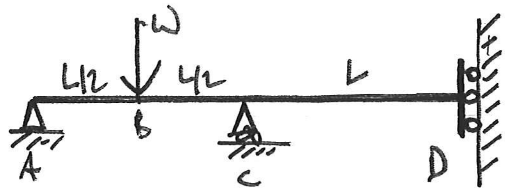
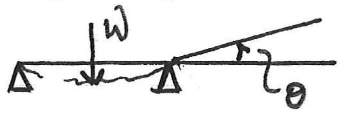
DR KA SEFFEN

Qu 1 4D10/2010.

a) Moment @ D is the redundancy: using deflection coefficients from structures databook

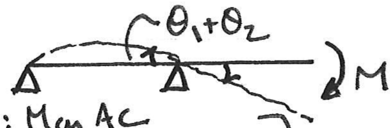


Remove M



$$\theta = \frac{WL^2}{16EI}$$

Restore M



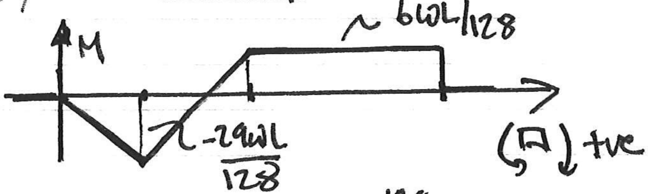
θ_1: M on AC
θ_2: M on CD if AC rigid } superpose.

$$\theta_1 + \theta_2 = \frac{4}{3}\frac{ML}{EI}$$

By compatibility: $\theta = \theta_1 + \theta_2 \Rightarrow \frac{WL}{16} = \frac{4}{3}M \Rightarrow \underline{\underline{M = \frac{3WL}{64}}}$

$\Rightarrow$ reactions as $\underline{\underline{R_A = 29W/64,\ R_C = 35W/64}}$

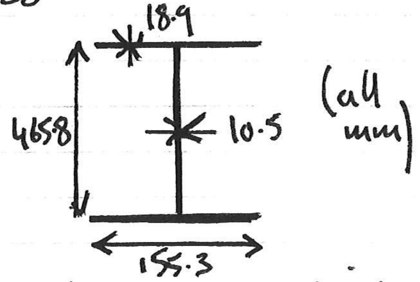
Bending moment diagram.



b) $457 \times 152 \times 82$ kg/m : struct. databk $\Rightarrow$

$\underline{I_{yy} = 1185\text{ cm}^4,\ J = 89.2\text{ cm}^4,\ Z_p = 1811\text{ cm}^3}$

$G = 81$ GPa, $E = 205$ GPa, $\sigma_y = 275$ MPa



Total span = 20m $\Rightarrow$ $\underline{L = 10m}$: buckle by DS2, two critical spans, 5m (AB, BC) and 10m (CD)

	L=5m	L=10m
M_1 (kNm)	263.2	131.6
M_2 (kNm)	214.3	53.6
M_E (kNm)	339.4	142.1
$M_p = Z_p\sigma_y$ (kNm)	498.0	498.0
$\lambda_{LT} = 75\sqrt{M_p/M_E}$	90.8	140.4
$\bar{M}_c$ (DS2)	~0.53	~0.25

$M_E = \sqrt{M_1^2 + M_2^2}$

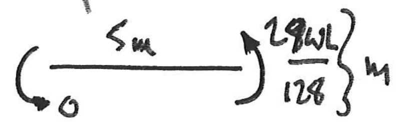
$M_1 = \frac{\pi}{L}\sqrt{GJEI_{yy}}$

$M_2 = \frac{\pi^2}{L^2} ED \frac{I_{yy}}{2}$

$D = 465.8 - 18.9$
$= 446.9$ mm

Qu 1 | 4D10

AB span



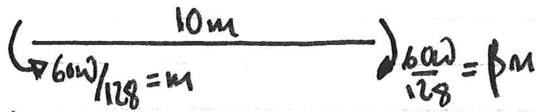
$L = 10\,\text{m}$ (not 5!) $\Rightarrow m = \frac{290W}{128}$, $\beta = 0$ by DS2.

