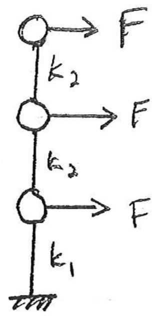
PART IIB 2010 4D6 - DYNAMICS IN CIVIL ENGINEERING DR GOPAL MADABHUSHI

1 (a) Estimate mode shape



$$\bar{u}_1 = \frac{3F}{k_1}$$

$$\bar{u}_2 = \frac{3F}{k_1} + \frac{2F}{k_2} = \frac{F(3k_2 + 2k_1)}{k_1 k_2}$$

$$\bar{u}_3 = \frac{3F}{k_1} + \frac{2F}{k_2} + \frac{F}{k_2} = \frac{3F(k_1 + k_2)}{k_1 k_2}$$

Normalize:

$$\left.\begin{aligned} \bar{u}_1 &= 1 \\ \bar{u}_2 &= \frac{\frac{1}{3}(3k_2 + 2k_1)}{k_2} = 1 + \frac{2}{3}\frac{k_1}{k_2} \\ \bar{u}_3 &= \frac{k_2 + k_1}{k_2} = 1 + \frac{k_1}{k_2} \end{aligned}\right\} \quad \phi_1 = \begin{bmatrix} 1 \\ 1 + \frac{2}{3}\ell \\ 1 + \ell \end{bmatrix}$$

where $\ell = \dfrac{k_1}{k_2}$

Estimate Natural Frequency:

$$m_{eq} = \sum m \bar{u}^2$$

$$= m\left(1 + \left(1 + \tfrac{2}{3}\ell\right)^2 + (1+\ell)^2\right)$$

$$= m\left(3 + \tfrac{10}{3}\ell + \tfrac{13}{9}\ell^2\right)$$

$$K_{eq} = k_1(\bar{u}_1^2) + k_2(\bar{u}_2 - \bar{u}_1)^2 + k_3(\bar{u}_3 - \bar{u}_2)^2$$

$$= k_1 + k_2 \tfrac{4}{9}\ell^2 + k_2\left(\tfrac{1}{9}\right)\ell^2$$

$$= k_1 + \tfrac{5}{9}k_2\ell^2$$

$$\omega_n = \sqrt{\frac{K_{eq}}{M_{eq}}}$$

- Actual mode shapes and frequencies:

$$\underline{M} = \begin{bmatrix} m & & \\ & m & \\ & & m \end{bmatrix}$$

$$\underline{K} = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \overset{k_3=k_2}{=} \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & 2k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$$

Equation of Motion: $\underline{M}\,\ddot{u} + \underline{K}\,u = 0$

solution : $u(t) = \phi_n \left(A_n \cos \omega_n t + B_n \sin \omega_n t\right)$

$\det\left[\underline{k} - \omega_n^2 \underline{M}\right] = 0 \;\rightarrow\;$ Find three natural frequencies: $\omega_1, \omega_2, \omega_3$

Plug ω_n back into following equation to get mode shapes:

$$\left[\underline{k} - \omega_n^2 \underline{M}\right] \phi_n = 0$$

$\boxed{1}$ (b) $k_1 = k_2 \rightarrow l = 1$

$$M_{eq} = \left(3 + \frac{10}{3} + \frac{13}{9}\right) m = \frac{70}{9} m$$

$$K_{eq} = k_1 + \frac{5}{9} k_1 = \frac{14}{9} k_1$$

$$\omega_n = \sqrt{\frac{14}{70}} \sqrt{\frac{k_1}{m}} = \sqrt{\frac{1}{5}} \sqrt{\frac{k_1}{m}} = \underline{\underline{0.45 \sqrt{\frac{k_1}{m}}}}$$

$$\underline{\underline{\phi_1 = \begin{bmatrix} 1 \\ 5/3 \\ 2 \end{bmatrix}}}$$

- The actual natural frequency is lower (Rayleigh's principle)

(c) $5k_1 = k_2 \rightarrow l = 1/5 \rightarrow$ $\underline{\underline{\phi_1 = \begin{bmatrix} 1 \\ 17/15 \\ 6/5 \end{bmatrix}}}$

$$M_{eq} = m\left(3 + \frac{10}{15} + \frac{13}{9}\frac{1}{25}\right)$$

$$= 3.72\ m$$

$$K_{eq} = k_1 + 5k_1\left(\frac{2}{15}\right)^2 + 5k_1\left(\frac{1}{15}\right)^2$$

$$= \frac{10}{9} k_1$$

$$\omega_n = \sqrt{\frac{1.11\ k_1}{3.72\ m}} = \underline{\underline{0.55 \sqrt{\frac{k_1}{m}}}}$$

