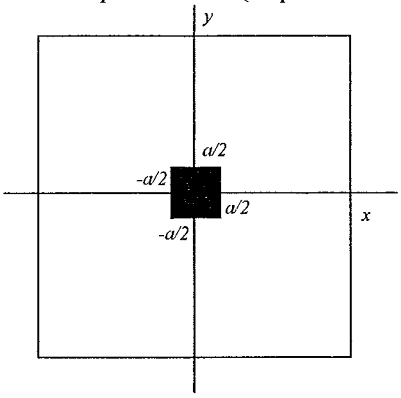
Q1 a) Fraunhofer region = Far field pattern = FT{ Aperture function}



$$F(u,v)=\iint_{\pm\infty} f(x,y)\,e^{2\pi j(ux+vy)}dxdy$$

$$=\int_{-a/2}^{a/2}\int_{-a/2}^{a/2} A\,e^{2\pi j(ux+vy)}dxdy = A\int_{-a/2}^{a/2} e^{2\pi j(ux)}dx\int_{-a/2}^{a/2} e^{2\pi j(vy)}dy$$

$$=A\left[\frac{e^{2\pi j(ux)}}{2\pi ju}\right]_{-a/2}^{a/2}\left[\frac{e^{2\pi j(vy)}}{2\pi jv}\right]_{-a/2}^{a/2}$$

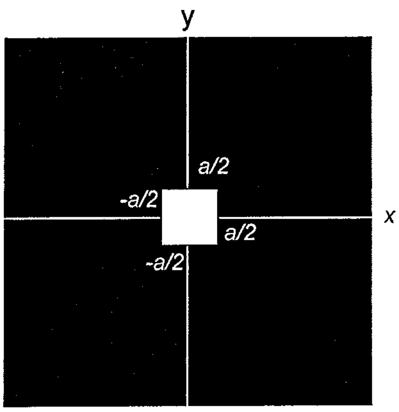
$$=A\left[\frac{1}{2\pi jv}\left(e^{\pi jva}-e^{-\pi jva}\right)\right]$$

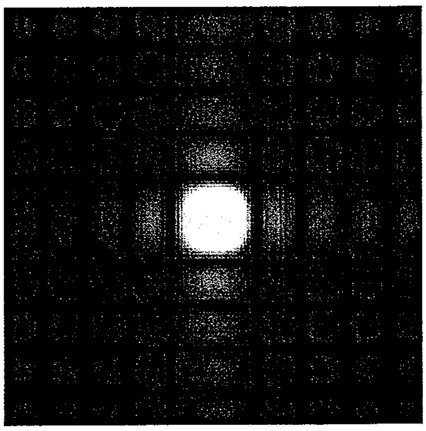
$$=A\left[\frac{1}{u}\sin(\pi ua)\right]\left[\frac{1}{v}\sin(\pi va)\right]=Aa^2\mathrm{sinc}(\pi au)\,\mathrm{sinc}(\pi av)$$

Use Goodman's rule

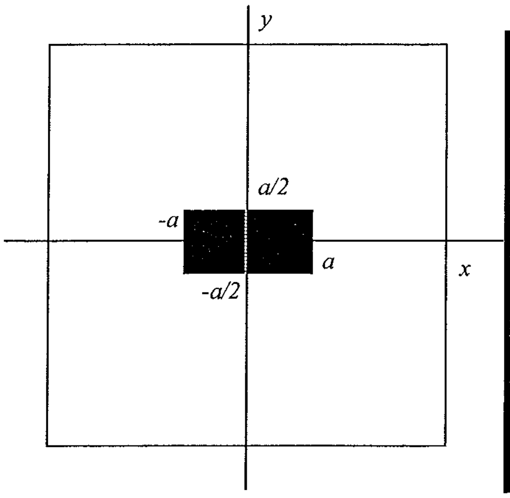
$$z>>\frac{k\left(x_{\max}^2+y_{\max}^2\right)}{2}=\frac{2\pi\left(a^2+a^2\right)}{2\lambda}=\frac{2\pi a^2}{\lambda}$$

