PART IIB 2010 4C8 APPLICATIONS OF DYNAMICS

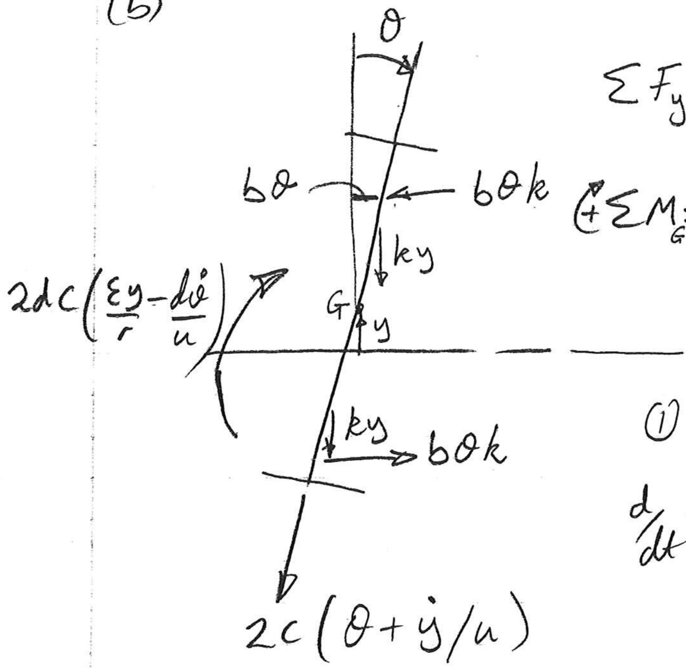
PROF D CEBON

Part IIB PAPER 4C8, 2010.

Q1

(a) See lecture notes for derivation of force & moment

(b)



$$\Sigma F_y=0:\ \cancel{2}c\left(\theta+\frac{\dot y}{u}\right)+\cancel{2}ky=0 \quad (1)$$

$$\circlearrowleft\Sigma M_G=0:\ \cancel{2}dc\left(\frac{\varepsilon y}{r}-\frac{d\dot\theta}{u}\right)-\cancel{2}b^2k\theta=0 \quad (2)$$

$$\left.\begin{array}{l} (1)\Rightarrow\ \theta=-\frac{k}{c}y-\dot y/u \\ \frac{d}{dt}\Rightarrow\ \dot\theta=-\frac{k}{c}\dot y-\ddot y/u \end{array}\right\}\quad (3)$$

(3) into (2) gives

$$\frac{\varepsilon y}{r}+\frac{d}{u}\left(\frac{k}{c}\dot y+\frac{\ddot y}{u}\right)+\frac{b^2k}{dc}\left(\frac{k}{c}y+\frac{\dot y}{u}\right)=0$$

$$\Rightarrow\quad \ddot y+\dot y\left[\frac{ku}{d^2c}(d^2+b^2)\right]+y\frac{u^2}{d}\left[\frac{\varepsilon}{r}+\frac{b^2k^2}{dc^2}\right]=0 \quad (4)$$

(c) Hunting wavelength:

(4) is damped SHM with $\omega_n^2=\frac{u^2}{d}\left[\frac{\varepsilon}{r}+\frac{b^2k^2}{dc^2}\right]$

$$\lambda=\frac{2\pi u}{\omega_n}=\frac{2\pi \cancel{u}}{\cancel{u}\sqrt{\frac{\varepsilon}{dr}+\frac{b^2k^2}{d^2c^2}}}$$

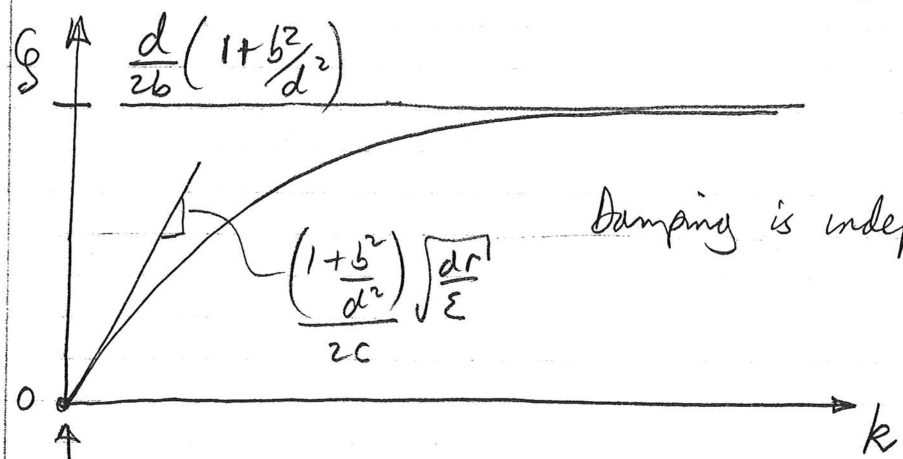
for $k=0$, $\lambda=2\pi\sqrt{\frac{dr}{\varepsilon}}$ as expected for a free wheelset.

