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Part IIB, 2010 **Crib for 4C2: Designing with Composites**

Q1. (a)

(i) The lamina stress-strain of the epoxy-glass fibre composite lamina can be written as

$$\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \begin{bmatrix} \bar{S}_{11} & \bar{S}_{12} & \bar{S}_{16} \\ \bar{S}_{12} & \bar{S}_{22} & \bar{S}_{26} \\ \bar{S}_{16} & \bar{S}_{26} & \bar{S}_{66} \end{bmatrix} \begin{pmatrix} 0.1 \\ 0 \\ 0 \end{pmatrix}$$

Using the supplied values of the elastic constants of the composite, for $\phi = 30°$

$$\bar{S}_{16} = \left(\frac{2}{50} + \frac{2(0.3)}{50} - \frac{1}{10}\right)\cos^3 30 \sin 30 - \left(\frac{2}{5} + \frac{2(0.3)}{50} - \frac{1}{10}\right)\cos 30 \sin^3 30$$

$$= -0.049 \text{ GPa}^{-1}$$

The shear strain is therefore given by

$$\gamma_{xy} = \bar{S}_{16}\sigma_x = -0.049(0.1) = -4.9\times10^{-3}$$

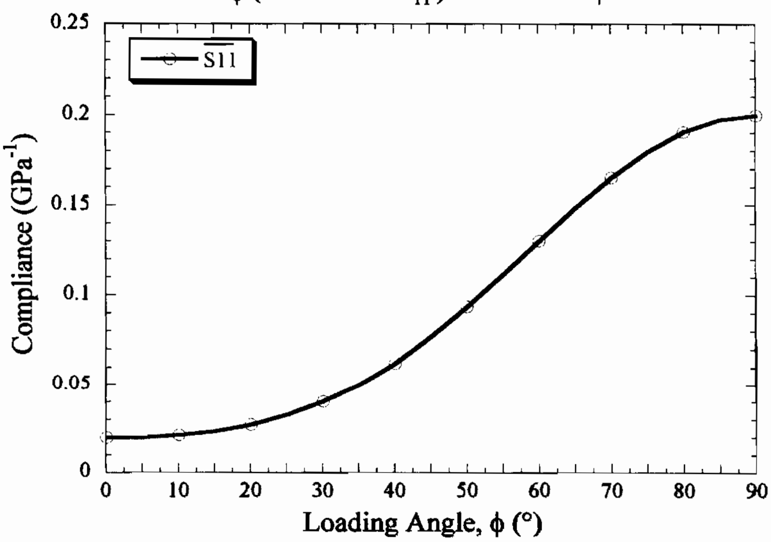
(ii) From Datasheet

$$\bar{S}_{11} = S_{11}c^4 + S_{22}s^4 + (2S_{12} + S_{66})c^2s^2$$

in which $c = \cos\phi$ and $s = \sin\phi$. Applied $\sigma_x \rightarrow \varepsilon_x = \bar{S}_{11}\ \sigma_x$

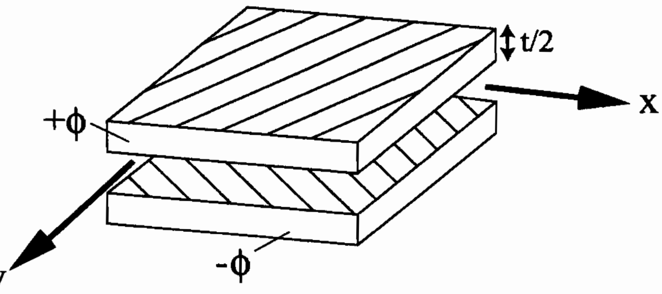
$$E_\phi = \frac{\sigma_x}{\varepsilon_x} = \frac{1}{\bar{S}_{11}}$$

From the plot, it can be seen that there is a fairly sharp increase in $\bar{S}_{11}$ compliance with misorientation. The minimum value of E_ϕ (maximum $\bar{S}_{11}$) appears from the plot to be 5 GPa at ϕ=90 and the maximum value of E_ϕ (minimum $\bar{S}_{11}$) 50 GPa at ϕ=0 . The ratio is therefore 10.



(b) A balanced laminate ($A_{16} = A_{26} = 0$) is one in which the laminate as a whole exhibits no tensile-shear interactions ie. the tension-shear interaction terms contributed by the individual laminae all cancel out each other.

