

Q1

(a)

One-group diffusion theory assumes that all neutrons are at a single constant energy. This is, of course, not the case in practice, even in a fast reactor.

Diffusion theory also assumes the reactor is homogeneous, whereas in reality the various core components (fuel elements, cladding, coolant, moderator, control rods etc.) have very different neutronic properties.

The neutron flux  $\phi$  is defined as  $\phi = nv$  where  $n$  is the neutron density (number per unit volume) and  $v$  is the average neutron speed.

By Fick's law of diffusion, the neutron current density is given by  $\underline{j} = -D\nabla\phi$  where  $D$  is the diffusion coefficient. [15%]

(b)

Using the 4M16 data sheet (p6), in cylindrical symmetry the one-group, steady-state, source-free neutron diffusion equation becomes

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) + \frac{\partial^2 \phi}{\partial z^2} + B_m^2 \phi = 0$$

where the material buckling  $B_m^2 = \frac{(\eta - 1)\Sigma_a}{D}$ .

Assuming that  $\phi(r, z) = F(r)Z(z)$  and substituting

$$\begin{aligned} \frac{Z}{r} \frac{\partial}{\partial r} \left( r \frac{\partial F}{\partial r} \right) + F \frac{\partial^2 Z}{\partial z^2} + B_m^2 FZ &= 0 \\ \therefore \frac{1}{Fr} \frac{\partial}{\partial r} \left( r \frac{\partial F}{\partial r} \right) + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + B_m^2 &= 0 \end{aligned}$$

This implies that

$$\frac{1}{Z} \frac{d^2 Z}{dz^2} + \gamma^2 = 0 \quad (1)$$

and

$$\frac{1}{Fr} \frac{d}{dr} \left( r \frac{dF}{dr} \right) + \alpha^2 = 0 \quad (2)$$

with

$$\alpha^2 + \gamma^2 = B_m^2 \quad (3)$$

Equation (1) is an SHM equation. Invoking symmetry its solution is

$$Z = C \cos(\gamma z)$$

Using the boundary condition that  $\phi = 0$  at  $z = \pm \frac{H}{2}$ , i.e. assuming extrapolation distances are negligible:

$$\gamma = \frac{\pi}{H} \Rightarrow Z = C \cos\left(\frac{\pi z}{H}\right)$$

Equation (2) is Bessel's equation of 0<sup>th</sup> order. From the 4M16 data sheet its general solution is

$$F(r) = A_1 J_0(\alpha r) + A_2 Y_0(\alpha r)$$

$$Y_0(0) = -\infty, \text{ so for physical reality } F(r) = A_1 J_0(\alpha r)$$

Using the boundary condition that  $\phi = 0$  at  $r = R$ , i.e. assuming extrapolation distances are negligible again, and the fact that  $J_0(2.405) = 0$

$$\alpha = \frac{2.405}{R} \Rightarrow F(r) = A_1 J_0\left(\frac{2.405r}{R}\right)$$

Thus, the general solution is

$$\phi(r, z) = \phi_0 J_0\left(\frac{2.405r}{R}\right) \cos\left(\frac{\pi z}{H}\right)$$

where  $\phi_0$  is the flux at the centre of the reactor, with

$$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = B_m^2$$

from equation (3).

[60%]

(c)

The radial form factor is defined by

$$F_R = \frac{\text{the flux at the centre of the core}}{\text{the average flux on the central plane at } z = 0}$$

$$\therefore F_R = \frac{\phi_0}{\frac{1}{\pi R^2} \int_0^R \phi_0 J_0\left(\frac{2.405r}{R}\right) 2\pi r dr}$$

$$\therefore F_R = \frac{R^2}{2 \int_0^R J_0\left(\frac{2.405r}{R}\right) r dr} = \frac{2.405}{2 J_1(2.405)} = \frac{2.405}{2 \times 0.5183} = 2.320$$

using the results for  $\int_0^R r J_0(\alpha r) dr$  and  $J_1(2.405)$  on the 4M16 data sheet.

[15%]

(d)

In practice form factors can be improved by surrounding the core by a *reflector*, a material with a high scattering cross-section, so that neutrons leaking from the core have a reasonable probability of being scattered back in. The presence of a reflector flattens the flux profile.

The flux profile can also be improved by distributing the fuel so that the fresher, more reactive fuel is towards the edge of the core and older, less reactive fuel is towards the middle, and/or by use of control materials (control rods or burnable poisons) to suppress flux peaks.

