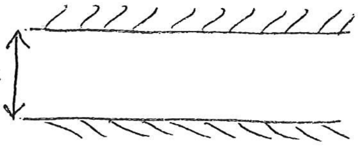
(a) See L. Notes

(b)



Considering only non-retarded L-J term

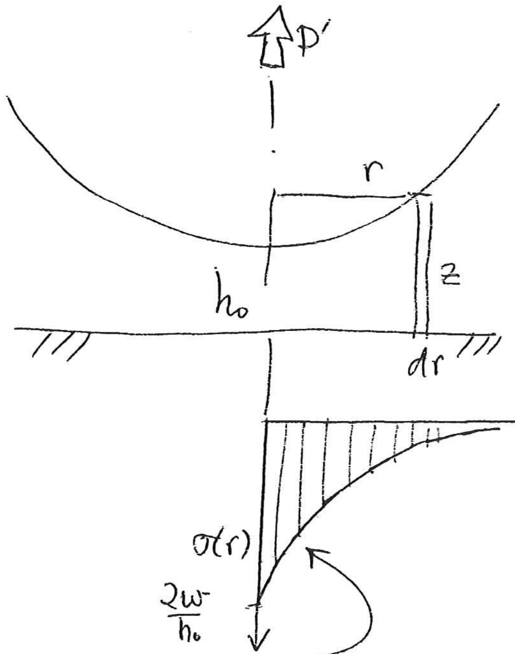
attractive force / unit area

$$\sigma(z) = \frac{A}{6\pi z^3}$$

$$w = \int_{h_0}^{\infty} \sigma(z)\,dz = \frac{A}{6\pi}\int \frac{dz}{z^3} = -\frac{A}{6\pi}\left[\frac{z^{-2}}{2}\right]_{h_0}^{\infty}$$

$$= \frac{A}{6\pi}\cdot\frac{1}{2h_0^2}$$

$$w = \frac{A}{12\pi h_0^2}$$



$$dP' = 2\pi r\,\sigma(r)\,dr$$

$$= 2\pi r\,\frac{A}{6\pi z^3}\,dr$$

But $z = h_0 + \dfrac{r^2}{2R}$

$$\therefore dz = \frac{r\,dr}{R}$$

$$\therefore dP' = \frac{2\pi A R\,dz}{6\pi z^3}$$

$$\sigma(r) = \frac{2w/h_0}{(1 + r^2/2Rh_0)^3}$$

$$P' = \int_{h_0}^{\infty} \frac{AR}{3z^3}\,dz$$

they should remember this approx quite possible to do (c) & (d) without S

$$P' = \frac{AR}{3}\left[\frac{-z^{-2}}{2}\right]_{h_0}^{\infty} = \frac{AR}{6h_0^2}$$

But $A = 12\pi h_0^2 w$ $\therefore$ $P' = \dfrac{12\pi h_0^2 w R}{6h_0^2} = 2\pi R w$

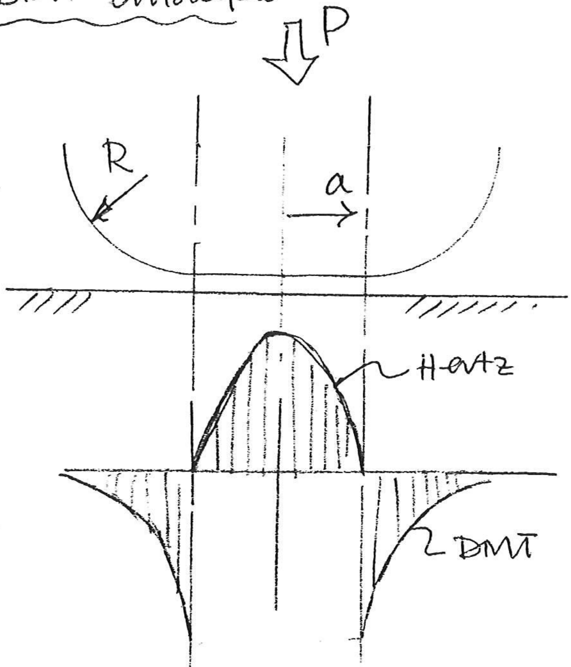
$$\sigma(r) = \frac{A}{6\pi z^3} \qquad z = h_0 + \frac{r^2}{2R} \qquad A = 12\pi h_0^2 w$$

