PART II B
2010

4C16 Advanced Machine Design
Crib for 2010 Exam
Dr D J Cole

1. a) Follows immediately from the data sheet when $\bar{u} = \frac{U}{2}$.

b) i) $$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial y}\right) = \frac{U}{2}\frac{\partial h}{\partial x}$$

now $\left|\frac{\partial p}{\partial x}\right| \ll \left|\frac{\partial p}{\partial y}\right|$ so $$\frac{\partial}{\partial y}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial y}\right) = \frac{U}{2}\frac{\partial h}{\partial x}$$

η is const and $h = h(x)$ so $$\frac{\partial^2 p}{\partial y^2}\cdot\frac{h^3}{12\eta} = \frac{U}{2}\frac{\partial h}{\partial x}$$

$$\underline{\underline{\frac{\partial^2 p}{\partial y^2} = \frac{6U\eta}{h^3}\frac{dh}{dx}}}$$

ii) Integrating w.r.t. y $$\frac{h^3}{6U\eta}\frac{\partial p}{\partial y} = \left(\frac{dh}{dx}\right)y + A_1$$

by symmetry $\frac{\partial p}{\partial y} = 0$ when $y = 0$ $\therefore A_1 = 0$

integrate again w.r.t. y $$\frac{h^3}{6U\eta}p = \left(\frac{dh}{dx}\right)\frac{y^2}{2} + A_2$$

but $p = 0$ when $y = \pm L$ $\therefore A_2 = -\frac{L^2}{2}\frac{dh}{dx}$

so $$\frac{h^3 p}{6U\eta} = \left(\frac{dh}{dx}\right)\left\{\frac{y^2}{2} - \frac{L^2}{2}\right\}$$

or $$\underline{\underline{p = 3U\eta\left\{y^2 - L^2\right\}\frac{dh/dx}{h^3}}} \quad (1)$$

c) If $h = h_0 + \dfrac{x^2}{2R}$ and $x \leq 0$

R is the local radius of curvature

$$\frac{dh}{dx} = \frac{2x}{2R} = \frac{x}{R}.$$

$$\therefore p = 3U\eta\{y^2 - L^2\}\frac{x/R}{(h_0 + x^2/2R)^3}$$

by inspection, p is maximum on centreline of bearing, that is, $y = 0$

$$\therefore \quad p = -\frac{3U\eta L^2}{R}\frac{x}{(h_0 + x^2/2R)^3}$$

$$\frac{dp}{dx} = -\frac{L^3 3U\eta}{R}\left\{-x\,3\left(h_0 + \frac{x^2}{2R}\right)^{-4}\frac{2x}{2R} + \left(h_0 + \frac{x^2}{2R}\right)^{-3}\right\}$$

max p when $\dfrac{dp}{dx} = 0$

$$\frac{3x^2}{R} = \left(h_0 + \frac{x^2}{2R}\right)$$

$$x^2 = h_0\frac{2R}{5}$$

$$x = -\sqrt{\frac{2}{5}h_0 R}$$

d)

$$W = \iint p\,dx\,dy$$

$$= 3U\eta\iint (y^2 - L^2)\frac{dh/dx}{h^3}\,dx\,dy$$

$$= 3U\eta\left[\frac{y^3}{3} - L^2 y\right]_{-L}^{L}\int_{h_0}^{\infty}\frac{dh}{h^3}$$

$$W = -3U\eta\frac{4}{3}L^3\left[\frac{1}{2h^2}\right]_{h_0}^{\infty} = \frac{2U\eta L^3}{h_0^2}$$

Assessor's comment Q1:

Part (a) and part (b)(i) were answered well by most candidates. Many solutions to part (ii) involved definite integration whereas an indefinite integration was required. There were few correct solutions to parts (c) and (d), with many candidates demonstrating little insight into the relationship between maximum pressure and pressure gradient.

2. a)

- IC engine should operate at maximum efficiency point or be turned off.
- need for energy storage should be minimised.
- ensure sufficient energy for required operating cycle.