Q1 (Cont)

(d) Damping ratio of hunting mode:

from ⑧
$$\zeta = \frac{\dfrac{k\mu}{d^2c}(d^2+b^2)}{2\mu\sqrt{\dfrac{\varepsilon}{dr}+\dfrac{b^2k^2}{d^2c^2}}} = \frac{\left(1+\dfrac{b^2}{d^2}\right)}{2c}\left[\frac{k}{\sqrt{\dfrac{\varepsilon}{dr}+\dfrac{b^2k^2}{d^2c^2}}}\right]$$

For large k,
$$\zeta \rightarrow \frac{\left(1+\dfrac{b^2}{d^2}\right)}{2c}\left[\frac{\cancel{k}}{\dfrac{b\cancel{k}}{dc}}\right] = \frac{d}{2b}\left(1+\frac{b^2}{d^2}\right)$$



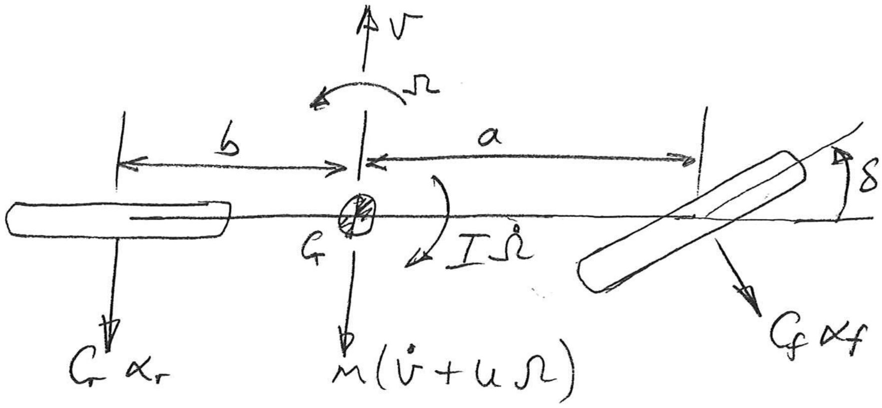
Damping is independent of speed.

Damping is zero for $k=0$ as for free wheelset

Assessor's comment:

Railway wheelset hunting. Part (a) was bookwork, generally well done. In Part (b), it was remarkable how few students could get the correct expression for the moment of the longitudinal spring forces about the centre of the axle. Many signs were wrong.

Q1



(a) ΣF: $m(\dot{v}+u\Omega) + C_f\alpha_f + C_r\alpha_r = 0$

$\circlearrowleft +\ \Sigma M_G$ $\quad I\dot{\Omega} + aC_f\alpha_f - bC_r\alpha_r = 0$ $\quad$ (1)

Slip angles: $\alpha_f = \dfrac{v + a\Omega}{u} - \delta$, $\quad \alpha_r = \dfrac{v - b\Omega}{u}$ — (2)

Combining (1) & (2):

$$\left.\begin{aligned} m(\dot{v}+u\Omega) + (C_f + C_r)\frac{v}{u} + (aC_f - bC_r)\frac{\Omega}{u} &= C_f\delta \\ I\dot{\Omega} + (aC_f - bC_r)\frac{v}{u} + (a^2C_f + b^2C_r)\frac{\Omega}{u} &= aC_f\delta \end{aligned}\right\} \quad (3)$$

Assumptions:
- All angles are small
- Neglect tyre realigning moment
- Tyres behave linearly
- δ is average of 2 steered front wheels
- Neglect motion of sprung mass on suspension