hence
$$\sigma(r) = \frac{2 w h_0^2}{h_0^3 (1 + r^2/2Rh_0)^3}$$

$$\sigma(r) = \frac{2w}{h_0 (1 + r^2/2Rh_0)^3}$$

when $r = 0$, $\sigma(0) = 2w/h_0$; $r \to \infty$ $\sigma(r) \to 0$

(c) DMT analysis

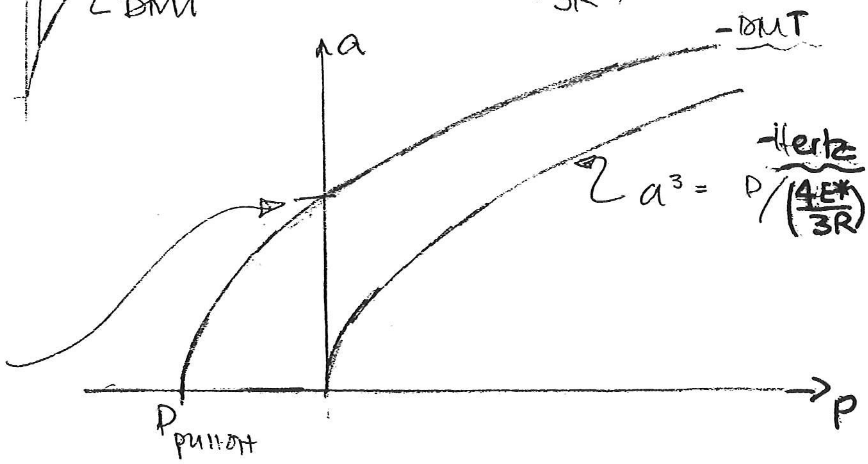


Hertz load $= \frac{4E^*}{3R} . a^3$

adhesive load $P' = 2\pi R w$

$$\therefore \; P = \frac{4E^*}{3R} a^3 - 2\pi R w$$

or $a^3 \left(\frac{4E^*}{3R}\right) = P + 2\pi R w$



when $P = 0$
$$a^3 = 2\pi R w . \frac{3R}{4E^*}$$

ie. $a = \left(\frac{3\pi R^2 w}{2E^*}\right)^{1/3}$

$P_{pull\,off}$
$\|$
$-2\pi R w$ (ie. tensile)

(d) JKR allows for adhesive forces to bring about further elastic deformation within material though at cost of singularity of stress at edge of the contact. JKR pull-off force is of magnitude $\frac{3}{2}\pi R w$. For a further discussion of the application of these analyses see, for example, Tabor D J. Colloid & I'face Sci 58 (1977) 113, or Johnson KL & Greenwood JA, J. Colloid Sci 192 (1997) 326-333.

Q2 (a) Conditions for equilibrium:

$$F_{NET} = F_{MECH} - F_{ELEC} = 0$$

$$F_{NET} = k(g_0 - g) - \frac{\varepsilon_0 A V^2}{2g^2} = 0.$$

$$k(g_0 - g) = \frac{\varepsilon_0 A V^2}{2g^2}$$

The stability criterion is :-

$$\frac{\partial F_{NET}}{\partial g} < 0$$

or pull-in is observed for:

$$\frac{\partial F_{NET}}{\partial g} = 0.$$

$$k = \frac{\varepsilon_0 A V_{PI}^2}{g_{PI}^2} \quad \text{— (1)}$$

V_{PI} - Pull-in voltage and g_{PI} - Pull-in gap.

$$k(g_0 - g_{PI}) = \frac{\varepsilon A V_{PI}^2}{2g_{PI}^2} \quad \text{— (2).}$$

From (1) and (2):

$$g_{PI} = \frac{2}{3} g_0$$

$$V_{PI} = \sqrt{\frac{8 k g_0^3}{27 \varepsilon_0 A}}$$

(b) The pull-out condition is:

$$F_{NET} = 0$$

$$\frac{\varepsilon_D A V_{POUT}^2}{2t^2} = k g_o$$ ④

$$V_{POUT}^2 = \frac{2 k g_o t^2}{\varepsilon_D A}$$

$$\frac{V_{PIN}^2}{V_{POUT}^2} = \frac{4}{27}\left(\frac{g_o}{t}\right)^2 \frac{\varepsilon_D}{\varepsilon_A}$$