Consider a $\pm\phi$ angle ply laminate with a ply thickness $t / 2$, as illustrated in the figure below.



It can be easily found that the tension-shear interaction terms for the $-\phi$ ply have the opposite sign from the corresponding terms for the $+\phi$ ply.

$$(\bar{Q}_{16})_{+\phi} = -(\bar{Q}_{16})_{-\phi}$$

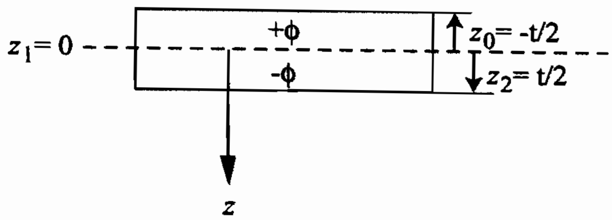
$$(\bar{Q}_{26})_{+\phi} = -(\bar{Q}_{26})_{-\phi}$$

The laminate stiffness matrices A_{16} and A_{26} can be written as

$$A_{16} = (z_1 - z_0)(\bar{Q}_{16})_{+\phi} + (z_2 - z_1)(\bar{Q}_{16})_{-\phi} = 0$$

$$A_{26} = (z_1 - z_0)(\bar{Q}_{26})_{+\phi} + (z_2 - z_1)(\bar{Q}_{26})_{-\phi} = 0$$

where z_0, z_1, and z_2 are the distances as shown below.



This laminate is balanced when loaded along the x -direction, or along the y-direction.

(c) The transformed lamina stiffness matrices $[\overline{Q}]$ for the +45° and - 45° plies are given by

$$\left(\overline{Q}_{11}\right)_{+45} = \left(\overline{Q}_{11}\right)_{-45}$$
$$\left(\overline{Q}_{12}\right)_{+45} = \left(\overline{Q}_{12}\right)_{-45}$$
$$\left(\overline{Q}_{22}\right)_{+45} = \left(\overline{Q}_{22}\right)_{-45}$$
$$\left(\overline{Q}_{16}\right)_{+45} = -\left(\overline{Q}_{16}\right)_{-45}$$
$$\left(\overline{Q}_{26}\right)_{+45} = -\left(\overline{Q}_{26}\right)_{-45}$$
$$\left(\overline{Q}_{66}\right)_{+45} = \left(\overline{Q}_{66}\right)_{-45}$$

The transformed stiffness matrix for the 0° ply is

$$\left[\overline{Q}\right]_0 = Q$$

The transformed stiffness matrix for the 90°ply ($c = \cos 90 = 0,\ s = \sin 90 = 1$)

$$\left(\overline{Q}_{11}\right)_{90} = Q_{22}$$
$$\left(\overline{Q}_{12}\right)_{90} = Q_{12}$$
$$\left(\overline{Q}_{22}\right)_{90} = Q_{11}$$
$$\left(\overline{Q}_{16}\right)_{90} = \left(\overline{Q}_{26}\right)_{90} = 0$$
$$\left(\overline{Q}_{66}\right)_{90} = Q_{66}$$

The components of the laminate extensional stiffness matrix are therefore given by

$$A_{11} = \frac{t}{4}\left(2\left(\overline{Q}_{11}\right)_{+45} + Q_{11} + Q_{22}\right) = \frac{t}{8}(3Q_{11} + 3Q_{22} + 4Q_{66} + 2Q_{12})$$
$$A_{12} = \frac{t}{2}\left(\left(\overline{Q}_{12}\right)_{+45} + Q_{12}\right) = \frac{t}{8}(Q_{11} + Q_{22} - 4Q_{66} + 6Q_{12})$$
$$A_{22} = \frac{t}{4}\left(2\left(\overline{Q}_{22}\right)_{+45} + Q_{11} + Q_{22}\right) = A_{11}$$
$$A_{16} = \frac{Q_{16}\ t}{4} = 0$$
$$A_{26} = \frac{Q_{26}\ t}{4} = 0$$
$$A_{66} = \frac{t}{2}\left(\left(\overline{Q}_{66}\right)_{+45} + Q_{66}\right) = \frac{t}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}) = \frac{A_{11} - A_{12}}{2}$$

$$[A] = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & (A_{11} - A_{12})/2 \end{bmatrix}$$

This stacking sequence gives complete in-plane isotropy and should give a Young's modulus of the same value for any loading angle.