$M_u = 0.6m$ (DS2) $\Rightarrow M_u = \frac{174}{128}\cdot W$

Strength: $m \leq M_p \Rightarrow 290/128 \cdot W \leq M_p \Rightarrow \underline{W \leq 220\,\text{kN}}$

Stability: $M_u \leq M_c \Rightarrow 174/128 \cdot W \leq \underbrace{0.53M_p}_{M_c \text{ for } 5\text{m span}} \Rightarrow \underline{W \leq 194\,\text{kN}}$ *

CD span.

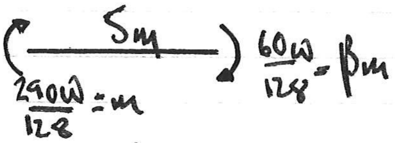


$\beta = 1 \Rightarrow M_u = m$ by DS2

Strength: $m \leq M_p \Rightarrow 60W/128 \leq M_p \Rightarrow \underline{W \leq 1062\,\text{kN}}$

stability: $M_u \leq M_c \Rightarrow 60W/128 \leq \underbrace{0.25M_p}_{M_c \text{ for } 10\text{m span}} \Rightarrow \underline{W \leq 266\,\text{kN}}$

BC span.



$-60W/128 = \beta \cdot \frac{290W}{128} \Rightarrow \beta = -\frac{6}{29} \Rightarrow M_u = [0.6 + 0.4\beta]m$

$\Rightarrow M_u = 0.516\,m$

Strength: $m \leq M_p \Rightarrow 290W/128 \leq M_p \Rightarrow \underline{W \leq 220\,\text{kN}}$

Stability: $M_u \leq M_c \Rightarrow 0.51 \cdot 290W/128 \leq 0.53M_p \Rightarrow \underline{W \leq 228\,\text{kN}}$

*Lowest $\Rightarrow$ $\underline{\underline{W_{max} = 194\,\text{kN}: \text{stability in AB governs}}}$.

Shear check

No S.F. in CD due to uniform B.M: the greatest change in B.M. is in BC where

$$S.F, V = \left[(29+6)\frac{W \times 10}{128}\right]/5 \simeq 106\,\text{kN}$$

$$V_c = A_{web} \times \underbrace{\tau_y}_{\sigma_y/\sqrt{3}} = \underbrace{0.465 \times 0.0105}_{A_{web}} \times \underbrace{275 \times 10^6}_{\sigma_y}/\sqrt{3} = 452\,\text{kN}$$

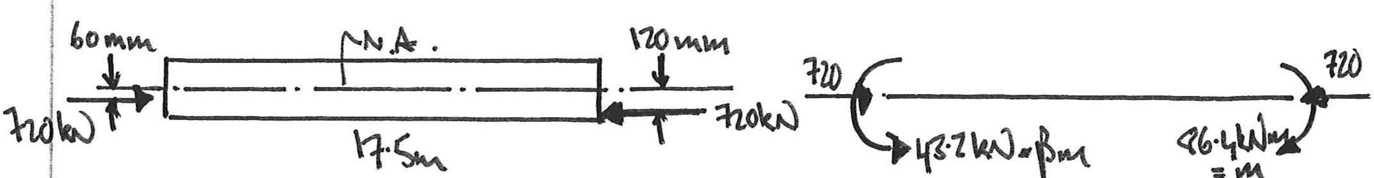
Thus $V \ll V_c/2$ [condition for using M_p in analysis].

Common mistakes: not checking all three spans AB, BC + CD. Algebraic errors for β: wrong B.M. diagram.

Assessor's comment Q1:

Most candidates were able to obtain the bending moment profile along the beam under the indicate loads and support conditions. The beam had three susceptible spans, each tackled by determining the equivalent uniform moment applied to the ends of the same length of span. Surprisingly, many candidates considered only two out of three spans: all three spans had different limitations, and thus, all had to be checked. Very few solutions verified that the shear force was not excessive (so that the appropriate datasheet formulae could be used).