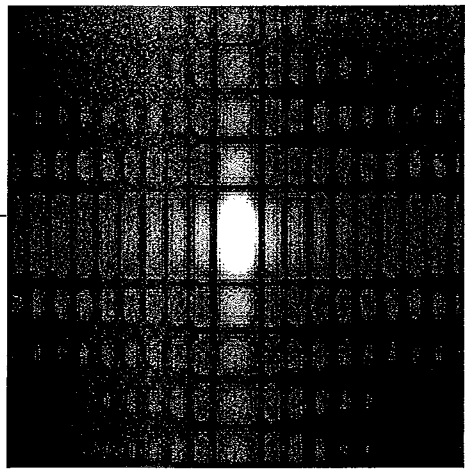
b) The original aperture gives a far field sinc function





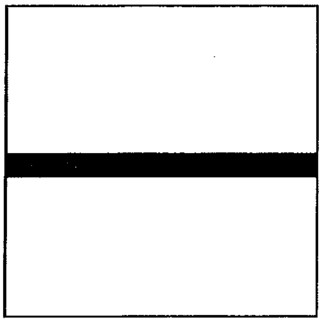
We can then extend the aperture into the x dimension and calculate a new F field

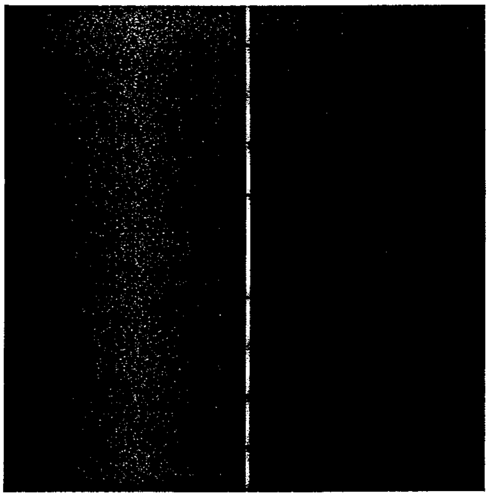




$$F(u,v) = 2Aa^2 \mathrm{sinc}(2\pi au)\mathrm{sinc}(\pi av)$$

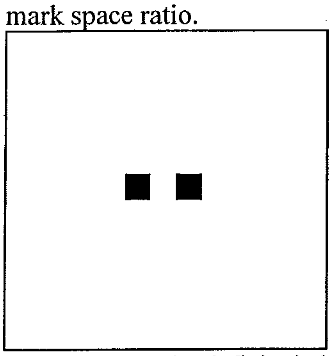
If we keep extending it to infinity it becomes a delta function in the RPF.

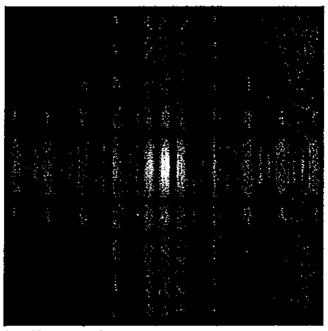




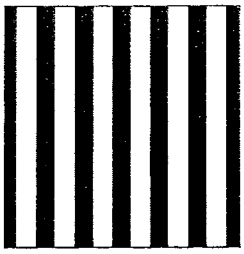
The original rectangular aperture is defined as a single pixel. By combining an array of these pixels at various positions on a regular grid, it is possible to generate a complex amplitude function in the far field. Such a 2-D combination of these pixels in various positions is defined as a Hologram and the pattern generated by the hologram if the far field is the Replay Field.

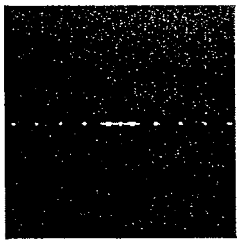
c) We can generate a grating using a 1D analogy as before but this time we include a 50/50 mark space ratio.





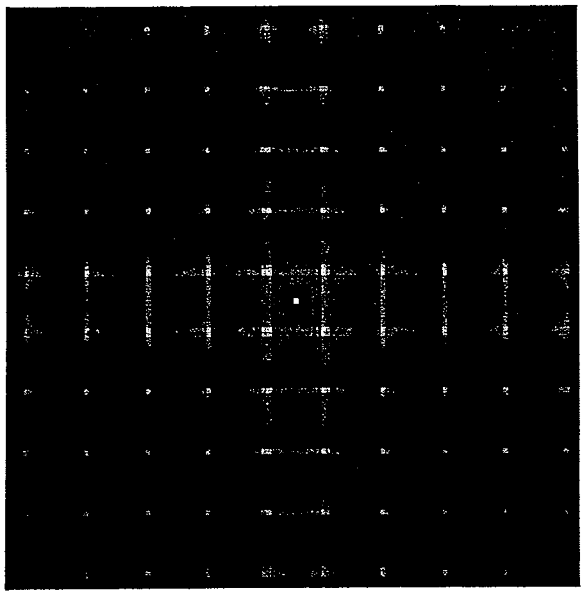
which we extend to infinity in both dimensions





We can then make a chequerboard a the product of these two functions

Hence in the RPF the result wil be the convolution of the FPFs of the two factors (H and V gratings).