A 2(a).

(i) Degradation at elevated temperature suggests that a matrix-dominated property is affected.

Failure in bending is probably due to compressive failure by plastic microbuckling, a matrix-governed failure mode.

(ii) Examine the fracture surface of the failed specimen with a scanning electron microscope (SEM). Expect to see kink bands:

To reproduce the failure, heat up the specimen and perform a bend test in 3-pt. bending or 4-pt. bending, or a compression test.

(iii) The microbuckling compressive strength σ_c scales with the shear yield strength τ_Y of the matrix and the fibre misalignment angle $\bar{\phi}$ according to

$$\sigma_c \approx \frac{\tau_Y}{\bar{\phi}}$$

A 2(a) (iii) contd.

So, increase σ_c by
- choosing a resin which does not soften much at elevated temperature, i.e. choose a high value of τ_y
- improve processing to reduce the fibre waviness angle $\bar{\phi}$.

A 2 (b).

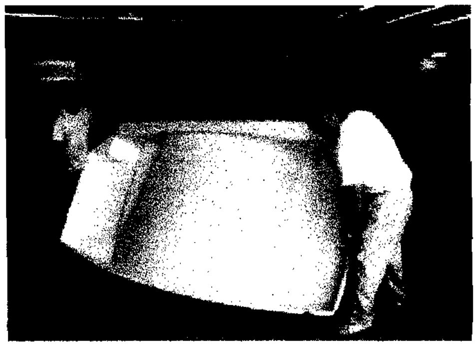
Injection moulding is a high rate process but gives poor mechanical performance. It is suitable for the mass production of automotive parts.

R.T.M. can achieve lower production rates but requires a lower investment in tooling and results in better quality than Injection moulding. It is appropriate for bespoke components (eg. magnetic resonance casings) of moderate mechanical performance.

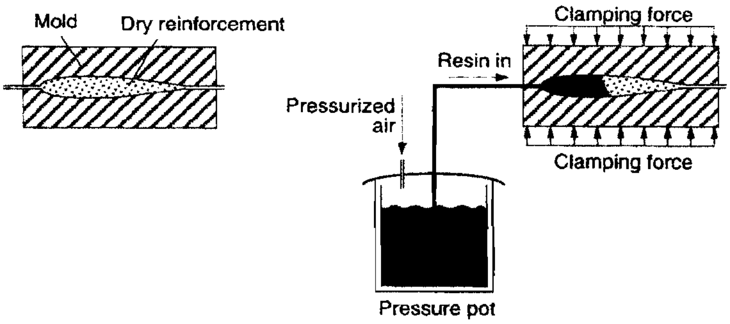
2(b)

Resin transfer moulding (RTM)

- Closed mould process
- Charge mould with dry fabric or preforms
- Close mould
- Inject thermoset resin at relatively low pressure
- Low tooling cost due to low pressure
- Intermediate rate, low cost



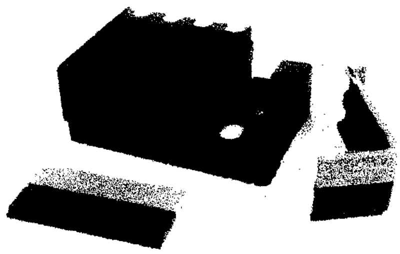
Roof moulding for MRI equipment - JHM Technologies



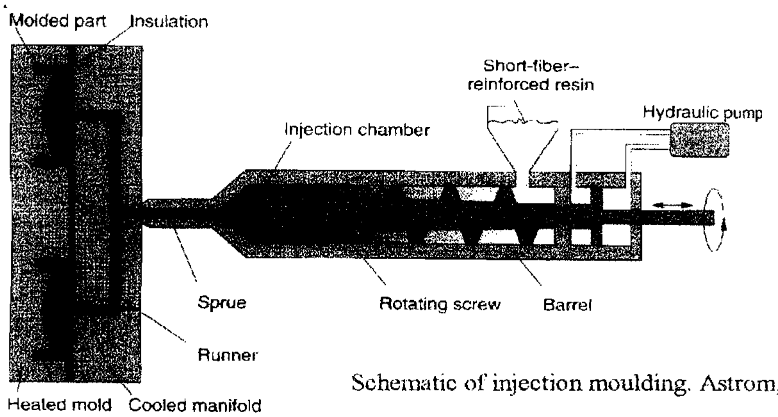
Schematic of resin transfer moulding. Astrom, 1997