Qu2 4D10/2010



Ratio of end moments ⇒ $\beta = 1/2$: ⇒ $M_u = (0.6 + 0.4 \times 1/2)m = 0.8m$

Beam is 254×254×107 kg/m UC, S275 : bending about major axis
[eccentricity of load lies on minor axis]

$I_{xx} = 17510\,cm^4$, $A = 136\,cm^2$, $r = 113.47\,mm$: $y = h/2 = 266.7/2 = 133.4$

a) CDC method : find σ_c, equivalent buckling load of strut w/o end moments

$r/y = 113.47/(133.4) = 0.85 \geq 0.7$ (DS1) ⇒ use curve A

$\sigma_c \cdot A = 720$ (applied, kN) ⇒ $\sigma_c = 720 \times 10^6 / 136 \times 10^{-4} = 52.9\,MPa$

⇒ $\bar{\sigma}_c = \sigma_c / \sigma_y = 52.9/275 = 0.193$: curve A, DS1 gives

$\lambda \simeq 160$ of $\bar{\sigma}_c = 0.193$

$\lambda = \frac{L_c}{r}\sqrt{\frac{275}{355}} = 160$ ⇒ $L_c \simeq 20.63\,m$ ⇒ $L/L_c = \frac{17.5}{20.63} = 0.85$ (equiv. beam; actual)

From DS3, choose UB major axis bending chart, $\beta = 1/2$

⇒ M_c/M_p' (reduced M_p) $= 0.25$ (to accuracy of m'ment)

$M_p = Z_p \cdot \sigma_y = 275 \times 10^6 \times [\underbrace{1484 \times 10^{-6}}_{Z_p,\ SWB}] = 408.1\,kNm$

Assume that the compressive core is confined to the web alone (size $b \times d$) ⇒ $M_p' = M_p - \frac{bd^2}{4}\sigma_y$

$b \cdot d \cdot \sigma_y = 720\,kN$ (axial capacity of core) ⇒ $\underbrace{12.8 \times 10^{-3}}_{\text{web thick}} \times d \times 275 \times 10^6 = 720 \times 10^3$

⇒ $d = 204.5\,mm$

This is just inside the web ⇒ $M_p' = 371\,kNm$

$M_c = 0.25 \times M_p' = 92\,kNm$: this is larger than the largest applied moment of 86.4 kNm, so beam-column safe by CDC method.

Qu 2 4M10/2010

b) By interaction equation.

(i) local strength check: $\frac{P}{P_p} + \frac{M_{max}}{M_p} \leq 1$ $P_p = A \cdot \sigma_y$ = squash load

$$\Rightarrow \frac{720\times10^3}{136\times10^{-4}\times275\times10^6} + \frac{86.4\times10^3}{408.1\times10^3} = \underline{0.4} \leq 1 \Rightarrow \underline{\underline{OK}}$$

(ii) stability: $\frac{P}{P_c} + \frac{M_u}{M_c} \leq 1$: N.B. $M_c = M_p$ when no LTB : as question restricts minor axis bending $\Rightarrow$ no LTB.

From before $M_u = 0.8\,\overset{\text{largest moment}}{m}$, $\beta = \frac{1}{2}$. For P_c; $\lambda = \frac{L_E}{r}\sqrt{\frac{275}{355}}$

$$\Rightarrow \lambda = \frac{17500}{113.4}\sqrt{\frac{275}{355}} = \underline{\underline{136}} : \text{curve A } \left(\frac{\sigma}{\sigma_y} = 0.85\right), \text{ BS1} \Rightarrow \underline{\underline{\bar{\sigma}_c = 0.3}}$$

$$\Rightarrow P_c = 0.3 \times 275\times10^6 \times A$$

$$\Rightarrow \frac{720}{0.3\times275\times10^6\times136\times10^{-4}} + \frac{0.8\times86.4\times10^3}{\underbrace{408.1\times10^3}_{M_p}} = \underline{\underline{0.81 \leq 1}}$$

So OK by <u>I.E. method also</u>

S.F. in beam = $(86.4-43.2)/17.5 = \underline{\underline{2.47\text{kN}}} = V$

Shear capacity $= V_c = A_{web} \times \underbrace{\tau_y}_{\sigma_y/\sqrt{3}} = \underbrace{0.0128 \times \frac{266.7}{1000}}_{A_{web}} \times \frac{275\times10^6}{\sqrt{3}}$