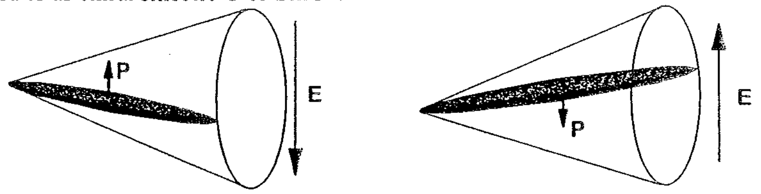


The central 4 dots (in a square) are the first order the next are the 3rd order as expected from a square wave based geometry. The central dot is the zero order. If the chequerboard is binary phase then this will not be present as the pattern is fully DC balanced (zero average)

Examiner's comment:

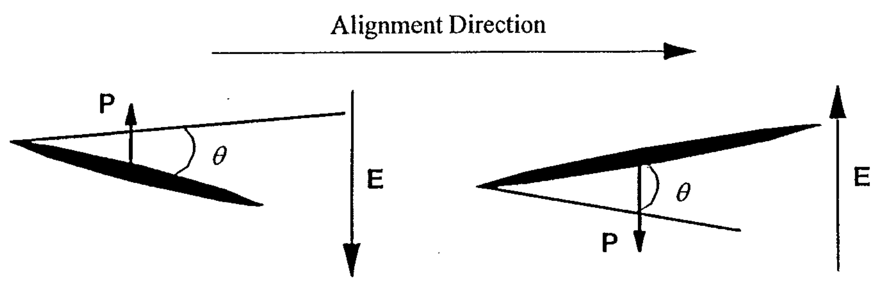
This question was mostly book work to derive the effects of diffraction theory on an aperture. The majority of the candidates answered it well and it was perhaps a little to straightforward..

Q2 a) One of the most useful smectic mesophases is the smectic C (SmC) phase as the molecules are highly ordered and form layers with the molecules tilted within each layer. When an electric field is applied across the molecules there is little interaction as the field is almost perpendicular to the main axis of the LC molecules. The smectic C structure can be improved by adding chirality to the molecular structure which adds an extra dipole perpendicular to the molecular axis of the LC material. This is often is referred to as chiral smectic C or SmC*.



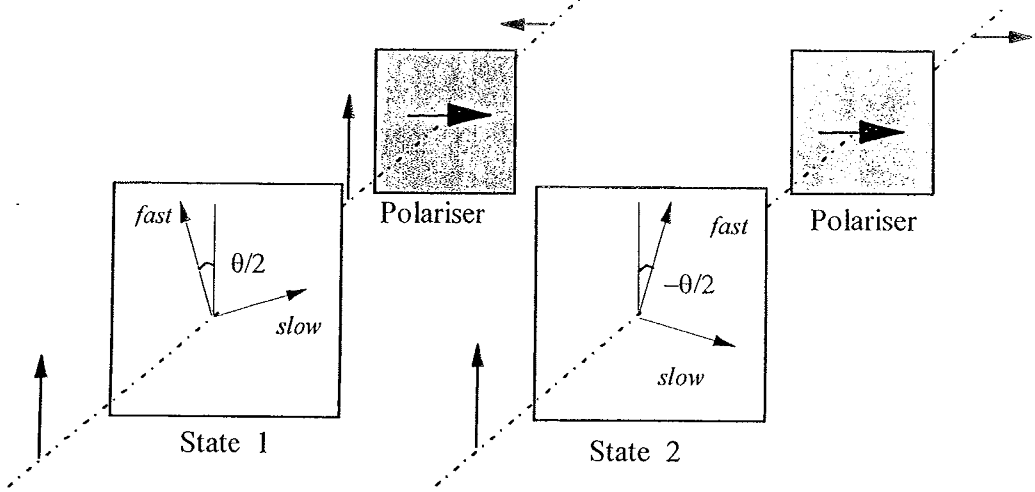
The addition of this dipole due to the chirality of the molecules means that the molecules are in an ordered structure but they are free to move. An applied field across the molecules is now parallel to the extra dipole and the interaction will exert a force on the molecules causing them to rotate about a cone of angles as the molecules are designed so that one end is fixed in the layered structure. The dipole **P** of the molecule, which is perpendicular to its length is often referred to as the Spontaneous Polarisation. When an electric field **E** is applied to the cell, there is an interaction between the **E** and **P**, which forces the molecule to move around the cone to a point of equilibrium. The SmC* phase in thick cells is not ferroelectric because in the equilibrium state the **P** dipoles of the molecules interact with each other forming a helix along the axis of the cell which results in no overall retardance.