Injection moulding

- Thermoplastic and thermoset resins, including recycled thermoset
- Short glass fibre lengths of 0.5-5mm survive screw
- Fibres aligned by flow near surfaces
- High tooling costs, highly automated, short cycle times
- Relatively poor mechanical performance and small components e.g. automotive, reflector assemblies for headlights, housings

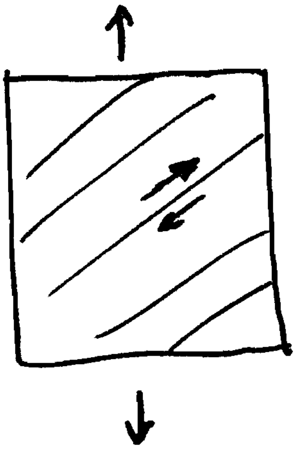


Medical instrument housing - Exothermic



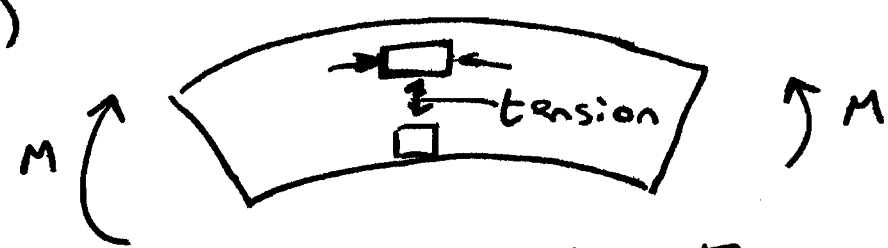
Schematic of injection moulding. Astrom, 1997

A3. (a) (i)



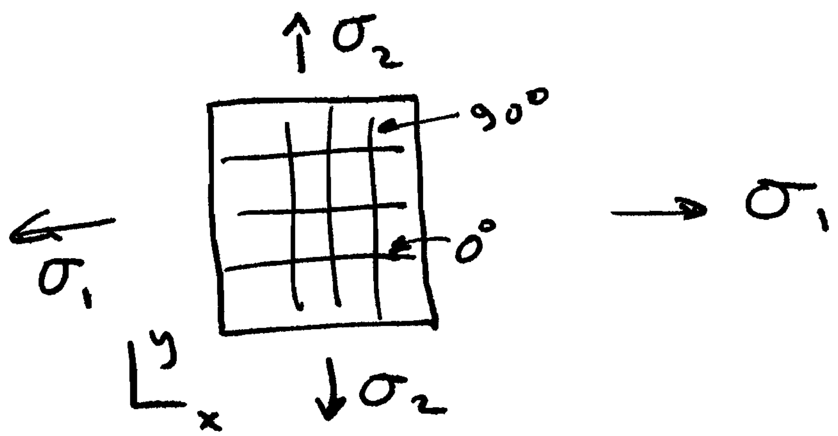
Shear in-plane leads to interface failure between matrix & fibre, and to shear failure within ~~the~~ matrix.

(ii)



get through-thickness tension, leading to inter-laminar failure.

A3. (b)



(i) First ply failure.

$\varepsilon^0 = A^{-1} N \implies$ we obtain the strain in all plies in $x-y$ system.

To get the strain in material co-ordinates, use

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = T^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

A3 (b)(i) contd.

Then obtain the stress in each ply, in the material co-ordinates, using

$$\begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} = Q\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix}$$

Then apply the failure criterion.