$\Rightarrow V_c \simeq 542\text{kN}$

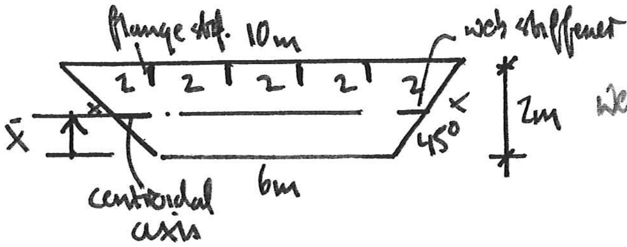
Thus $V \ll V_c/2$; <u>shear OK also</u>

Common mistakes: using M_c via LTB when not required for I.E. approach; not calculating M_u properly; misreading BS3

Assessor's comment:

Most solutions were of a high standard of correctness and completeness. Some candidates did not ignore lateral torsional buckling, as instructed, when considering the performance according to the interaction equation approach, resulting in extra and unnecessary working. Some also failed to be sufficiently accurate in reading properties from the appropriate curves in the datasheet.

Qu3 4M10/2010



All stiffeners 100×10 mm (top flange, webs only): vertical cross frames every 2m. All other plates are 20 mm thick.

Smeared thickness of top flange: $10\times0.02 + \overset{\text{\# stiff.}}{4}\times0.1\times0.01 = 10h$

$\Rightarrow h = 20.4\,\text{mm}$

Inclined web: $2\sqrt{2}\times0.02 + 1\times0.1\times0.01 = 2\sqrt{2}h \Rightarrow h = 20.35\,\text{mm}$

c.g. ht above btm flange: $\left(6\times0.02 + \underbrace{2\sqrt{2}\times0.02035\times2}_{\text{webs}} + \underbrace{10\times0.0204}_{\text{top}}\right)\times\bar{X}$

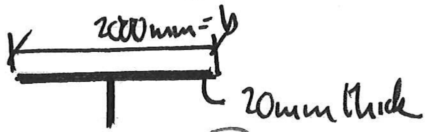
$= 6\times0.02\times0.01 + 2\sqrt{2}\times0.02035\times2\times1 + 10\times0.0204\times2$

$\Rightarrow \bar{X} = 1.194\,\text{m}$

$I_{xx} = (10\times0.0204)\times(2-1.194)^2 + 2\times\left[2^3\times\dfrac{\overbrace{(0.02035\times\sqrt{2})}^{\text{equiv thickness}}}{12} + 2\times0.02035\sqrt{2}\times(1.194-1)^2\right]$
$+ 6\times0.02\times1.194^2$

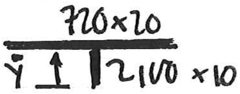
$\Rightarrow I_{xx} = 0.3459\,\text{m}^4 \quad [\text{area} = 0.4391\,\text{m}^2]$.

a) Adequacy of the top flange.



λ (compactness) $\Rightarrow \dfrac{b}{t}\sqrt{\dfrac{275}{355}} = \dfrac{2000}{20}\sqrt{\dfrac{275}{355}} = 88 \Rightarrow K_c = 0.36$ via BS3

Effective section has $b_e = K_c\cdot b$ as width $= 720\,\text{mm}$



Position of N.A. in $\bar{Y}$ from bottom $\Rightarrow$

$(720\times20 + 100\times10)\bar{Y} = (100\times10)\times50 + (720\times20)\times110$

$\Rightarrow \bar{Y} = 106.1\,\text{mm}$.

$I = 720\times20^3/12 + (720\times20)\times(10-6.1)^2 + \dfrac{100\times10^3}{12} + (100\times10)\times56.1^2$

$\Rightarrow I = 3.855\times10^6\,\text{mm}^4$

$r = \underbrace{\sqrt{\dfrac{I}{A}}}_{\text{T-strut}} = \sqrt{\dfrac{3855\times10^3}{720\times20+1000}} = 15.82\,\text{mm} \Rightarrow \dfrac{r}{y} = \dfrac{15.82\,\text{mm}}{106.1\,\text{mm}} = 0.1477$

P.T.O.