Possible difficulties are:

- finite energy storage.
- constraints on power for charging & discharging the energy store.
- operating cycle not usually known in advance.

b) i) data sheet $\omega_s = (1+R)\omega_c - R\omega_a$

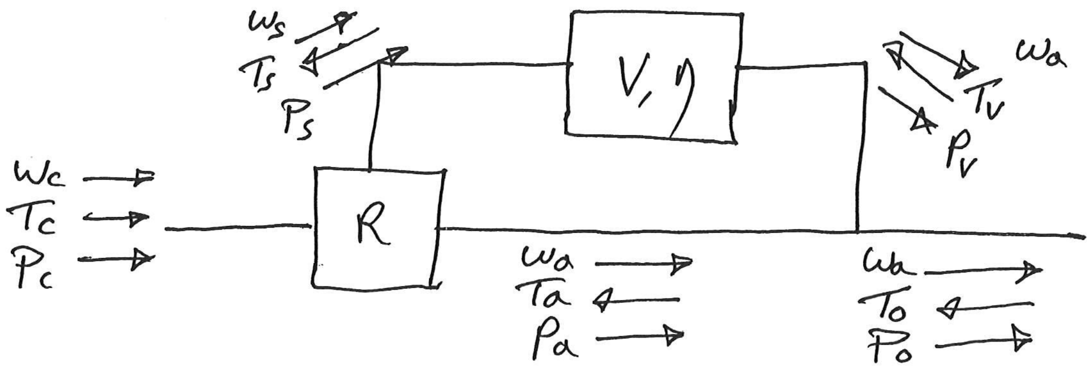
but $V = \dfrac{\omega_a}{\omega_s}$ $\therefore$ $\omega_s = \dfrac{\omega_a}{V}$

so $$\frac{\omega_a}{V} = (1+R)\omega_c - R\omega_a$$

$$\omega_a\left(\frac{1}{V} + R\right) = (1+R)\omega_c$$

$$\underline{\underline{\frac{\omega_a}{\omega_c} = \frac{(1+R)}{\left(\frac{1}{V} + R\right)}}}$$

ii) torques - choose signs to be consistent with powers & speeds shown on figure:



need to find $\frac{T_v}{T_a}$, which is $\frac{T_s}{T_a} \cdot \frac{T_v}{T_s}$

for $\frac{T_v}{T_s}$ consider power through V

$$\eta = \frac{P_v}{P_s} = \frac{\omega_a T_v}{\omega_s T_s}$$

(assuming power flows left to right)

$$\therefore \frac{T_v}{T_s} = \eta \frac{\omega_s}{\omega_a} = \eta/V$$

for $\frac{T_s}{T_a}$ use virtual power on epicyclic gear

$$P_c = P_s + P_a$$

$$T_c \omega_c' = T_s \omega_s' + T_a \omega_a'$$

elim' T_c by setting $\omega_c' = 0$

$$\therefore \frac{T_s}{T_a} = -\frac{\omega_a'}{\omega_s'} = \frac{1}{R}$$

$$\therefore \frac{T_v}{T_a} = \frac{T_s}{T_a} \cdot \frac{T_v}{T_s} = \frac{1}{R} \cdot \frac{\eta}{V} = \underline{\underline{\eta/RV}}$$

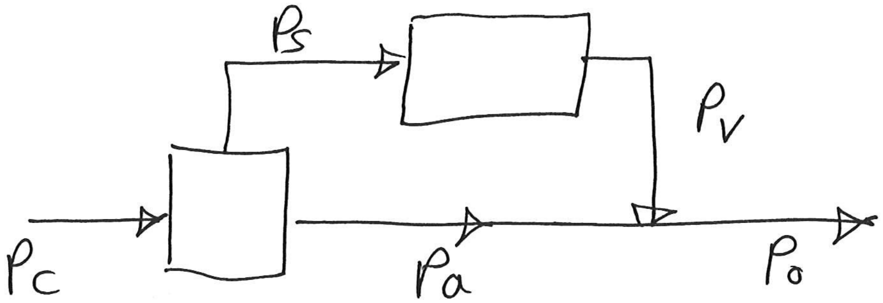
Now consider powers

$$\frac{P_v}{P_0} = \frac{P_v}{P_v + P_a} = \frac{\omega_a T_v}{\omega_a T_v + \omega_a T_a} = \frac{1}{1 + \frac{T_a}{T_v}}$$