Here, $N = \begin{pmatrix}\sigma\\ \sigma\\ 0\end{pmatrix} t$. $\gamma_{xy} = 0$ by symmetry

$$\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\end{pmatrix} = \begin{pmatrix}0.0135 & -5\times10^{-4}\\ -5\times10^{-4} & 0.0135\end{pmatrix}\begin{pmatrix}\sigma\\ \sigma\end{pmatrix} t$$

$$= \begin{pmatrix}0.0130\\ 0.0130\end{pmatrix} t.\sigma$$, units of (GPa mm)

Note that both ply types (0° and 90°) have the same loading, and so only consider unrotated plies : $\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix} \equiv \begin{pmatrix}\varepsilon_x\\ \varepsilon_y\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}\sigma_1\\ \sigma_2\end{pmatrix} = \begin{bmatrix}139 & 2.7\\ 2.7 & 9\end{bmatrix}\begin{pmatrix}0.013\\ 0.013\end{pmatrix}\sigma t \quad (\text{mm})$$

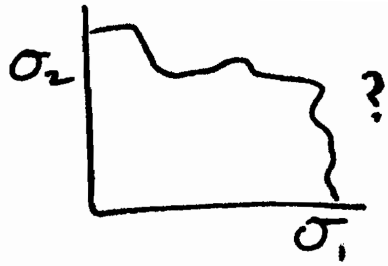
$$= \begin{pmatrix}1.84\\ 0.15\end{pmatrix}\sigma$$

Now apply Tsai-Hill :

$$\underset{\uparrow S_L^+}{\frac{1.84^2}{1448^2}} - \underset{\uparrow S_L^+}{\frac{1.84\times0.15}{1448^2}} + \underset{\uparrow S_T^+}{\frac{0.15^2}{48.3^2}} = \frac{1}{\sigma^2}$$

$\Rightarrow$ $\underline{\sigma = 300\text{ MPa}}$

A3 (b) (ii)



? Failure surface

First, find the uniaxial strength : $\sigma_1 = \sigma$, $\sigma_2 = 0$

— Consider the 0° plies, and then the 90° plies.

$$\Rightarrow \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \end{pmatrix} = \begin{pmatrix} 0.0135 \\ -0.0005 \end{pmatrix} \sigma \quad (\text{GPa})^{-1}$$

$$\Rightarrow \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 139 & 2.7 \\ 2.7 & 9 \end{pmatrix} \begin{pmatrix} 0.0135 \\ -0.0005 \end{pmatrix} \sigma$$

$$= \begin{pmatrix} 1.875 \\ 0.032 \end{pmatrix} \sigma$$

Feed these stresses into the Tsai - Hill criterion, to give

$$\frac{1.875^2}{1448^2} - \frac{1.875 \times 0.032}{1448^2} + \frac{0.032^2}{48.3^2} = \frac{1}{\sigma^2}$$

$$\Rightarrow \quad \underline{\sigma = 692 \text{ MPa}}$$

— Now for the 90° plies :

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{pmatrix} -0.0005 \\ 0.0135 \end{pmatrix} \sigma \quad (\text{GPa})^{-1}$$

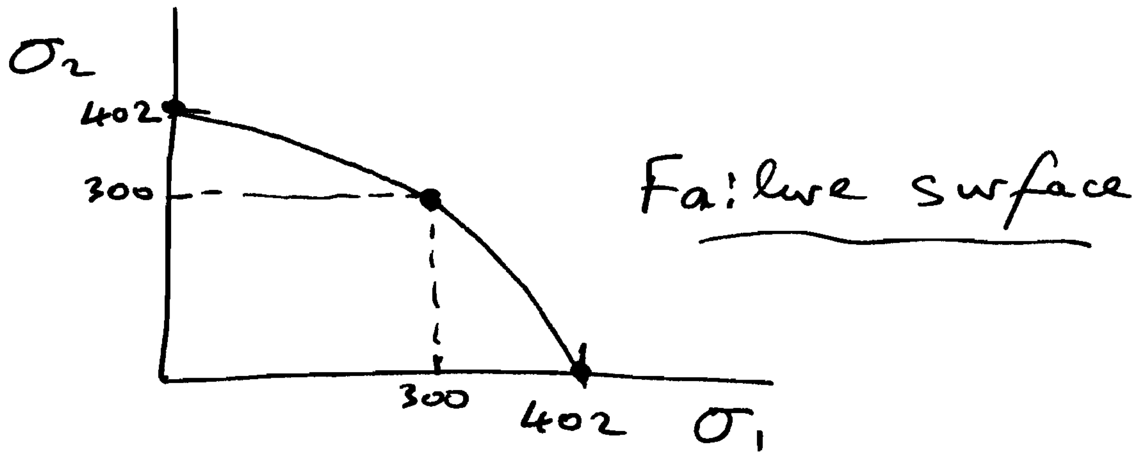
$$\Rightarrow \begin{pmatrix} \sigma_1 \\ \sigma_2 \end{pmatrix} = \begin{pmatrix} 139 & 2.7 \\ 2.7 & 9 \end{pmatrix} \begin{pmatrix} -0.0005 \\ 0.0135 \end{pmatrix} \sigma = \begin{pmatrix} -0.033 \\ 0.12 \end{pmatrix} \sigma$$