[10%]

### Assessor's comment:

Question 1: A very popular question on neutron diffusion theory and its application to simple reactor analysis. This was well answered in general with no systemic failings apparent in the answers.

Q2

(a)

Neglecting the delay associated with the decay of Th-233 (as instructed in the question) the production rate of the Pa-233 will equal the rate at which Th-232 captures neutrons.

The reaction rate will be  $R = N\sigma_{cT}\phi$

Number density of Th-232 atoms:

$$N = \frac{\rho}{\text{Au}} = \frac{11724}{232.04 \times 1.661 \times 10^{-27}} = 3.042 \times 10^{28} \text{ m}^{-3}$$

$$\therefore R = 3.042 \times 10^{28} \times 0.309 \times 10^{-28} \times 10^{15} = 0.940 \times 10^{15} \text{ m}^{-3}\text{s}^{-1} \quad [15\%]$$

(b)

$T\sigma_{cT}\phi$  is the rate at which Th-232 is transmuted into Pa-233 through neutron capture (and assumed instantaneous decay);

$\lambda_P P$  is the rate at which Pa-233 decays into U-233;

$P\sigma_{cP}\phi$  is the rate at which Pa-233 is transmuted into Pa-234 through neutron capture. [10%]

(c)

Although these equations describe an unfamiliar scenario they are identical in form to the equations that govern the variation of the I-135 and Xe-135 populations after a shutdown.

By inspection  $\frac{dT}{dt} = -T\sigma_{cT}\phi$  has a solution

$$T = T_0 \exp(-\sigma_{cT}\phi t) \quad (1)$$

Thus, the Th-232 population decays away from its initial value of  $T_0$ .

$\frac{dP}{dt} = T\sigma_{cT}\phi - \lambda_P P - P\sigma_{cP}\phi$  can be rewritten as

$$\frac{dP}{dt} + (\lambda_P + \sigma_{cP}\phi)P = T\sigma_{cT}\phi \quad (2)$$

Noting the form of the solution for  $T$  above, the particular integral will be of the form

$$P_{PI} = A \exp(-\sigma_{cT}\phi t)$$

Thus, substituting in equation (2) and using equation (1)

$$-A\sigma_{cT}\phi + (\lambda_P + \sigma_{cP}\phi)A = T_0\sigma_{cT}\phi$$

$$\therefore A = \frac{T_0\sigma_{cT}\phi}{\lambda_P + (\sigma_{cP} - \sigma_{cT})\phi}$$

The complementary function is the solution of the homogeneous form of equation (2)

$$\frac{dP}{dt} + (\lambda_P + \sigma_{cP}\phi)P = 0$$

which, by inspection, has a solution of the form

$$P_{CF} = B \exp(-[\lambda_P + \sigma_{cP}\phi]t)$$

Thus, the general solution for  $P$  is

$$P = P_{CF} + P_{PI} = B \exp(-[\lambda_P + \sigma_{cP}\phi]t) + \frac{T_0 \sigma_{cT} \phi}{\lambda_P + (\sigma_{cP} - \sigma_{cT})\phi} \exp(-\sigma_{cT}\phi t)$$

If  $P = 0$  at  $t = 0$  then

$$B = -\frac{T_0 \sigma_{cT} \phi}{\lambda_P + (\sigma_{cP} - \sigma_{cT})\phi}$$

Thus 
$$P = \frac{T_0 \sigma_{cT} \phi}{\lambda_P + (\sigma_{cP} - \sigma_{cT})\phi} \left[ \exp(-\sigma_{cT}\phi t) - \exp(-[\lambda_P + \sigma_{cP}\phi]t) \right] \quad [50\%]$$

(d)

The equation for  $P$  can be written as

$$P = Q \left[ \exp(-\lambda_1 t) - \exp(-\lambda_2 t) \right]$$

where  $Q = \frac{T_0 \sigma_{cT} \phi}{\lambda_P + (\sigma_{cP} - \sigma_{cT})\phi}$ ,  $\lambda_1 = \sigma_{cT}\phi$  and  $\lambda_2 = \lambda_P + \sigma_{cP}\phi$