If the FLC is restricted to a cell thickness of 2-5μm then the helix of along the cell is suppressed and the molecules are bounded into two stable states either side of the director cone. The angle between these two states is defined as the switching angle θ. This is referred to as a surface stabilised FLC geometry an creates a high degree of ferroelectricity and creates a large birefringent electro-optical effect. The penalty for doing this is that the molecules are only stable in the two states an therefore the modulation will only be binary. The up side to this binary modulation is that it can be very fast (~10μsec) and that the stability can lead to the molecules remaining in the two states in what is known as bistable switching.



b) Binary Phase Modulation

If the light is polarised so that its direction bisects the switching angle and an analyser (polarisor) is placed after the pixel at 90° to the input light, then phase modulation is possible. If we start with vertically polarised light, then the FLC pixel fast axis positions must bisect the vertical axis and will be oriented at angles of $\theta/2$ and $-\theta/2$ respectively for each state.



State 1.

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2}\cos^2\frac{\theta}{2} + e^{j\Gamma/2}\sin^2\frac{\theta}{2} & -j\sin\frac{\Gamma}{2}\sin(\theta) \\ -j\sin\frac{\Gamma}{2}\sin(\theta) & e^{j\Gamma/2}\cos^2\frac{\theta}{2} + e^{-j\Gamma/2}\sin^2\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix}$$

$$= \begin{pmatrix} -V_y j \sin\frac{\Gamma}{2}\sin(\theta) \\ 0 \end{pmatrix}$$

State 2

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2}\cos^2\frac{\theta}{2} + e^{j\Gamma/2}\sin^2\frac{\theta}{2} & j\sin\frac{\Gamma}{2}\sin(\theta) \\ j\sin\frac{\Gamma}{2}\sin(\theta) & e^{j\Gamma/2}\cos^2\frac{\theta}{2} + e^{-j\Gamma/2}\sin^2\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix}$$

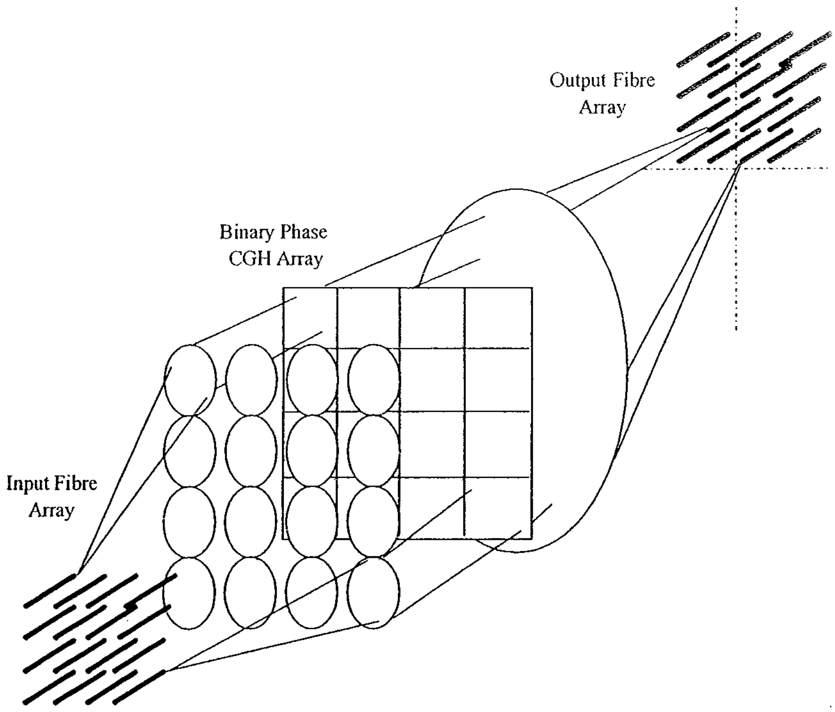
$$= \begin{pmatrix} V_y j \sin\frac{\Gamma}{2}\sin(\theta) \\ 0 \end{pmatrix}$$