Qu3 4/10/2010

$\Rightarrow r/y \leq 0.45 \Rightarrow$ use curve D in DS1

$\lambda = \frac{L}{r}\sqrt{\frac{\sigma_y}{355}}$: L = crossframe separation = 2m

$\Rightarrow \lambda = \frac{2000}{15.82}\sqrt{\frac{275}{355}} = 111.2 \Rightarrow \frac{\sigma_c}{\sigma_y} \simeq 0.33 \Rightarrow \sigma_c \simeq 91\,\text{MPa}$

$P_{crit} = \sigma_c \times A_{T\text{-}STRUT} = 91\times10^6 \times (720\times20 + 1000)\times10^{-6} = \underline{\underline{1.401\,\text{MN}}}$

[Well with capacity of applied loads].

b). Adequacy of most heavily stressed panel.

For each web, bending stresses differ top and bottom:

$\sigma_{TOP} = M.y_{top}/I_{xx} = (50\times10^6)\times\frac{(2-1.194)}{0.3459} = 116.51\,\text{MPa}$

$\sigma_{BTM} = M\,y_{BTM}/I_{xx} = (50\times10^6)\times\frac{1.194}{0.3459} = 172.60\,\text{MPa}$

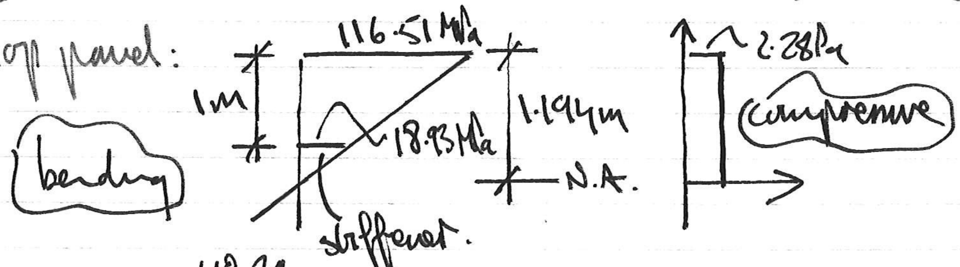
all below strength limit (check)

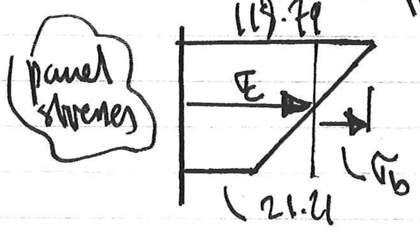
Compressive stress due to axial load: $1\times10^6/0.4391 = 2.28\,\text{MPa}$ (almost negligible)

For each panel $\lambda = \frac{b}{t}\sqrt{\frac{275}{355}}$ actual length $= \sqrt{2}$m ; $t = 0.02$m

$\Rightarrow \lambda = \frac{\sqrt{2}}{0.02}\sqrt{\frac{275}{355}} = 62.2$: not compact $\Rightarrow \underbrace{K_c = 0.5,\ K_b = 1.13}_{\text{via DS4}}$

Top panel:





$\sigma_c = 70.0\,\text{MPa}$, $\sigma_b = 48.78\,\text{MPa}$

$$\frac{\sigma_c}{\underbrace{\sigma_{cc}}_{K_c\sigma_y}} + \left(\frac{\sigma_b}{\underbrace{\sigma_{bc}}_{K_b\cdot\sigma_y}}\right)^2 + \left(\frac{\tau}{\tau_y}\right)^2 \leq 1$$

$\tau = 0\,\text{MPa}$ (max b.m.) $\Rightarrow \frac{70.0}{0.5\times275} + \left(\frac{48.78}{1.13\times275}\right)^2 = 0.533 \leq 1$

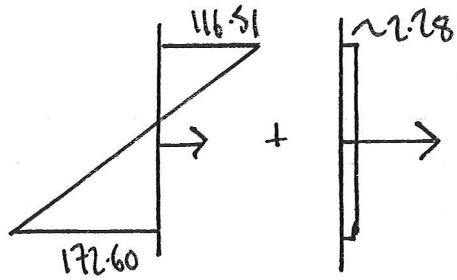
$\underline{\underline{\text{OK}}}$

Qu3 4D10/2010

Btm panel is in tension, largely; not liable to buckle.
∴ all panels OK.

c) If central web stiffener removed, entire web susceptible to buckling.