$$= \frac{1}{1 + \frac{RV}{\eta}}$$

$$\underline{\underline{\frac{P_v}{P_0} = \frac{\eta}{\eta + RV}}}$$

iii) Recirculation



for no recirculation $\quad 0 < \frac{P_v}{P_o} < 1$

i.e. $\quad 0 < \frac{\eta}{\eta + RV} < 1$

so $\quad RV > 0$

$\quad V > 0 \quad$ since R is +ve

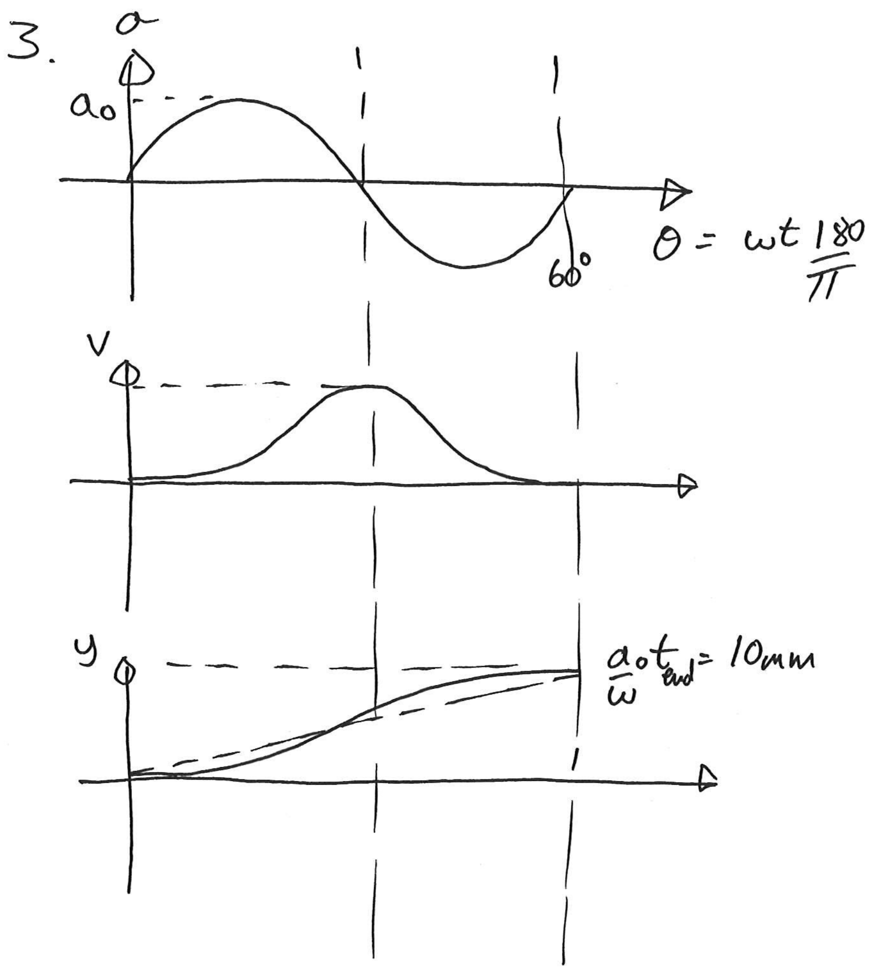
from (i) $\quad \frac{\omega_a}{\omega_c} = \frac{1+R}{\frac{1}{V}+R} = \frac{V+VR}{1+VR}$

if $V>0$ then $\quad \underline{\underline{0 < \frac{\omega_a}{\omega_c} < \frac{1}{R}+1}}$

Assessor's comments:

Generally well answered. Most candidates successfully answered parts (a) and (b)(i). In part (b)(ii) correct answers were less frequent, due to problems with signs, use of the virtual power equation, and general algebraic manipulation. There were some good attempts at finding the ratio of speeds in part (b)(iii) although no one got the correct answer.