From these two expression we can see that the difference between the two states is just the minus sign, which means that the light has been modulated by 180° (π phase modulation). Moreover, the phase modulation is independent of the switching angle θ and the retardation Γ. These parameters only effect the loss in transmission through the pixel which can be gained by squaring the above expressions.

$$T = V_y^2 \sin^2(\theta)\sin^2\left(\frac{\Gamma}{2}\right)$$

Hence maximum transmission (and therefore minimum loss) occurs when $\Gamma = \pi$ and $\theta = \pi/2$.

c)



In this case, each fibre in the input array is collimated by a lenslet (part of a lenslet array) and illuminates a portion of the SLM, which contains the routing hologram for that input to a particular output fibre. The loss factors are similar to those of the one to *n* switch, but fan-in loss becomes more dominant. We are limited however, by the binary phase modulation of the FLC SLM, which means that a symmetric copy of the desired replay field always appears rotated by 180°. For this reason, we have to accept a 3dB penalty in the power. Due to the binary phase modulation, the distribution of the background power is not uniform and there tends to be small peaks of intensity, which may occur at fibre positions in creasing crosstalk. The optima condition for binary phase is to be a half wave plate which at telecoms wavelengths means that the SLM is quite thick making surface stabilisation difficult

Examiner's comment:

Well answered. A good understanding of FLCs and their properties. Good derivation of the modulation scheme but part c) was not so well answered with some including a crossbar switch from a part of the course removed 3 years ago!

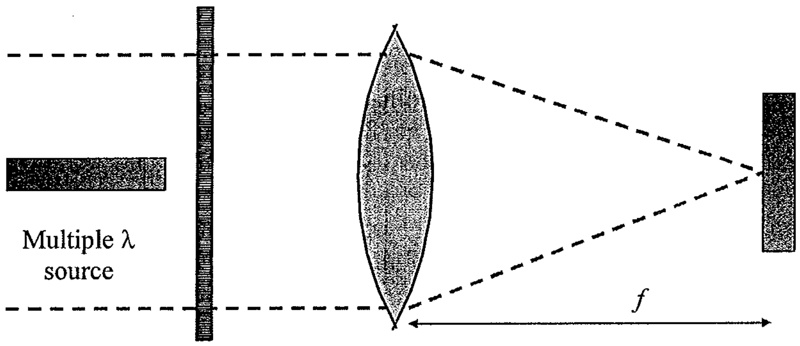
3 a) The manipulation of spatial frequencies in the replay field implies that the wavelength is held constant, however as can be seen in the normalized spatial frequency variables [u,v], the spatial frequency is also a function of wavelength λ. Hence in Fourier holograms, spatial and wavelength properties are interchangeable.

$$u = \frac{k\alpha}{2\pi f} \qquad v = \frac{k\beta}{2\pi f}$$

If a hologram or grating is created to generate fixed positions or orders in the replay field, then if the wavelength is varied, then the positions of the orders from the hologram will vary. This is known as a wavelength sensitive or dispersive hologram or grating. This concept can also be expressed by classical grating theory which relates the pitch of the grating d to the angle at which the light that passes though it is diffracted. The experiment of Young's slits shows how light is diffracted through a given angle and that theory can be extended to gratings. This shows how the diffracted angle of the light is dependent on the grating pitch and the wavelength. In the grating above, illuminated with a wavelength of λ, we have

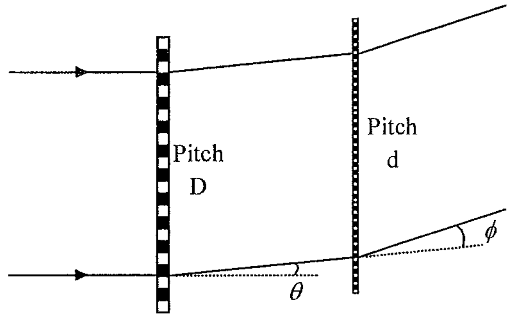
$$\sin\alpha - \sin\beta = \frac{m\lambda}{d}$$

where d is the grating pitch, α and β are the angles shown and m is the integer order of the diffracted light. In general, we are only interested in the first diffracted order ($m = +1$). By changing the grating pitch d, we can vary the angle of the diffracted light, β for a wavelength λ. By placing a positive focal length lens after the grating, we can view the far field and see that the diffracted angle is converted into the position of the diffracted order (as we would expect from a grating). If we change the pitch of the grating, then the position of the spot generated by the grating will sweep across the far field. If we have a multiple wavelength input source illuminating the grating then each wavelength has a different angle of diffraction and will lead to a different position in the far field.



