}$ as follows:

$$\frac{E}{RT} = \frac{E}{RT_P} \frac{T_P}{T} = \frac{E}{RT_P} \frac{T_P}{T_P - (T_P - T)}$$

$$= \frac{E}{RT_P} \frac{1}{1 - \epsilon} \approx \frac{E}{RT_P} (1 + \epsilon)$$

The reaction for the integral can then be simplified as by assuming that :

$$\int_{T_I}^{T_P} w dT = \int_{T_I}^{T_P} A \exp\left(-\frac{E}{RT_P}(1 + \epsilon)\right) dT$$

$$= A \exp\left(-\frac{E}{RT_P}\right) \int_0^{\beta_I} \exp(-\beta), d\beta$$

where $\beta = \frac{E}{RT_P} \epsilon$ and $dT = -T_P d\epsilon$. Finally We assume that $\beta_I \gg 1$ based on the fact that the activation energy is large, so we take $\beta_I \rightarrow \infty$ in the evaluation of the integral to yield the desired equation.

Note: only a couple of students remembered this expansion, and incorrectly integrated the exponential, forgetting that T appears in the denominator rather than the numerator of the integrand.

(d) [20%] The equation denotes a significant dependence of the laminar flame speed on the product temperature, with a dependence proportional to $\frac{T_P}{T_P - T_R} \exp(-E/(RT_P))$. Peak flame temperatures in hydrocarbon-oxygen-carbon dioxide mixtures are obtained around the stoichiometric ratio for oxygen, and at the minimum carbon dioxide concentrations.

(e) [20%] Running systems with oxygen for combustion of hydrocarbons minimizes the amount of diluent nitrogen in the exhaust stream, thus simplifying the operation of carbon dioxide removal systems for sequestration. In addition, the absence of nitrogen eliminates the problem of nitric oxide formation at high temperatures. However, practical materials constraints limit the temperatures that can be operated, so that carbon dioxide in such a system can be used as a diluent. The price to be paid in such oxyfuel

system is that the separation of oxygen and nitrogen must be done upfront (but typically easier than removal of CO₂ from hot gases).

Note: several students misinterpreted the question, writing instead about the O₂/CO₂ mixtures after burning, rather than the effect of O₂/CO₂ in the *reactants*.

Examiner's comment:

Most students attempted this question, as it was a straightforward application of the theory developed in class. Only one candidate remembered the high activation energy expansion of the exponential term in the integral.

Question 2

Consider a non-premixed flamelet with single step reaction $F + sO \rightarrow P$, which can be characterised parametrically as a strained non-premixed flame. The mixture fraction is defined as $Z = (\beta - \beta_2)/(\beta_1 - \beta_2)$, where $\beta = sY_F - Y_O$, 1 and 2 are the two different streams, Y_F and Y_O the mass fractions of fuel and air, and s the mass stoichiometric ratio of oxidizer to air.

(a) [20%]

Determine the stoichiometric mixture fraction Z_s for (i) streams of methane on one side pure air on the other, (ii) methane on one side pure oxygen on the other

For methane, $\nu = c + h/4 = 1 + 4/4 = 2$, $s = 2W_{O_2}/W_{CH_4} = 2 \times 32/16 = 4$. We have $Z = \frac{(sY_F - Y_O) - (sY_F - Y_O)_2}{(sY_F - Y_O)_1 - (sY_F - Y_O)_2}$. At stoichiometric conditions, $sY_f - Y_o = 0$. For case (i), we have $Y_{F,1} = 1$, $Y_{F,2} = 0$, $Y_{O,1} = 0$, $Y_{O,2} = 0.23$, so that :

$$Z_s = \frac{0 - (4 \times 0 - 0.23)}{(4 \times 1 - 0) - (4 \times 0 - 0.23)} = 0.0543$$

For case (ii), we have $Y_{F,1} = 1$, $Y_{F,2} = 0$, $Y_{O,1} = 0$, $Y_{O,2} = 1$, so that:

$$Z_s = \frac{0 - (4 \times 0 - 1)}{(4 \times 1 - 0) - (4 \times 0 - 1)} = 0.20$$

(b) [30%] We start from:

$$\rho v \frac{\partial Z}{\partial y} = \frac{\partial}{\partial y} \left(\rho D \frac{\partial Z}{\partial y} \right)$$

and substitute $v = -ay$ to yield

$$\rho(-ay) \frac{\partial Z}{\partial y} = \frac{\partial}{\partial y} \left(\rho D \frac{\partial Z}{\partial y} \right)$$