(b) Steady turning: $\dot{\Omega} = \dot{v} = 0$ & $\Omega = u/R$ $\quad$ (4)

(4) into (3) gives

$$\begin{bmatrix} C & Cs + mu^2 \\ Cs & Cq^2 \end{bmatrix} \begin{Bmatrix} \beta \\ 1/R \end{Bmatrix} = C_f\delta \begin{Bmatrix} 1 \\ a \end{Bmatrix} \quad (5)$$

where $C = C_f + C_r$, $\quad s = \dfrac{aC_f - bC_r}{C_f + C_r}$, $\quad q = \dfrac{a^2C_f + b^2C_r}{C_f + C_r}$, $\quad \beta = v/u$

$l = a + b$

Solving (5) for $1/R$ gives

$$\frac{1/R}{\delta} = \frac{C\,C_f(a - s)}{C_fC_r l^2 - Csmu^2} = \frac{l\,C_fC_r}{C_fC_r l^2 - Csmu^2} \quad (6)$$

Q2 Cont

ie $\delta = \frac{l}{R}\left(1 - \frac{C s m u^2}{l^2 C_f C_r}\right)$ // ——— (7)

Differentiate (7): $\frac{d\delta}{du} = -\frac{2l}{R}\left(\frac{C s m}{l^2 C_f C_r}\right)u$ ——— (8)

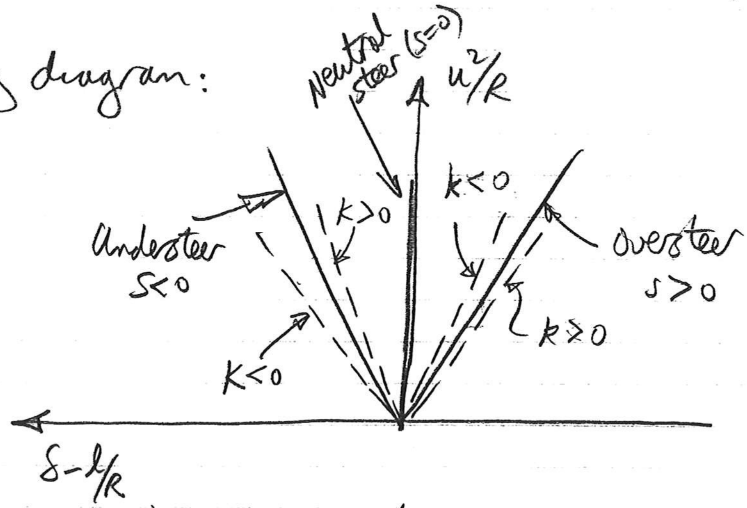
Neutral steer $(s=0) \rightarrow \delta = l/R$ & $d\delta/du = 0$

Understeer $(s<0) \rightarrow d\delta/du > 0$ all speeds

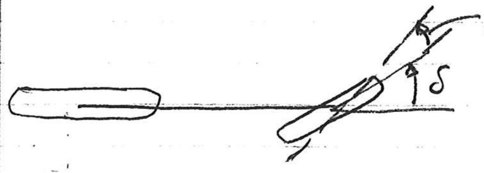
Oversteer $(s>0) \rightarrow d\delta/du < 0$ all speeds

Vehicle becomes unstable when $\delta \leq 0 \Rightarrow u \geq \sqrt{\frac{C_f C_r l^2}{C s m}}$

Handling diagram:



(c) Roll steer induces additional steer angle: $K u^2/R$



$$\left[\delta - l/R = -\frac{K u^2}{R} - \frac{l}{R}\left(\frac{C s m u^2}{l^2 C_f C_r}\right)\right]$$

If $K>0$ Roll steer generates <u>greater</u> steer angle than the vehicle without roll-steer.