Thus,  $P$  reaches its maximum when

$$\frac{dP}{dt} = Q \left[ -\lambda_1 \exp(-\lambda_1 t) + \lambda_2 \exp(-\lambda_2 t) \right] = 0$$

$$\therefore \lambda_1 \exp(-\lambda_1 t) = \lambda_2 \exp(-\lambda_2 t)$$

$$\therefore \ln(\lambda_1) - \lambda_1 t = \ln(\lambda_2) - \lambda_2 t$$

$$\therefore t = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$$

$$\lambda_1 = \sigma_{cT}\phi = 0.309 \times 10^{-28} \times 10^{15} = 3.09 \times 10^{-14} \text{ s}^{-1}$$

$$\lambda_P = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{27 \times 86400} = 2.971 \times 10^{-7} \text{ s}^{-1}$$

$$\therefore \lambda_2 = \lambda_P + \sigma_{cP}\phi = 2.971 \times 10^{-7} + 0.976 \times 10^{-28} \times 10^{15} = 2.971 \times 10^{-7} \text{ s}^{-1}$$

$$\therefore t = \frac{\ln(3.09 \times 10^{-14} / 2.971 \times 10^{-7})}{3.09 \times 10^{-14} - 2.971 \times 10^{-7}} = 54.1 \times 10^6 \text{ s} = 627 \text{ days} = 1.72 \text{ years} \quad [25\%]$$

## Assessor's comment:

Question 2: A popular question on the breeding of uranium-233 from thorium-232. This required the application of standard reactor kinetics methods but in an unfamiliar context. The assessor was pleased with how well the question was answered. A handful of candidates tried to bluff their way to a proof of the result quoted in part (c) but the assessor was not fooled.

Q3

(a)

CANDU reactors are refueled on-line. As the fuel is natural uranium the reactivity change in replacing a burnt assembly with a fresh one is comparatively small, and the reactor is naturally kept close to critical.

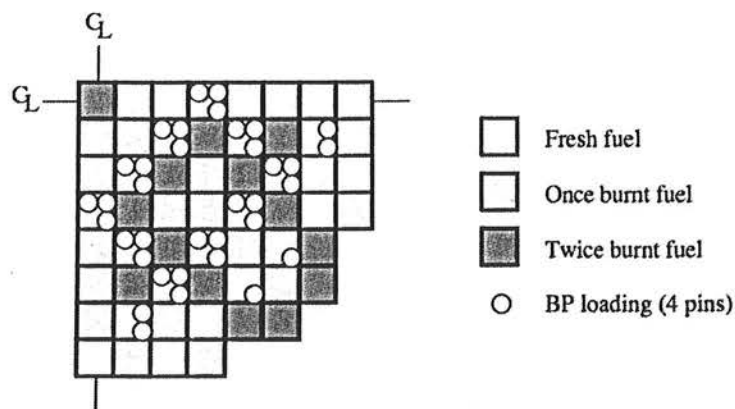
PWRs (typical fuel enrichment 3-4%) are refueled off-line at 12, 18 or even 24 month intervals. The procedure requires the reactor to be depressurised so that the top of the pressure vessel can be removed, allowing fuel assemblies to be removed or shuffled about. Refueling takes about two months. The more often a reactor is refueled, the less electricity it will generate per annum. Thus, some sort of compromise must be struck between the conflicting goals of achieving high fuel burnup and high reactor availability. In practice, most PWRs are operated using a 3- or 4-batch strategy.

For reactors, like CANDU, that are refueled on-line there is no real scope for in-core fuel management, but for reactors that are refueled off-line the design of the core loading pattern (LP) is an important issue. The purpose of in-core fuel management is to control local spatial effects resulting from the actual physical arrangement of the fuel. For instance, fresh fuel will tend to amplify the local flux distribution above the overall 'cosinusoidal' shape, while fuel with a large accumulated burnup will tend to depress it. Thus, if too many fresh fuel elements are clustered together, there is likely to be a local power peak (a hot spot).

The LP designer can have several (conflicting) objectives to meet:

- to limit the magnitude of any local power peaking,
- to limit the amount of radial neutron leakage from the core so as to reduce the irradiation damage to the pressure vessel,
- to maximise the average burnup accumulated by the fuel, or
- to maximise the overall reactivity of the core (which is, in fact, configuration dependent) so as to reduce the amount of fresh fuel required.

To strike an appropriate balance between these competing objectives some sort of 'checkerboarding' is usually used, and it is also common to load burnable poisons (BPs) with fresh fuel assemblies to overcome power peaking problems in otherwise desirable core designs. A typical PWR reload core using BPs is



[40%]

(b)

Assume the variation of reactivity with time/burnup can be adequately modeled using the *partial reactivity model* and the *linear reactivity model*. Also assume that the reactor operates at constant power.

Linear reactivity model:  $\rho(\tau) = \rho_0 \left[ 1 - \frac{\tau}{T_1} \right]$

From the information in the question  $T_1 = 18 \text{ MWd kg}^{-1}$  – the length of the first cycle.