For constant ρ and D , we have:

$$-ay \frac{\partial Z}{\partial y} = D \frac{\partial^2 Z}{\partial y^2}$$

Using the coordinate transform suggested: $\eta = y/\sqrt{2D/a}$, we have:

$$\begin{aligned} \frac{\partial Z}{\partial y} &= \frac{\partial Z}{\partial \eta} \frac{\partial \eta}{\partial y} = \frac{1}{\sqrt{2D/a}} \frac{\partial Z}{\partial \eta} \\ \frac{\partial^2 Z}{\partial y^2} &= \frac{\partial^2 Z}{\partial \eta^2} \frac{1}{2D/a} \end{aligned}$$

so that

$$\begin{aligned} -a\eta \sqrt{2D/a} \frac{1}{\sqrt{2D/a}} \frac{\partial Z}{\partial \eta} &= D \frac{1}{2D/a} \frac{\partial^2 Z}{\partial \eta^2} \\ \frac{\partial^2 Z}{\partial \eta^2} + 2\eta \frac{\partial Z}{\partial \eta} &= 0 \end{aligned}$$

QED.

(c) [20%] The boundary conditions to be used are:

$$\begin{aligned} y = -\infty &\rightarrow Z = 1 \\ y = +\infty &\rightarrow Z = 0 \end{aligned}$$

The equation is separable, so that if $f = \frac{\partial Z}{\partial \eta}$ we have:

$$\begin{aligned}\frac{df}{d\eta} &= -2\eta f \\ \frac{df}{f} &= -2\eta d\eta \\ \ln f &= -\eta^2 + \ln C_1 \\ \frac{\partial Z}{\partial \eta} &= C_1 \exp(-\eta^2) \\ Z &= C_1 \underbrace{\int_0^\eta \exp(-\eta^2) d\eta}_{(\sqrt{\pi}/2)\text{erf}(\eta)} + C_2\end{aligned}$$

Using the boundary conditions, and the definite integrals $\text{erf}(-\infty) = -1$ and $\text{erf}(+\infty) = 1$, we have:

$$\begin{aligned}Z(+\infty) &= \frac{\sqrt{\pi}}{2}C_1 + C_2 = 1 \\ Z(-\infty) &= -\frac{\sqrt{\pi}}{2}C_1 + C_2 = 0\end{aligned}$$

We have

$$\begin{aligned}2C_2 &= 1 \rightarrow C_2 = 1/2 \\ C_1 &= \frac{2}{\sqrt{\pi}}C_2 = -\frac{1}{\sqrt{\pi}} \\ Z &= -\frac{1}{\sqrt{\pi}}(\sqrt{\pi}/2)\text{erf}(\eta) + 1/2 = \frac{1}{2}(1 - \text{erf}(\eta))\end{aligned}$$

QED.

This is a very straightforward plot of the solution for Z , where the intercept for Z_s corresponds to the flame location.

(e) [15%]

Examiner's comment:

Not a favourite question, even though it was as straightforward as Q1. Several students sketched the structure of the flame vs. mixture fraction rather than as a function of space, and this was given partial credit.

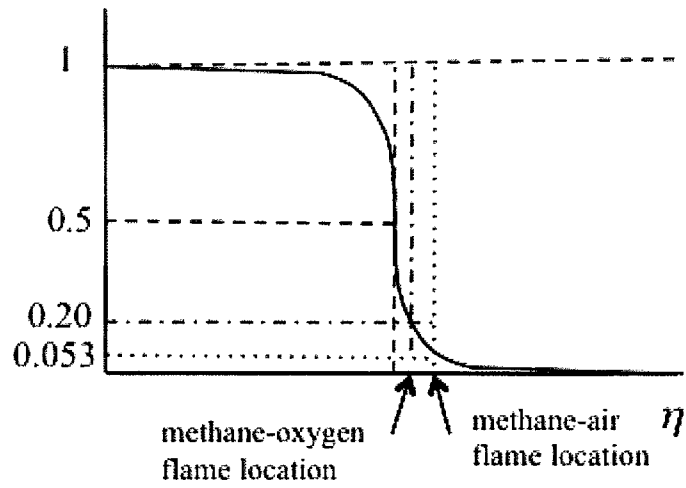
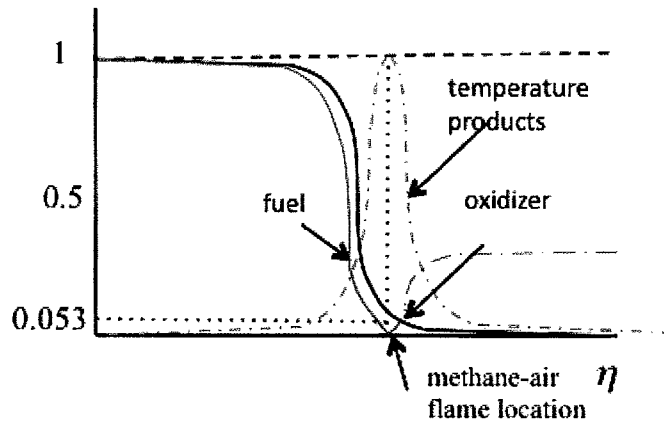