$\boxed{1}$ (d) For $l=1$:

$$\Gamma = \frac{m\left(1 + 5/3 + 2\right)}{\frac{70}{9} m} = 0.6$$

For $l=2$:

$$\Gamma = \frac{m\left(1 + 1.13 + 1.2\right)}{3.72\, m} = 0.9$$

Given: $f_1 = \Gamma m \phi_1 A_1$

- in both cases: $\phi_1 = 1$

- ω_n for (b) and (c) are quite similar, so the spectral acceleration A_1 will likely be similar.

∴ The relative increase at the bottom floor $= \frac{0.9}{0.6} = 150\%$

- The forces experienced by the first floor are 50% higher than expected by the designer.

Assessor's comment:

In general, the majority of students did well on parts (b) and (c) but found parts (a) and (d) more difficult. In part (d), many students struggled to interpret the results in parts (a) to (c), and did not come up with a rational explanation. About half of the students struggled to find the first mode shape from the static deflected shape in part (a). Also in part (a), the students were only required to set up the procedure for finding the actual mode shapes and corresponding natural frequencies of the frame, and many of the students did this correctly, although the question could have been better phrased.

Q 2 a) UB 254 × 102 × 28 $\quad m = 28.3$ kg/m

$I_x = 4005\ cm^4 = 4.005 \times 10^{-5}\ m^4$.

Steel $E = 210$ GPa

$EI = 8.4105 \times 10^6\ N m^2$.

using Data Sheets:

1st Mode $\quad K_{1eq} = \frac{(i\pi)^4 EI}{2L^3} = \frac{(1\times\pi)^4\ 8.4105\times10^6}{2\times12^3} = 237.054\times10^3$ N/m

$M_{eq} = \frac{mL}{2} = 28.3\times\frac{12}{2} = 169.8$ kg/m

$\omega_{1eq} = \sqrt{\frac{K_{1eq}}{M_{eq}}} = 37.36$ rad/s $\Rightarrow \underline{f_1 = 5.95\ Hz} \simeq 6$ Hz $\quad$ or

2nd mode $\quad K_{2eq} = \frac{(2\pi)^4 EI}{2L^3} = 3792.866\times10^3$ N/m

$M_{eq} = 169.8$ kg/m

$\omega_{2eq} = \sqrt{\frac{K_{2eq}}{M_{eq}}} = 149.4$ rad/s $\Rightarrow \underline{f_2 = 23.78\ Hz}$ $\quad$ [30%]

2 b) DAF for 1st Mode $\quad \frac{t_d}{T_1}$ $\quad \frac{t_d}{T_1} = \frac{0.1}{0.167} \simeq 0.6 \quad \underline{DAF = 1.4}$

$T_1 = \frac{1}{f_1} = 0.167\ s$

DAF for 2nd mode $\quad t_d/T_2$

$T_2 = 1/f_2 = 1/23.78 = 0.042 \quad \frac{t_d}{T_2} = \frac{0.1}{0.042} = 2.4$

$\Rightarrow \underline{DAF \simeq 1.0}$

Max dynamic deflection at 'P' in 1st mode

DAF = 1.4

At 'P' $\quad F_1 = F_0 \times U_{1P} = F_0 \times \sin\frac{3\pi}{12} = 5\times0.707 = 3.535$ kN.

$$U_1 \text{ static} = \frac{F_{1eq}}{K_{1eq}} = \frac{3.535 \times 10^3}{237.054 \times 10^3} = 0.0149 \text{ m}.$$

$$\therefore U_1 \text{ dynamic} = DAF \times U_1 \text{ stat} = 1.4 \times 0.0149$$

$$\text{Dynamic } \delta_P = 0.0208 \text{ m or } \underline{\underline{20.88 \text{ mm}}}.$$