The equation for pull-out gap (g_{POUT}) is :-

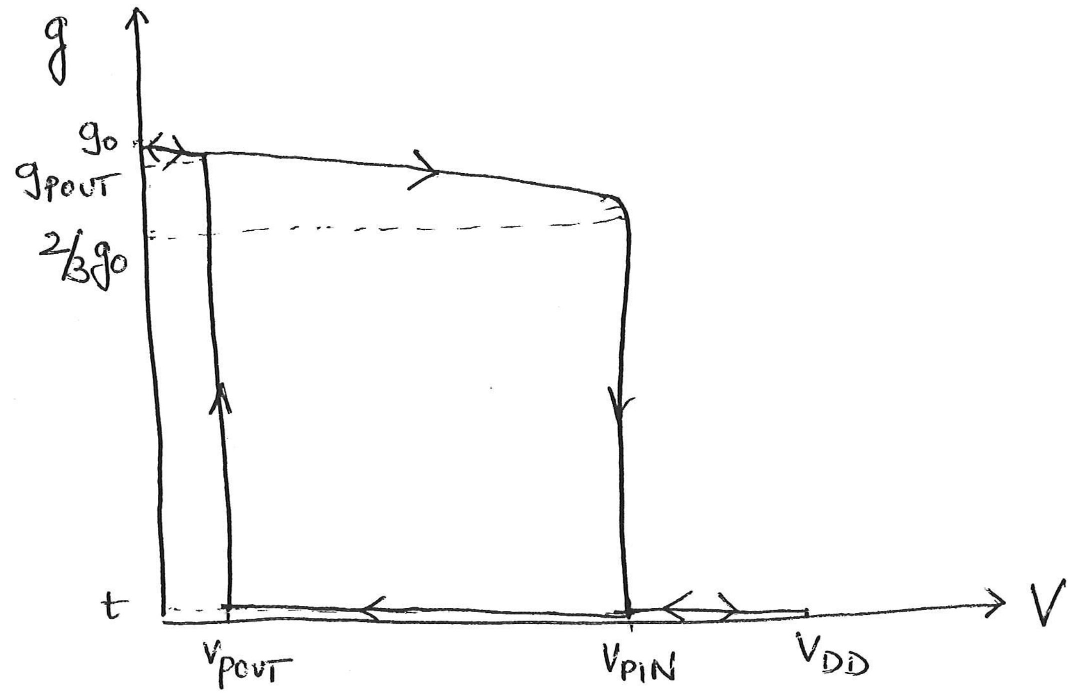
$$\frac{\varepsilon_o A V_{POUT}^2}{2 g_{POUT}^2} = k(g_o - g_{POUT}).$$

Substituting for V_{POUT}^2 and simplifying :

$$\left(\frac{\varepsilon_o A}{2K}\right)\left(\frac{2 K g_o t^2}{\varepsilon_D A}\right) = g_o g_{POUT}^2 - g_{POUT}^3$$

$$\left(\frac{\varepsilon_o}{\varepsilon_D}\right)\left(\frac{t}{g_o}\right)^2 = \left(\frac{g_{POUT}}{g_o}\right)^2 - \left(\frac{g_{POUT}}{g_o}\right)^3$$

(c)



3 (a) Displacement $x = \frac{ma}{k_x}$

$$= \frac{10^{-9} \times 9.8}{10} = 0.98 \text{ nm}.$$ (small relative to gap)

(b) $$\Delta C = \frac{\varepsilon_0 A}{g_0 - x} - \frac{\varepsilon_0 A}{g_0 + x}$$ (change in capacitance)

$$\simeq \frac{\varepsilon_0 A}{g_0}\left(\frac{2x}{g_0}\right)$$

$$\Delta C = \frac{8.85\times10^{-12} \times 2\times10^{-6} \times 200\times10^{-6} \times 2\times0.98\times10^{-9} \times 30}{(10^{-6})^2}$$

$$= 6.94 \text{ aF} \times 30 = 0.208 \text{ fF}$$ (assuming displacement is small)

Scale factor

$$SF|_x = \frac{\Delta C}{a} = 30\times6.94 \text{ aF}/g = 0.208 \text{ fF}/g$$

(c) $$z = \frac{ma}{k_z} = \frac{10^{-9} \times 9.8}{9} = 1.09 \text{ nm}$$

$$SF|_z = \left.\frac{\Delta C}{g}\right|_z \simeq SF|_x \cdot \frac{z}{t} \simeq SF|_x \cdot \frac{1.09\times10^{-3}}{1}$$

$$= 0.227 \text{ aF}/g.$$

(d) Self-test : This feature allows for the verification of mechanical and electrical reliability of the accelerometer before or after deployment. Self-test can be implemented by converting some of the capacitor electrodes into parallel plate actuators. When a voltage is applied, the mass deflects and this deflection is picked up by the electrode resulting in a proportional voltage. Thus both mechanical and electronic functionality can be verified at any point during operation.