Figure 1: (d) [15%]



- profiles are linearly proportional to Z
- fuel and oxidizer are zero at stoichiometric Z_s
- similar for O_2-CH_4 , different location

Figure 2: (e) [15%]

Question 3

Write brief notes which would convey to an engineer not versed in IC engine technology why:

(a) a throttle is required to vary the load of gasoline engines, [15%]

Gasoline engines work with a pre-mixed air/fuel mixture, and the start of combustion is initiated with a spark. These engines work with air/fuel mixtures around stoichiometric, for several reasons: (a) a successful ignition depends on a reliably growing flame kernel, and this will only occur if the mixture is close to stoichiometric, (b) burn up of the last parts of the mixture depends on a high flame temperature i.e. close to stoichiometric, and most importantly, (c) the 3-way catalyst requires operation within a few percent of stoichiometric. Therefore, since the total fueling rate is proportional to the air flow rate, the only way to vary the torque is to control the initial charge mass. In normally aspirated engines this means the inlet air has to be throttled (an exception would be a stratified charge engine, but these are not common).

(b) the crank angle over which combustion takes place is roughly independent of engine speed for gasoline engines. [15%]

The combustion in a gasoline engine proceeds as a turbulent flame front, and the burn rate (in terms of the mass burning rate) depends on the total area of flame locally the flame burns at the laminar rate. The energy in the inlet flow through the inlet valve, partly stored as bulk swirl (barrel, axial) during intake and compression, breaks up into turbulence before ignition, and causes the wrinkling of the flame front. This energy increases with engine speed, which in turn increases the turbulence levels, and hence the mass burning rate. It turns out that the increase in mass burning rate is almost proportional to the engine speed. (High turbulence engines have a faster burning rate, which leads to a higher efficiency, but the scaling still applies).

(c) the bore sizes of most automotive gasoline engines are similar. [20%]

The engine efficiency is partly dependent on the compression ratio, which should be as high as possible. The compression ratio is limited by the onset of knock, which occurs if the expanding flame front fails to burn the whole mixture before the chemical induction time is reached. Thus, the bigger the bore of the cylinder, the smaller the compression ratio achievable. This simple argument would suggest that, for a given engine capacity, the number of cylinders should be very high, with a small bore. However, the efficiency of the engine is also reduced due to heat transfer in the cylinder, and as the power per cylinder scales as volume, but the heat transfer as area, the largest possible number of cylinders should be used. (This also explains why the bore/stroke ratio tends to be about unity, as the surface area is minimised per unit volume). However, frictional losses (and cost) also tend to increase with many small cylinders. Thus an optimum bore size is expected and it happens to be about 80 mm for the octane rating of typical gasoline.

(d) diesel engines of a similar power output typically have a better sfc than gasoline engines [20%].

The diesel engine operates on a compression ignition cycle, where compression ratio is not limited by knocking, and can thus be higher than gasoline engines. Thus higher efficiencies are possible purely on thermodynamic grounds. In addition, the air fuel ratio is significantly leaner, leading to cooler combustion products, and hence less heat transfer. Finally, and probably most important for automobile engines, at part load operation, no throttling is required, and the high pumping losses found in gasoline engines at part load are avoided.

Note however that the parasitic losses associated with the diesel high pressure fuel pump are not insignificant, and emissions regulations are forcing the use of higher and higher fuel injection pressures (and hence increased parasitic losses), and lower compression ratios (to reduce combustion temperatures and hence NO_x). In addition, since most turbo diesel engines are turbocharged, there is also a parasitic loss due to inefficiencies in the turbocharger set.

(e) large cylinder volume diesel engines have a better sfc than small cylinder volume engines [15%].

As already referred to in (c), heat losses scale as surface area, while work per cycle scales as volume. Thus the heat transfer losses per unit output decrease as the bore size increase.

(f) Turbochargers are useful, but have disadvantages [15%].

The output from an engine is ultimately limited by the oxygen (i.e. air) supply more fuel can always be added, but if there is no oxygen left, there is no extra power. Turbochargers enable more air to be put in the cylinders, via a higher inlet pressure. The limit to this pressure is largely via mechanical stresses. Turbochargers allow a much smaller engine to be used for the same power output leading to significant efficiency improvements (smaller engine, less friction, operates with less throttling at light loads compared with the equivalent NA engine).