2c) At Q :-

$$U_1(x) = \sin \frac{\pi x}{L} = \sin \pi \times \frac{8}{12} = 0.866.$$

$$U_2(x) = \sin \frac{2\pi x}{L} = \sin 2\pi \times \frac{8}{12} = -0.866$$

$$\therefore F_{1eq} = 5\text{kN} \times 0.866 = 4.33 \text{ kN} \quad (\text{mode } 1)$$

$$F_{2eq} = 5\text{kN} \times -0.866 = -4.33 \text{ kN} \quad (\text{mode } 2).$$

$$\therefore U_{1\,max} = 1.4 \times \frac{4.33 \times 10^3}{237.054 \times 10^3} \times 1000 = 25.57 \text{ mm at Q}$$

$$U_{2\,max} = 1.0 \times \frac{-4.33 \times 10^3 \times 1000}{3792.866 \times 10^3} = -1.1416 \text{ mm}.$$

$\therefore$ Max deflection at Q.

$$|U_1| + |U_2| = 26.71 \text{ mm}$$

$$SQRR = \sqrt{U_1^2 + U_2^2} = \underline{25.59 \text{ mm}}$$

[20%]

2^{nd} mode has very little effect on the max deflection at Q.

Assessor's comment Q2:

This question was on the dynamic response of a simply supported beam and finding the natural frequencies for modes 1 and 2. The second and third parts of the questions tested the candidates on determining the response of the beam under time varying loading and superposition of different modal responses. This was a very popular question with all candidates attempted it. Generally the solutions were very good, but a few candidates made numerical errors.

Q 3) a) The finite element method is originally formulated for continuum. However it is possible two have two layers of different continuum be overlain in FE formulations. These continua occupy same spatial co-ordinates and are coupled in some fashion. In geotechnical engineering, the solid nodes forming a solid mesh, and fluid nodes forming a fluid mesh are overlain to represent saturated soils. The pore pressure and solid node displacements are taken as field variables.
[10]

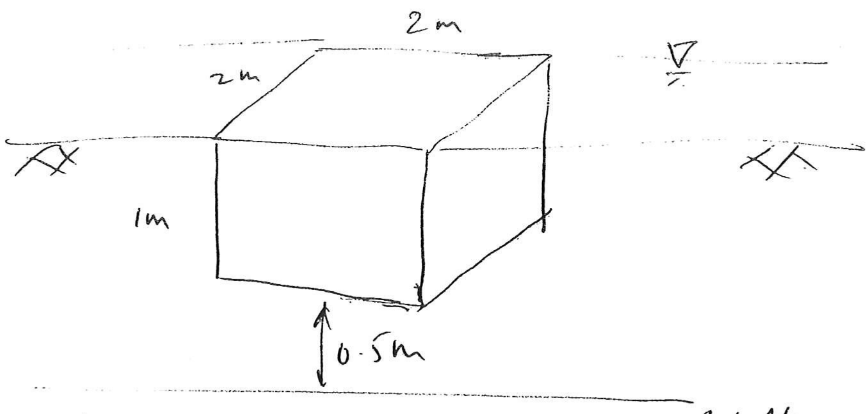
Q 3) b) The semi-infinite extent of soil usually poses a problem for dynamic problems. FE discretisations are usually limited, where as in nature soil extends laterally to infinity. In dynamic problems wave energy can be dissipated away into infinity in nature. However in a "limited size" of a FE mesh, this wave energy may have to be removed artificially by having numerical boundaries

a) absorbing boundary ⇒ organise a series of ~~dam~~ viscous dampers on the boundary nodes to absorb energy.

b) overlaying boundary ⇒ Smith-Cundall type boundaries which overlay two meshes with fixed & free boundaries

c) CPC boundary ⇒ Compound parabolic boundaries which collect and retain the incident wave energy until analyses are completed
[20%]

3 c)



From Data Book I

$$G_{max} = 100 \frac{[3-e]^2}{1+e} \sqrt{p'} \text{ MPa.}$$

$e = 0.9$

$K_0 = \frac{\nu}{1-\nu} = \frac{0.3}{0.7}$

Total stress on Ref Plane } $\sigma_v = \frac{24 \times 2 \times 2}{2 \times 2} + 19.5 \times 0.5$

$= 33.75$ kPa.

$u = (1+0.5)\gamma_w = 1.5 \times 10 = 15$ kPa.