Hence, wavelength and position will vary with the grating pitch in the far field. If we monitor a fixed point in the far field, then the wavelength will scan across that point with the changing pitch, making a wavelength filter. If the grating pitch d, were increased as an integer, we would only have a small number of fixed wavelengths that could be tuned. However, by using one dimensional holograms, we can select any point in the output plane along the single axis which means we can select a range of angles and hence a range of wavelengths. The use of an FLC SLM is ideal for the high-speed display of 1-D holograms to select the wavelength. Moreover, by using the phase modulation capabilities of the FLC, we can reduce the loss of the filter and prevent excess light being lost in the zero order of the hologram. As with other FLC phase applications, a switching angle of 90° would produce optimum results.

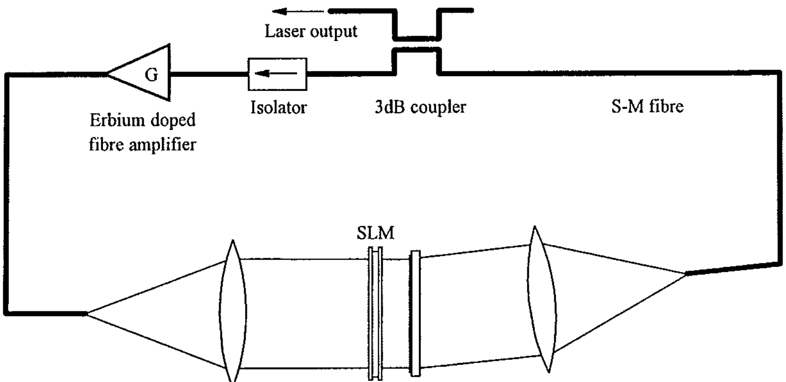
The most logical application of such a filter are telecommunications WDM systems which require channels separated by 0.8nm centred at a wavelength of 1550nm. Such a filter would require a 1-D SLM with a pixel pitch of about 5um, which is unlikely to be built in the near future and may never work properly due to the properties of the FLC domains.



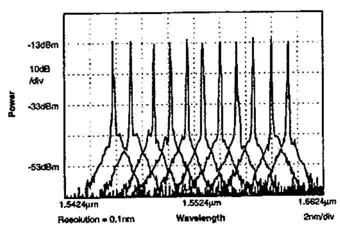
$$\sin\theta = \frac{\lambda}{D} \qquad \sin\phi = \frac{\lambda}{d}$$

From this system we can define the two angles of diffraction, where D and *d* are the respective grating pitches. Hence, for an SLM with pixel pitch D we can choose the second grating pitch d based on the desired centre wavelength and tuning range.

b) The next step from the wavelength tuneable filter is to use this filter as a tuning element to create a digitally tuneable laser. The relatively long cavity lengths of fibre lasers (from metres to a few centimetres) results in very narrow linewidths and closely spaced longitudinal modes, enabling almost continuous tuning. Such attributes are very desirable in telecommunications WDM systems.