(e) The Brownian motion of the mass results in a change in capacitance regardless of the applied acceleration – this change appears as a random output voltage symptomatic of white noise spectral behaviour.

$$Q_x = 10$$

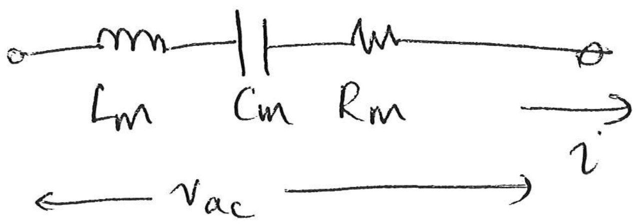
$$\bar{x}_n = \frac{\bar{F}_n}{k_x} = \frac{\sqrt{\frac{4k_BTm\omega_0}{Q}\cdot(\Delta f)}}{K_x}$$

$$= \sqrt{\frac{4\times1.38\times10^{-23}\times300\times10^{-9}\times10^{5}}{10}}\times\frac{1}{10}$$

$$= 4.07\times10^{-14}\ m/\sqrt{Hz}$$

$$\bar{a}_n = (0.98\times10^{-9})^{-1}\times4.07\times10^{-14}\ g/\sqrt{Hz}$$

$$= 41.5\ \mu g/\sqrt{Hz}$$



$$\frac{i}{V_{ac}} = \frac{1}{j\omega L_m + R_m + \frac{1}{j\omega C_m}}$$

This now gives us:

$$L_m = \frac{m}{\eta^2}, \quad C_m = \frac{\eta^2}{k}, \quad R_m = \frac{b}{\eta^2}$$

(d) R_m is the motional resistance of the resonator. It represents the effective electrical loss of the resonator.

$$R_m = \frac{b}{\eta^2}$$

To reduce R_m, reduce b (i.e. mechanical losses) and increase η (electro-mechanical coupling). This involves increasing V_{DC} and lowering the gaps (g) and increasing transduction area (A). Some of these parameters (e.g. A) can be set by careful design at a device level while others (g) are set at a process level.

Q4

(a)
$$F = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$
$$= \frac{\varepsilon A}{2g^2} (V_{DC} + v_{ac})^2$$
$$= \frac{\varepsilon A}{2g^2} \left(V_{DC}^2 + 2V_{DC}v_{ac} + v_{ac}^2 \right)$$
$$F(\omega) = \frac{\varepsilon A}{2g^2} \cdot 2V_{DC}v_{ac} = \frac{\varepsilon A V_{DC} v_{ac}}{g^2}$$

(b)
$$i = \text{capacitive current}$$
$$= \frac{d(CV_{DC})}{dt} = V_{DC} \frac{\partial C}{\partial x} \cdot \frac{\partial x}{\partial t}$$
$$i = \frac{V_{DC}\,\varepsilon A}{g^2} \cdot \dot{x} = \eta \dot{x} \quad \text{where } \eta = \frac{\varepsilon A V_{DC}}{g^2}$$

(c) For the electromechanical system
$$\frac{\dot{x}(j\omega)}{F(j\omega)} = \frac{1}{mj\omega + b + \frac{k}{j\omega}}$$
$$\frac{i/\eta}{\eta v_{ac}} = \frac{1}{mj\omega + b + k/j\omega}$$
$$\frac{i}{v_{ac}} = \frac{\eta^2}{mj\omega + b + \frac{k}{j\omega}}$$

consider the analogue to a series L-C-R circuit

PART II B 2010 4C15 MEMS DESIGN

Assessor's comments:

Questions:

Question 1, This question had a low average mark. Most students had difficulty with deriving the expression for the force of adhesion between the two solids in part b (ii) and with the sketch in part (c) with some answers demonstrating a worrying lack of understanding in the underlying concepts.

Question 2, The first two parts of this question were generally well done. Most answers to parts (c) and (d) had room for improvement with descriptive parts relating to self-test and Brownian noise answered to a minimum level of detail.

Question 3, %. This question was the least popular and also carried the lowest average mark indicating that students had difficulty with the subject matter. Most students had problems with deriving expression for the pull-in voltage from first principles in part (a) and visualising the pull-out condition in part (b). Some students used their physical intuition to sketch the operational cycle of the switch in part (c) correctly.

Question 4, . This question was the most popular and attempted by nearly all candidates. While parts (a) and (b) were generally well done; most students did not derive expressions for the resonator motional parameters from an equivalent circuit analysis in part (c) – simply writing down the final expressions in many cases.