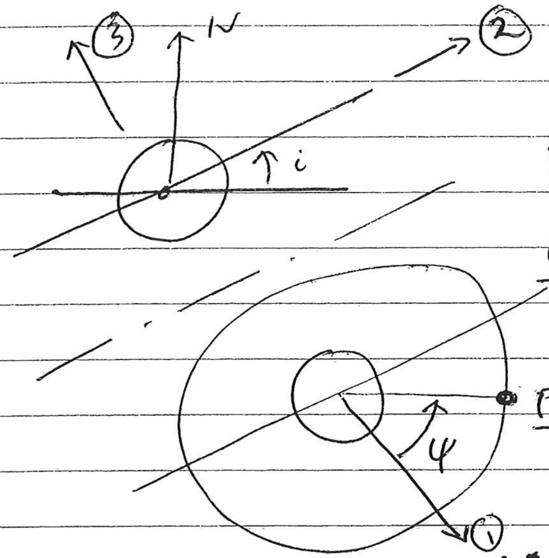
$\Rightarrow$ for the understeering vehicle, this additional steer angle increases the lateral acceleration – ie it makes the vehicle <u>less</u> understeering (more oversteering)

$\Rightarrow$ for the oversteering vehicle, the additional steer angle reduces the lateral acceleration – ie it make the vehicle <u>more</u> oversteering

Assessor's comment on Q2:

Car Handling: A popular question. Generally well done.

3 a)



By VW

$M \times \delta i = \text{mass} \times \delta U$
about ① axis

$$u(r,\theta) = \frac{\mu}{r}\left\{1-\left(\frac{R}{r}\right)^2 J_2 P_2(\cos\theta)\right\}$$

$$\cos\theta = \underline{e}_P \cdot \underline{e}_N = (\cos\psi, \sin\psi, 0)\cdot(0, \sin i, \cos i)$$

$$= \sin\psi \sin i$$

$$U_P = \frac{\mu}{a}\left\{1-\left(\frac{R}{a}\right)^2 \frac{J_2}{2}\left(3\sin^2\psi\sin^2 i - 1\right)\right\}$$

Spreading mass around the ring

$$M = \int_0^{2\pi} \frac{\mu R^2 J_2}{a^3}\, \frac{3}{2}\sin^2\psi \, 2\sin i\cos i \,\frac{m}{2\pi}\, d\psi$$

$$= \frac{3\mu R^2 J_2 m}{2a^3\pi}\sin i\cos i \int_0^{2\pi}\sin^2\psi\, d\psi$$

$$= \underline{\frac{3\mu R^2 J_2 m}{2a^3}\sin i\cos i}$$

b) Since $\omega^2 a = \mu/a^2$, $\underline{M = \frac{3R^2\omega^2 J_2 m}{2}\sin i\cos i}$

3 cont

Suppose precession is Ω about polar axis where $\Omega << \omega$

A.V. of reference frame, $\underline{\Omega} = [0, \Omega \sin i, \Omega \cos i]$

A.V. of ring is $[0, \Omega \sin i, \Omega \cos i + \omega]$

So $\underline{h} = [0, \frac{1}{2} m a^2 \Omega \sin i, m a^2 (\Omega \cos i + \omega)]$

For steady precession,

$$\underline{M} = \underline{\Omega} \times \underline{h} \simeq [m a^2 \omega \Omega \sin i, 0, 0]$$

(neglecting Ω^2 term)

$$\therefore \tfrac{3}{2} R^2 \omega^2 J_2 m \sin i \cos i \simeq m a^2 \omega \Omega \sin i$$

$$\rightarrow \underline{\underline{\Omega \simeq \tfrac{3}{2} \frac{R^2}{a^2} \omega J_2 \cos i}}$$

c) For satellite, $a = 23222 + R = \underline{29600 \text{ km}}$

$$\omega = \mu / a^3 = \frac{398603}{29600^3} = 15.37 \times 10^{-9} \text{ rad/s}$$

$$\Omega = \frac{3}{2} \times \left(\frac{6378}{29600}\right)^2 \times 15.37 \times 10^{-9} \times 1082 \times 10^{-6} \times \cos 56$$

$$= 647.6 \times 10^{-15} \text{ rad/s}$$

$$\text{No. of orbits} = \frac{\omega}{\Omega} = \underline{\underline{23,732}}$$

Assessor's comment:

Precession of non-equatorial orbits. This was a fairly standard 'bookwork' question, and candidates either scored very well on it (14 got 70% or more) or very poorly (5 attempts got 15% or less), presumably depending on whether they had studied the notes or not. Many fell into the trap of taking the altitude of the orbit in part (c) to be the radius.