$\Rightarrow \lambda = 124.4 \Rightarrow K_c = 0.27, \; K_b = 0.92$



$\sigma_c \simeq -25.77\,MPa$ (in tension; set to zero)

$\sigma_b \simeq 144.56\,MPa$

$$\Rightarrow \left(\frac{\sigma_b}{K_b \cdot \sigma_y}\right)^2 = \left(\frac{144.56}{0.92 \times 275}\right)^2 = 0.33 \leq 1$$

Yes it is possible to remove the central stiffener: the increase in bending stress amplitude is offset by the decrease in compressive amplitude.

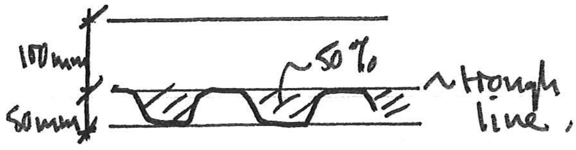
Comments: not a very popular question: solutions varied from a few poor efforts to exemplary solutions, and little in between.

Assessor's comment:

Not a popular question at all. Most solutions were partially completed, and no-one returned a full solution attempt owing to running out of time.

Qu 4 4D10/2010

a) Ribs running orthogonal to the floor: assume 50% area below rib line



Effective width of span = smaller of $\frac{14}{4}$ (span) or 3 (beam spacing) : $\underline{3m}$

Loads: slab self-weight $= 24$ (conc, kN/m³) $\times \left[3\times 0.1 + 0.05\times 3\times \tfrac{1}{2} \text{ (50\%)}\right] = 9\,kN/m$

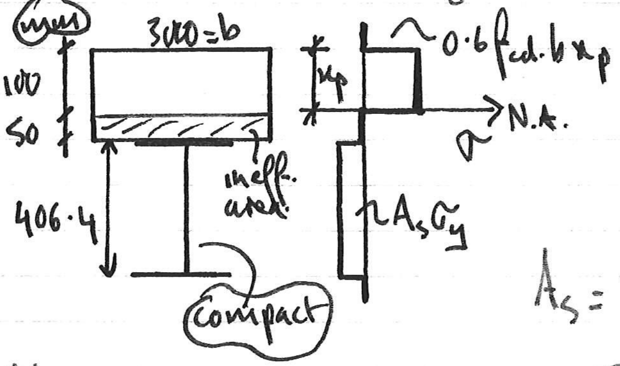
services $= 1\,kN/m^2 \times 3 = 3\,kN/m$

406×178×60 UB (S355) $= 60\,kg/m \times 9.81 = 0.59\,kN/m$

$12.59\,kN/m$

Total load $= \underbrace{12.59}_{\text{dead}} \times \underbrace{1.4}_{\text{give}} + (\underbrace{5.5\,kN/m^2}_{\text{give / live}} \times 3) \times \underbrace{1.6}_{\text{give}} = \underline{44.03\,kN/m = w}$

Design moment $= \frac{wL^2}{8}$ (span) $= \frac{44.03\times 14^2}{8} = \underline{1080\,kNm}$



Assume N.A. lies in the slab itself: axial eqm gives

$0.6 f_{cd}$ (~30MPa) $\cdot b \cdot x_p = A_s \sigma_y$

$A_s = 76.5\,cm^2 \Rightarrow \underline{x_p = 50.6\,mm}$ (in slab, < 100, OK)

M_d = design moment of floor $= A_s\sigma_y\left[\frac{D}{2} + h_c - \frac{x_p}{2}\right]$ from lectures.

$\Rightarrow M_d = 76.5\times 10^{-4} \times 355\times 10^6 \times \left[\frac{0.4064}{2} + \underbrace{0.15}_{\text{total depth}} - 0.0506\right]$

$\Rightarrow \underline{M_d = 890.1\,kNm}$.

i.e. structure does NOT meet requirements: have to have a larger UB or deeper slab, e.g. using 406×178×74 UB's gives $x_p = 62.1\,mm$, $M_d = 1091.4\,kNm$, which suffices.