$\therefore \sigma_v' = \sigma_v - u = 33.75 - 15 = 18.75$ kPa

$p' = \left(\frac{1+2k_0}{3}\right)\sigma_v'$

$= \left(\frac{1+2 \times 0.428}{3}\right) 18$

$p' = 11.61$ kPa

$$\therefore G_{max} = 100 \frac{[3-0.9]^2}{1.9} \times \sqrt{\frac{11.61}{1000}}$$

$$= \underline{\underline{25 \text{ MPa}}}.$$

Using Data Sheets :- $2l = 2m$ $2b = 2m$ $e = 1$ $l/b = 1$ $e/b = 1$

$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8 (l/b)^{0.65} + 2.4\right]\left[1 + 0.33 + \frac{1.34}{1+l/b}(e/b)^{0.8}\right]$$

$$= \frac{Gb}{2-\nu}[9.2][2] = \underline{\underline{270.58 \text{ MN/m}}}$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73 (l/b)^{2.4} + 2.7\right]\left[1 + e/b + \frac{1.6}{(0.35 + (l/b)^4)}(e/b)^2\right]$$

$$= \frac{Gb^3}{1-\nu} \times [6.43][3.185] = \underline{\underline{731.45 \text{ MN-m/rad}}}$$

[30%]

3d) Mass of Concrete block = $2 \times 2 \times 1 \times 24 = 96$ kN
$= 9600$ kg

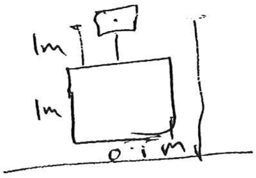
Mass of machine = 2000 kg

Horizontal Mode = $9600 + 2000 = 11600$ kg

$\therefore \omega_h = \sqrt{\frac{K_h}{m_h}} = \sqrt{\frac{270.58 \times 10^6}{11600}} = 152.72$ rad/s

$f_h = \underline{\underline{24.3 \text{ Hz}}}$

Rocking Mode:



$I = \cancel{80}\ 2000 \times 2.5^2 + 9600 \times 1^2$

$= 22100$ kg-m^2

$\therefore \omega_{rocking} = \sqrt{\frac{731.45 \times 10^6}{22100}} = 181.92$ rad/s

$f_{rock} = \underline{28.95}$ H[z]

[20%]

3e) Excess pore pressures can be generated in saturated sands + shear strains amplitude can cause a degradat[ion] in sand's shear modulus.

$-(f_h)_{actual} = 0.8 \times 24.3 = 19.44$ Hz.

$\Rightarrow (f_r)_{actual} = 0.8 \times 28.95 = \underline{\underline{23.16 \text{ Hz}}}$

[20%]

Assessor's comment:

The initial parts of the question were on the use of finite element method to study dynamic problems with mutli-phase materials such as soils. The later parts of the question were on the determination of rocking and horizontal mode vibrations of the machine foundations. The initial parts were done well by relatively few candidates while the calculations parts were done reasonably well by most candidates.

MA

4D6 Q4 2010

a) Marks will be obtained for descriptions of

i) Suspension bridge

- static drag, plus the dynamic response component from buffeting via the gust factor (with an explanation of what that is). There are horizontal and vertical buffeting components. Can be reduced by having streamlined deck.
- Flutter – tends to dominate ULS design of the main span. Marks will be obtained for a description of what flutter is (2 dof, catastrophic instability). Mention wind tunnel testing, aerodynamic derivatives, etc. Can be reduced by having perforated deck to equalise pressures.
- Vortex shedding – vertical excitation of deck – typically not catastrophic. Typically higher frequency and lower wind speeds than flutter. Can be reduced by guide vanes (e.g. Storebaelt). Also occurs on susupension cables – where Stockbridge dampers may reduce its effects.
- Galloping – exacerbated by non-circularity of cables (e.g. helical strands, or by ice, or rivulets of rain (wind-rain excitation)). Can be limited using Stockbridge dampers.
- Static divergence – tend to just design a way from it.
- Wake-buffetting – e.g. two adjacent bridge decks.

ii) Chimney

Main effects are alongwind excitation due to static drag plus buffeting, and across-wind excitation due to vortex-shedding (which can be reduced by helical strakes or shrouds (at the expense of extra drag). Can also get ovalling at the top of thin shell chimneys excited by vortex shedding, and this can be reduced by stiffening ring around top of chimney.

b) Marks will be obtained for description of robustness design (e.g. multiple load paths, detailing as per earthquake codes, etc.) , increased stand-off distances via bollards, even slanting (stealth-like) architecture. Students must mention the effect of blast on glazing, and the various methods available for reducing the damage to occupants from this, especially multilayer PVB.

Assessor's comment:

An essay question on wind-engineering and blast that was answered pleasingly well by most.