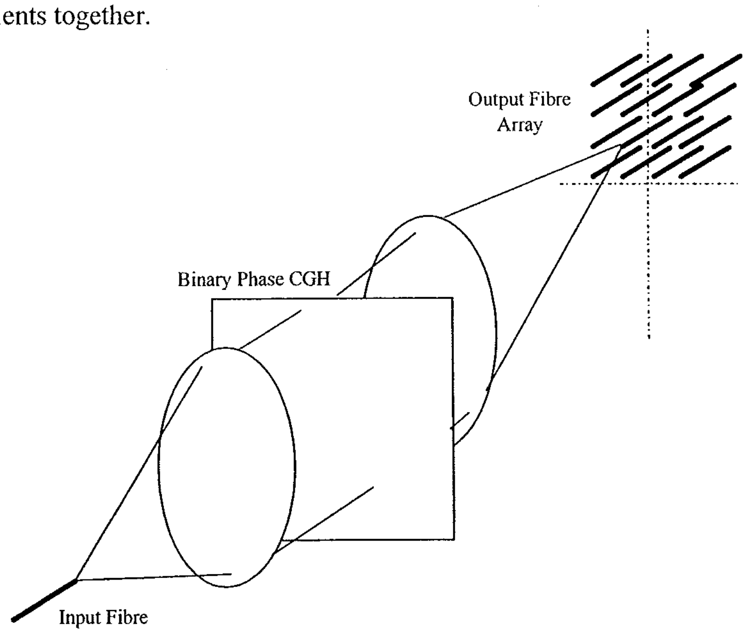


The FLC SLM and fixed grating form the tuning mechanism via the hologram calculated to select a lasing wavelength (via the fixed launch fibre in the RPF). The combination is set to tune lasing in the low loss window of silica SMF (around 1550nm). The SMF allows a clean monochromatic wave to propagate with a single well described mode. The coupler allows light to escape the laser and be utilised. The isolator prevents any reflections from propagating in the reverse direction as they may destabilise the laser (especially the EDFA). The EDFA is there to provide gain to compensate for the losses in the rest of the ring (especially the SLM and filter). Hence the total gain around the loop will be more than one leading to oscillation (lasing), with the wavelength being selected by the hologram on the FLC SLM.



There are many advantages (as listed above). A key advantage is that the hologram also allows you to control the intensity of each wavelength which means that the laser has an externally controllable spectral structure which can be used for equalisation or compensation in telecoms applications.

c) There are many different answers to this 'holy grail' question depending on the application and the perspective taken on the whole problem. In the context of a holographic WDM switch I would say no as the two requirements are not scalable. There is a fundamental mismatch of scaling due to the relative sizes of the spatial and wavelength domains. If an SLM was available with a massive number of very small pixels, then it might be possible to multiplex to two requirements together.



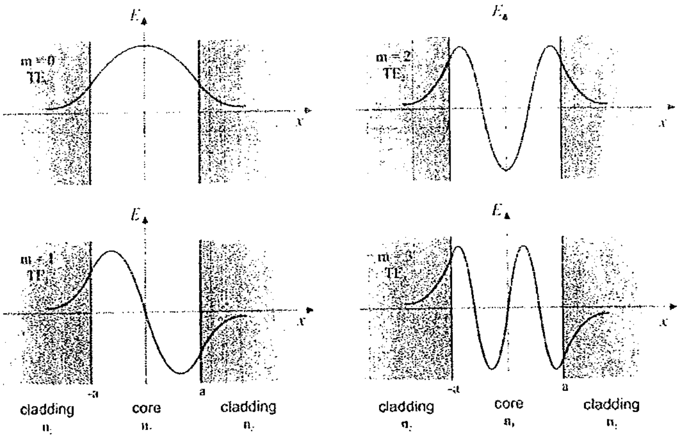
The effective scale of the fibres and their positions is several times larger than the wavelength, hence a hologram with one set of scales would not be suitable for the other application. There are some applications such as RGB where the separations of the wavelengths are comparable so more combined features are possible. It also adds another degree of freedom into the algorithms and minimisation energy space which makes the calculation of the holograms much more difficult.

Examiners comment:

The least popular and worst answered question. This was a section of the course that had not been asked previously, hence there were no past cribs. A but disappointing but one answer was excellent.

4\.

(a) Ensure that wavefunctions (and their derivatives) are continuous for all x, that they exponentially decay to zero in the cladding, and have sinusoidal functions within the core.



(b) *Intermodal dispersion*: Caused by different optical modes travelling at different speeds within the fibre. High order modes contain much of their wavefunction within the cladding, whilst low order modes are contained mostly within the core. The lower index of the cladding compared to the core means that higher order modes will therefore travel faster than lower order modes. A pulse that contains multiple modes will therefore suffer from temporal broadening, which limits the data carrying capacity of the fibre.

All fibres have a cut-off frequency, beyond which higher modes cannot be carried. Smaller diameter fibres have lower cut-off frequencies and are therefore capable of carrying fewer optical modes. At a critical radius, they will only be able to carry a single mode (at a single velocity) and they become a monomode fibre. Intermodal dispersion therefore cannot occur in these narrow fibres.

Three ways in which intermodal dispersion can be compensated:

1. Low index difference fibres
2. Gradient index (GRIN) fibres
3. Use of soliton pulses in non-linear fibres

Note: Narrow linewidth laser sources and the use of temporal pulse shapes with lower frequency content will only combat *intramodal* dispersion and NOT *intermodal* dispersion.