In the following, let's use 406×178×60 UB's as per question; equivalent solutions for a 406×178×74 UB will also be given in brackets.

P.T.O.

Qu 4 4/10/2010

b) # of studs $\geq 2 \times A_s \sigma_y / P_d$: P_d = stud design moment = 47kN for f_{cu} = 30MPa

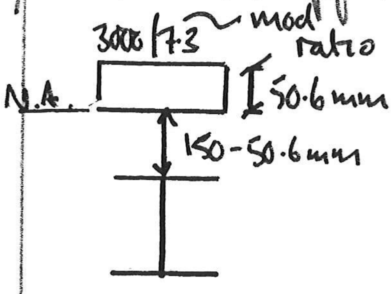
$\Rightarrow \# \geq 2 \times 76.5\times10^{-4} \times 355\times10^{6} / 47\times10^{3}$

$\Rightarrow$ **# studs = 116** (142 for 406×178×74)

Spacing = 14m/116 = 120 mm (< ϕ13, OK) [for heavier beam, spacing is 97mm, OK]

[Can also use studs in pairs with combined strength $1.6 P_d$/pair].

c) Short term application of load: use a transformed section.



$I_{xx}|_{UB} = 21508\ cm^4$

$$I_{xx}|_{transformed} = \frac{3000}{7.3} \times \frac{50.6^3}{12} + \overbrace{\left[\frac{3000}{7.3}\times 50.6\right] \times \left[150 - \frac{50.6}{2}\right]^2}^{\parallel \text{ axis}} + 21508\times10^4 + \underbrace{(76.5\times10^{-4})}_{A_s} \times \left[\frac{406.4}{2} + 150 - 50.6\right]^2$$

$\Rightarrow I_{xx}|_{transformed} = 0.84\times10^9\ mm^4$ ($1.46\times10^9\ mm^4$ for heavier beam)

Imposed loading w/o factors = 5.5×3 = 16.5 kN/m = w

$$\Rightarrow \delta = \frac{5wL^4}{384EI_{xx}} \text{ s/s} = \frac{5\times(16.5\times10^3)\times14^4}{384\times(205\times10^9)\times0.84\times10^{-3}} = \underline{47.8mm}$$

Allowable deflection = 14m/250 = <u>56mm</u> ∴ OK

[heavier 406×178×74 returns δ = 28mm].

Common mistakes: very few. The question was not set to deliberately catch out the students with a failing design: this was an accidental mistake. Efforts were rewarded fairly in terms of recognising the design methodology.

Most candidates found that the proposed floor design was inadequate, despite the expectation of the contrary. The rest of the question was answered well with relatively few problems.

ENGINEERING TRIPOS PART IIB 2010

4D10 STRUCTURAL STEELWORK

1a) Reactions Ra = 29W/64, Ra =35W/64: bending moment salient values Ma = 0, Mb = -29WL/128, Mc = Md = 6WL/128.

1b) AB span, W = 220 kN (strength), W = 194 kN (stability); BC span, W = 220 kN (strength), W = 228 kN (stability); CD span, W = 1062 kN (strength), W = 266 kN (stability). AB span therefore critical and governed by stability.

2a) Lc = 20.63m, reduced plastic moment = 371 kNm, critical moment = 92kN, max applied moment = 86.4 kNm; therefore safe by CDC method.

2b) Safe by Interaction Equation approach: margin on strength = 0.6; margin on stability = 0.19 (relative to limit of unity).

3) Ixx = 0.346 m^4, A = 0.439 m^2 (overall cross-section).

3a) Critical buckling load of compressive flange = 1.401 MN; safe.

3b) Most heavily stressed panel safe by stability formula (margin = 0.47 relative to unity).

3c) It is possible to remove central stiffener – the margin of safety increases to 0.57.

4a) Md= 890.1 kNm, applied moment – 1080 kNm, therefore proposed floor design not safe.

4b) 116 studs, spacing 120.7 mm.

4c) deflection = 47.8 mm, which is less than span/250.

K.A.S. May 2010