4 a) No term for $n=1$ because c of m of earth is at origin (by definition)

J_2 term is caused by earth's equatorial bulge, which (in turn) is due to the earth's spin.

b) $$U = \mu/r\left[1-\left(\frac{R}{r}\right)^2\frac{J_2}{2}(3\cos^2\theta-1)-\left(\frac{R}{r}\right)^3\frac{J_3}{2}(5\cos^2\theta-3\cos\theta\right.$$
$$\left.-\left(\frac{R}{r}\right)^4\frac{J_4}{8}(35\cos^4\theta-30\cos^2\theta+3)+\ldots\right]$$

At equator, $\cos\theta=0$ and $\sin\theta=1$

So force → earth's centre

$$=-\frac{\partial U}{\partial r}=\frac{\mu}{r^2}-\frac{3\mu R^2}{r^4}\frac{J_2}{2}(-1)-\frac{5\mu R^4}{r^6}\frac{J_4}{8}(3)+\ldots$$
$$=\mu/r^2\left(1+\frac{3}{2}\left(\frac{R}{r}\right)^2J_2-\frac{15}{8}\left(\frac{R}{r}\right)^4J_4+\ldots\right)$$

And force → North Pole

$$=-\frac{1}{r\sin\theta}\frac{\partial U}{\partial\theta}=\frac{\mu}{r^2\sin\theta}\left[\frac{R^2J_2}{2r^2}(-6\cos\theta\sin\theta)\right.$$
$$+\frac{R^3J_3}{2r^3}(-10\cos\theta\sin\theta+3\sin\theta)+\frac{R^4J_4}{8r^4}(-140\cos^3\theta\sin\theta$$
$$\left.+60\cos\theta\sin\theta)+\ldots\right.$$

$$=\frac{\mu}{r^2}\left[\frac{3}{2}\frac{R^3}{r^3}J_3+\ldots\right]$$ SOUTHWARDS

c) Earth spins 366.25 times in 365.25 days

$$\therefore \text{Angular velocity}=\frac{366.25\times2\pi}{365.25\times24\times3600}\ \text{rad/s}$$

$$=72.9211\times10^{-6}\ \text{rad/s}$$

4 Cont

Satellite must have same ω, and needs

$$acc^n = force / mass = r\omega^2$$

i.e. $\frac{\mu}{r^2}\left(1 + \frac{3}{2} J_2 \left(\frac{R}{r}\right)^2\right) = r\omega^2$

Letting $r = 42165$ km gives

$LHS = 224.200 \times 10^6 (1 + 37.1 \times 10^{-6}) = 224.209 \times 10^6$

$RHS = 42165 \times \omega^2 = 224.212 \times 10^{-6}$

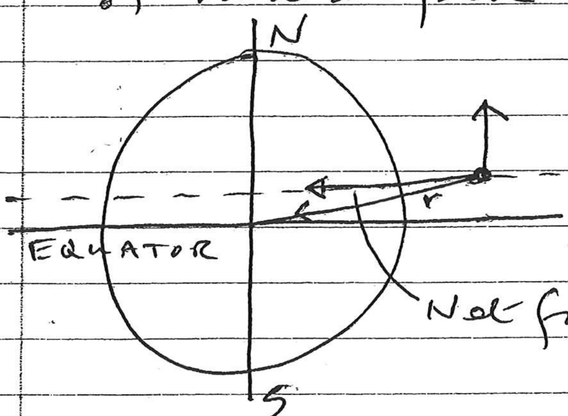
Agree to 5 sig figs, so value for r is good.

d) If $m = 100$ kg, force $\rightarrow$ South Pole

$$= \frac{300 \mu R^3 J_3}{2 r^5} = -297 \times 10^{-12} N$$

i.e. force is acting towards North Pole

Compensate by moving orbit into plane just North of equator, so component of radial pull cancels out polar pull:



actual distance is about 0.5 metres!

Net force

Assessor's comment:

Gravitational potential up to the J4 term. A fairly straightforward "show that…" type of question, requiring an understanding of potential theory and the definition of a sidereal day. The mark distribution was again strongly bimodal (14 candidates scored 70% or more, and 10 scored 30% or less), suggesting that candidates either had, or had not, prepared themselves for examination on this material. Most candidates who got that far made sensible suggestions for the last part.