$$\Rightarrow \frac{(-0.033)^2}{\underset{\sigma_L^{-}}{1172^2}} + \frac{0.033 \times 0.12}{1172^2} + \frac{0.12^2}{48.3^2} = \frac{1}{\sigma^2}$$

$$\Rightarrow \quad \underline{\sigma = 402 \text{ MPa}}$$

A 3 (b) (ii) contd.

The 90° ply fails before ~~the~~ 0° ply as 402 MPa is below 692 MPa.



A4 (a) – A high E and low ρ gives a high resonant frequency.

– Cheap to manufacture in prismatic form by pultrusion

– high corrosion resistance.

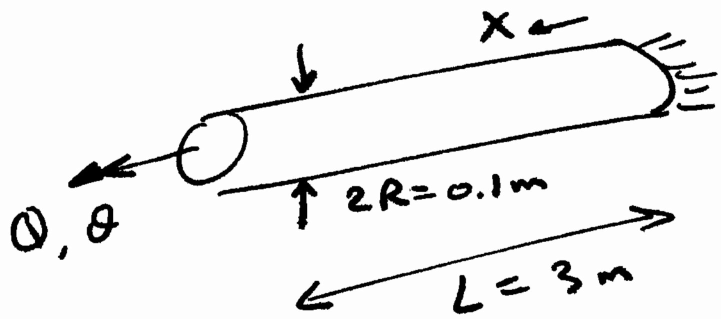
(b) Need to narrow down choice of material and laminate via conceptual/preliminary/detailed design.

At each step, it is necessary to quantify the laminate mass, cost, etc..

Consider, for example, stiffness, strength and the vibration equations and impose constraints in order to optimise the design. Use carpet plots or similar in order to choose the laminate that meets all the constraints.

Optimise the material and lay-up progressively to minimise mass, cost, etc.

(c)



Apply $Q = 200\,\text{Nm}$

Constraints :

$\theta \leq 0.03$ radians

$f \leq 10$ Hz

A 4 (c) Contd.

<u>Torsion</u> : $\theta = \frac{L\gamma}{R} = \frac{L\tau}{GR} = \frac{LQ}{2\pi G t R^3}$

$\Rightarrow \quad Gt > \frac{LQ}{2\pi R^3 \theta} = \frac{3 \cdot 200}{2\pi\,(0.05)^3\,0.03} = 2.7 \times 10^7 \text{ N m}^{-1}$

<u>Bending</u>

The relevant E is along the beam axis — call this the x-direction, aligned with the 0° direction.

$$f \le 0.56 \left(\frac{E_x \, \pi R^3 t}{2\pi R t \rho L^4} \right)^{1/2}$$

since $m = 2\pi R t \rho$. Now substitute values:

$\rho = 1900$ kg/m³, $f = 10$ Hz, $R = 0.05$ m, $L = 3$ m

$\Rightarrow \quad E_x > 39$ GPa (independent of t)

So, we seek a satisfactory lay-up & laminate overall thickness t to satisfy

$Gt > 2.5 \times 10^7 \text{ N m}^{-1}$ and $E_x > 39$ GPa.

(i) Use the carpet plot of Fig. 2a to obtain $E_x = 39$ GPa $\Rightarrow$ 80% 0°
20% ±45°

(No need to include 90° fibres in the design

4. (c) contd.

(ii) Fig. 2a $\Rightarrow$ $G = 6.5$ GPa

$$\Rightarrow t = \frac{2.7 \times 10^7}{6.5 \times 10^9} = 4.0 \text{ mm}$$

$\Rightarrow$ 8 plies (each ply is of thickness 0.5 mm).

So, choose 8 plies of 0° & 2 plies of ±45 to give $[0_4/\pm 45/0_4]$.

- We have 10 plies (thickness of $t = 5$ mm) in order to achieve ~~the~~ required value of E_x.
- This solution almost is a balanced, symmetric laminate.