The demerits are:

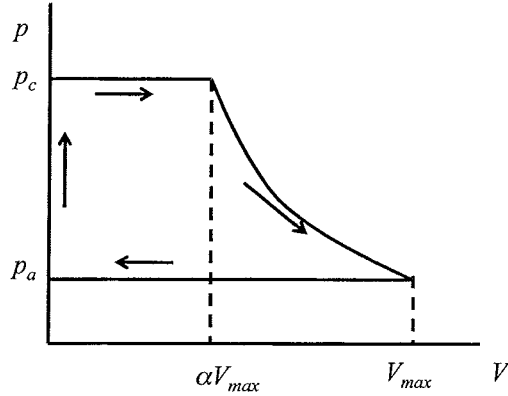
(i) Cost (ii) For gasoline engines, even with intercooling, the inlet temperature is increased, and along with the higher pressure, knocking combustion limits the compression ratio, or alternatively non-optimal spark timings are required. (iii) Sizing of the turbocharger is a big compromise a small turbo will give good boost at low engine rpm (low air flows) but will produce too much boost at high air flows, requiring bypassing of the turbine to limit the power input this is very inefficient (it is essentially a throttle). Vice versa for big turbos low boost at low rpm. (iv) It takes time for the turbo to spin up leading to turbo lag compared with an NA engine, the driver experiences a significant delay in response to a torque demand. This time lag also leads to higher emissions during transients when air is not yet available to consume the extra fuel injected.

Examiner's comments:

Many candidates gave long-winded answers that were well off the mark, but a select few had paid close attention to the lectures and notes and gave sensible and correct answers.

Question 4

(a) [10%]



(b) [10%]

For the reversible adiabatic expansion, $pV^\gamma = p_c(\alpha V_{max})^\gamma = p_a V_{max}^\gamma$. Thus,

$$\alpha = \left(\frac{p_a}{p_c}\right)^\gamma$$

(c) [30%]

The work done in a cycle is found from $W = \oint p dV = p_c \alpha V_{max} + \int_{\alpha V_{max}}^{V_{max}} p dV - p_c V_{max}$. But,

$$\int_{\alpha V_{max}}^{V_{max}} p dV = \frac{V_{max}(\alpha p_c - p_a)}{\gamma - 1}$$

so that

$$W = \frac{\gamma}{\gamma - 1} V_{max} p_a \left[\left(\frac{p_c}{p_a}\right)^{(\gamma-1)/\gamma} - 1 \right]$$

(d) [10%]

The air mass flow rate is the volume induced times the density times the number of cycles per unit time, N :

$$\dot{m} = \alpha V_{max} \frac{p_c}{RT_c} N$$

(e) (i) [20%]

The power output is $P = W N$. As the reservoir pressure drops, there is a minimum reservoir pressure at a maximum speed that will continue to deliver the minimum power. thus we need to find the value of $p_{c,min}$ that satisfies:

$$P = W N_{max} = \frac{\gamma}{\gamma - 1} V_{max} p_a \left[\left(\frac{p_{c,min}}{p_a}\right)^{(\gamma-1)/\gamma} - 1 \right] N_{max}$$

Which can be solved for $p_{c,min} = 1.60$ bar.

(e) (ii) [20%]

The pressure in the reservoir will decrease according to:

$$\frac{dp_c}{dt} = -\dot{m} \frac{RT_c}{V_{res}}$$

and $\dot{m} = \alpha V_{max} \frac{p_c}{RT_c} N$.

The speed is now *variable*, for constant power and decreasing work per unit cycle as p_c decreases,
 $N = P/W$

Substituting for \dot{m} and N in the expressions above, we have:

$$\left(1 - \left(\frac{p_c}{p_a}\right)^{(1-\gamma)/\gamma}\right) dp_c = -\frac{P}{V_{res}} \frac{\gamma-1}{\gamma} dt$$

Substituting,

$$\left(\frac{p_c}{p_a} - \frac{p_{c,i}}{p_a}\right) - \gamma \left[\left(\frac{p_c}{p_a}\right)^{1/\gamma} - \left(\frac{p_{c,i}}{p_a}\right)^{1/\gamma}\right] = -\frac{P}{p_a V_{res}} \frac{\gamma-1}{\gamma} t$$

For $p_c = 1.60$ bar, we have

$$(1.6 - 300) - 1.4[(1.6^{1/1.4} - (300)^{1/1.4})] = -\frac{5000}{(10^5)(0.34)} \frac{0.4}{1.4} t$$

and thus $t = 5189$ s.

Examiner's comment:

Most candidates were able to handle the bulk of the question well, except for the questions that required some thinking about what remained constant (power, not speed).