3.



$$a = a_0 \sin \omega t$$

$$\therefore \quad v = \int a\,dt = -\frac{a_0}{\omega}\cos \omega t + A$$

but $v = 0$ at $t = 0$ $\quad \therefore \quad A = \frac{a_0}{\omega}$

$$y = \int v\,dt = -\frac{a_0}{\omega^2}\sin \omega t + \frac{a_0 t}{\omega} + B$$

$y = 0$ at $t = 0$ $\quad \therefore \quad B = 0$

max lift $\quad y = \frac{a_0 t_{end}}{\omega} = 10mm$ at end of lift

what is t_{end}? $\quad$ 60° rotation at 6000 rpm

takes $\quad \frac{60° \times 60 \text{ secs/min}}{360°/\text{rev} \times 6000 \text{ revs/min}} = \underline{\underline{\frac{1}{600}\,s}}$

what is ω? $\quad$ one cycle of 2π rad in $\frac{1}{600}$ s

gives $\underline{\underline{\omega = 1200\pi \text{ rad/s}}}$

so. $\dfrac{a_0}{\omega} t_{end} = 0.01$

$a_0 = 0.01 . \dfrac{\omega}{t_{end}} = 0.01 . \dfrac{1200\pi}{1/600}$

$\underline{\underline{a_0 = 7200\pi \ m/s^2}}$

b) $E^* = 115 \text{ GPa}$

max accn. $a_0 = 7200\pi \ m/s^2$

so max force $F_{max} = m . a_0$

$= 2 . 7200\pi$

$=$

Hertz line contact $p_0 = \left(\dfrac{P' E^*}{\pi R} \right)^{\frac{1}{2}}$

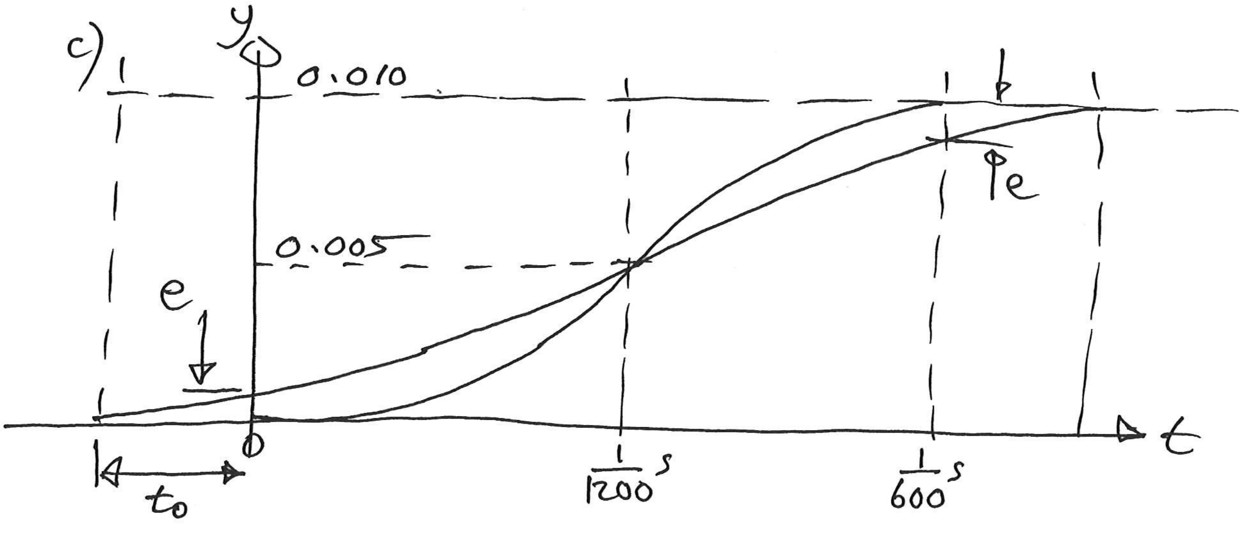
$R = \dfrac{0.005 . 0.02}{0.005 + 0.02} = \underline{0.004 m}$

$P' = \dfrac{2 . 7200\pi}{0.01}$ ← face width.