(c) The maximum amount of temporal pulse broadening (Δt) over a distance L can be roughly estimated by assuming that the highest and lowest possible frequencies travel at speeds determined by the refractive indices of the cladding and the core respectively.

$$v_{min} = \frac{c}{n_1} \qquad v_{max} = \frac{c}{n_2}$$

The times taken for these maximum and minimum velocity modes to travel a distance L are therefore given by:

$$t_{max} = \frac{L}{v_{min}} \qquad t_{min} = \frac{L}{v_{max}}$$

Substituting for the velocities from the first two equations into the second two, we can write the following expression for the maximum time difference ($t_{max} - t_{min}$), which is equal to the maximum temporal pulse broadening (Δt) that can occur.

Maximum temporal pulse broadening: $$\Delta t \leq \frac{L}{c}(n_1 - n_2)$$

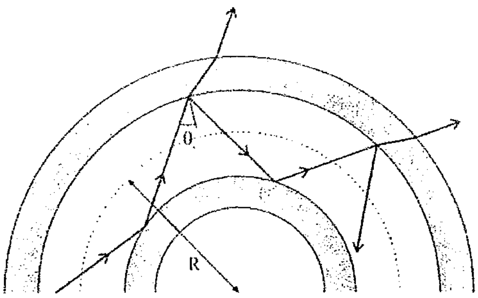
Substituting values: $L = 100$ km, $n_1 = 1.51$, $n_2 = 1.50$, gives: **$\Delta t = 3.33$ μs**

Bandwidth-Length product: $$BL\ [bits/s \cdot m] = \frac{c}{2(n_1 - n_2)}$$

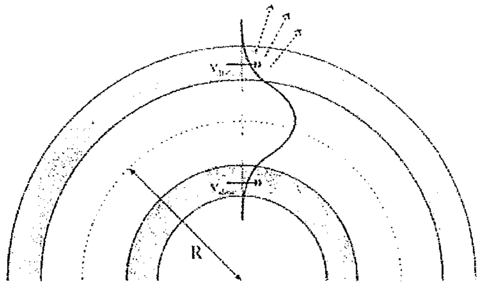
Divide both sides by B: $$L = \frac{c}{2B(n_1 - n_2)}$$

Substitute values: $B = 150$ MBits/s, $n_1 = 1.51$, $n_2 = 1.50$, gives: **$L = 100$ m**

(d) Bending losses can be explained two ways (either will suffice):



Ray model



EM model

1. *Ray model*: Bending the fibre causes the angle of incidence of some (higher order mode) rays to fall below the critical angle. These rays are then refracted out of the fibre and are lost.

2. *Electromagnetic model*: Within any single mode of propagation, photons travelling along the outside edge of a bent fibre will need to travel faster than those travelling along the inside edge, in order to maintain the correct field amplitude distribution across the fibre. At the critical radius, light at the outside edge is required to travel faster than the speed of light, which it cannot do. It is instead radiated out through the cladding as it no longer is permitted to remain part of the optical mode.

Critical bending radius: $$R_c \approx \frac{a}{(NA)^2}$$

Numerical Aperture: $$NA = \sqrt{n_1^2 - n_2^2}$$

Substitute for *NA* into critical bending radius formula, gives: $R_c \approx \dfrac{a}{n_1^2 - n_2^2}$

Then substitute values: Core radius, $a = 50$ μm, $n_1 = 1.51$, $n_2 = 1.50$

Gives: $\mathbf{R_c \approx 1.66}$ **mm**

(e) Other sources of optical loss (any 3 of the following):

1. **Intramodal dispersion**: Different frequencies are contained within the optical light source (either due to finite linewidth of source, or the temporal shape of the optical pulse). These travel at different speeds along the fibre. Temporal broadening of optical pulses result, which limits the data carrying capacity of the fibre.
2. **Absorption**: Extrinsic - impurities in the fibre material (eg: water, transition metals). Intrinsic - optical transmittance of material may not be 100% at desired wavelength.
3. **Insertion losses**: Includes reflection losses at the entrance (and exit) of the fibre, and also acceptance angle losses.
4. **Scattering**: Due to imperfections and inhomogeneities in the structure. Also including reflections at fracture interfaces. Dominated by Rayleigh scattering, but also includes Mie scattering, Brillouin scattering and Raman Scattering.

Examiner's comments:

This question was too easy but it did encourage people not to bin this section of the course. The maths section was a bit too simple but the basic knowledge of waveguides was very encouraging..