$\therefore \quad p_0 = \sqrt{\dfrac{2.7200\pi . 115.10^9}{0.01 . \pi . 0.004}}$

$\underline{\underline{p_0 = 6.43 \text{ GPa}}}$

c)



starting with $y = \dfrac{a_0 t}{\omega} - \dfrac{a_0 \sin \omega t}{\omega^2}$

consider lift starting early:

$$0.005 = \frac{a_{new}}{\omega_{new}}\left(\frac{1}{1200} + t_0\right) - \cancelto{0}{\frac{a_{new}}{\omega_{new}^2} \sin \omega_{new}\left(\frac{1}{1200} + t_0\right)}$$

so $$\frac{a_{new}}{\omega_{new}} = \frac{0.005}{\left(\frac{1}{1200} + t_0\right)}$$

at $t = 0$, $y = 0.0001\,m$

$$0.0001 = \frac{a_{new}}{\omega_{new}} t_0 - \frac{a_{new}}{\omega_{new}^2} \sin \omega_{new} t_0$$

subst. $\dfrac{a_{new}}{\omega_{new}}$ to give:

$$0.0001 = \frac{0.005\, t_0}{\left(\frac{1}{1200} + t_0\right)} - \frac{1}{\omega_{new}} \frac{0.005}{\left(\frac{1}{1200} + t_0\right)} \sin \omega_{new} t_0$$

where $\omega_{new} = \dfrac{\pi}{\frac{1}{1200} + t_0}$ (half cycle in $\frac{1}{1200} + t_0$)

so $$0.0001 = \frac{0.005\, t_0}{\frac{1}{1200} + t_0} - \frac{\cancel{\frac{1}{1200} + t_0}}{\pi} \cdot \frac{0.005}{\cancel{\frac{1}{1200} + t_0}} \sin \frac{t_0 \pi}{\frac{1}{1200} + t_0}$$

then $$0.0001 = 0.005\alpha - \frac{0.005}{\pi} \sin \pi\alpha$$

where $$\alpha = \frac{t_0}{\frac{1}{1200} + t_0}$$

assume small α so:

$$0.02 = \alpha - \frac{1}{\pi} \sin \pi\alpha$$
$$= \alpha - \frac{1}{\pi}\left(\pi\alpha - \frac{(\pi\alpha)^3}{3!} + \dots\right)$$
$$0.02 = \frac{1}{\pi} \frac{\pi^3}{6} \alpha^3$$
$$\underline{\underline{\alpha = 0.23}}$$

hence
$$t_0 = \frac{0.23}{1200} + 0.23\, t_0$$
$$t_0 = \frac{0.23}{1200(1-0.23)}$$
$$\underline{\underline{t_0 = 2.488 \cdot 10^{-4}\, s \approx 2.5 \cdot 10^{-4}\, s}}$$

Now
$$y_{max} = \frac{a_0 t_{end}}{\omega} = 0.01 = \frac{a_{new}\, t_{end\,new}}{\omega_{new}}$$

$$\therefore \frac{a_{new}}{a_0} = \frac{\omega_{new}}{\omega} \cdot \frac{t_{end}}{t_{end\,new}}$$
$$= \frac{\dfrac{\pi}{\frac{1}{1200} + 2.5 \cdot 10^{-4}}}{1200\pi} \cdot \frac{\frac{1}{600}}{\frac{1}{600} + 2 \cdot 2.5 \cdot 10^{-4}}$$
$$= 0.593$$

pressure varies with sq. root of accn

so
$$\frac{p_{new}}{p_0} = \sqrt{0.593} = 0.77$$

so a $\underline{\underline{23\%\ \text{reduction}}}$

Assessor's comment Q3:

Even though part (a) of the question did not require it, surprisingly few candidates sketched graphs of lift, speed and acceleration. Perhaps because of this many candidates incorrectly deduced that ω was the camshaft angular speed. The contact pressure calculation in part (b) was generally performed well. The modified cam profile in part (c) proved to be a significant challenge and there were no correct answers; marks were awarded for correctness of the approach taken.