Using the partial reactivity model, the end-of-cycle condition for the second cycle is

$$0 = \frac{2}{3} \rho_0 \left[ 1 - \frac{T_1 + \psi_2}{T_1} \right] + \frac{1}{3} \rho_0 \left[ 1 - \frac{\psi_2}{T_1} \right]$$

$$\therefore 0 = \frac{2}{3} \left[ -\frac{\psi_2}{T_1} \right] + \frac{1}{3} \left[ 1 - \frac{\psi_2}{T_1} \right]$$

$$\therefore \psi_2 = \frac{T_1}{3} = 6 \text{ MWd kg}^{-1}$$

Similarly, the end-of-cycle condition for the third cycle is

$$0 = \frac{1}{3} \rho_0 \left[ 1 - \frac{T_1 + \psi_2 + \psi_3}{T_1} \right] + \frac{1}{3} \rho_0 \left[ 1 - \frac{\psi_2 + \psi_3}{T_1} \right] + \frac{1}{3} \rho_0 \left[ 1 - \frac{\psi_3}{T_1} \right]$$

$$\therefore 0 = \frac{1}{3} \left[ -\frac{\psi_2 + \psi_3}{T_1} \right] + \frac{1}{3} \left[ 1 - \frac{\psi_2 + \psi_3}{T_1} \right] + \frac{1}{3} \left[ 1 - \frac{\psi_3}{T_1} \right]$$

$$\therefore \psi_3 = \frac{2}{3} T_1 - \frac{2}{3} \psi_2 = \frac{2}{3} [18 - 6] = 8 \text{ MWd kg}^{-1} \quad [35\%]$$

(c)

From the 4M16 data sheet  $T_M = \frac{2}{M+1} T_1$

Thus  $T_3 = \frac{2}{3+1} T_1 = \frac{1}{2} \times 18 = 9 \text{ MWd kg}^{-1}$

The discharge burnup  $B_M = M T_M \Rightarrow B_3 = 27 \text{ MWd kg}^{-1} \quad [10\%]$

(d)

If a fraction  $f$  of the fuel is replaced, the end-of-cycle condition for the second cycle becomes for a cycle length  $\psi_2 = T_3$

$$0 = (1-f) \rho_0 \left[ 1 - \frac{T_1 + T_3}{T_1} \right] + f \rho_0 \left[ 1 - \frac{T_3}{T_1} \right]$$

$$\therefore 0 = (1-f) \left[ -\frac{T_3}{T_1} \right] + f \left[ 1 - \frac{T_3}{T_1} \right]$$

$$\therefore f = \frac{T_3}{T_1} = \frac{9}{18} = 0.5 \quad [15\%]$$

### Assessor's comment:

Question 3: A question on in-core and out-of-core fuel management. The least popular question but nevertheless done by more than half the candidates. Part (a), which asked for discussion of the fuel management strategies for two different reactor types, was surprisingly poorly answered with many candidates getting basic details wrong or confused. The numerical parts of the question were well done by many.

Q4

(a)

*Advantages*

- Reprocessing recovers uranium and plutonium for re-use.
- Allows the separation of fission products for safe storage.
- There are to date no full-scale disposal facilities for the direct disposal of spent fuel.
- As supplies of gas and oil run out and environmental pressures increase the cost of power generation from coal, the price of uranium will rise.
- This will re-awaken interest in plutonium as a reactor fuel and make the recovery of waste uranium an economic proposition again.

*Disadvantages*

- When the decisions to build the large reprocessing plants at Sellafield and Cap la Hague (France) were made in the early 1980s it was predicted that the price of uranium would rise to \$100 per pound by 1990 but it never reached that level.
- Some environmentalists argue against reprocessing on the grounds that it produces waste and results in stockpiles of plutonium.
- The waste argument is not proven because reprocessing actually concentrates the highly active fission products and it is a well proven process.
- The plutonium produced from civil power reactors is largely made up of the 240 and 241 isotopes which cannot readily be made into weapons but can be used as a reactor fuel. [25%]

(b)

Reprocessing wastes are classified as:

High level wastes

Medium active wastes

Low level wastes

- High level wastes arise entirely at the primary separation stage and contain the bulk of the fission products and the higher actinides. They are very highly radioactive and heat generating and, because they contain actinides, have a long half-life. The normal treatment process is evaporation, decay storage and vitrification for long-term storage.
- Medium active wastes arise from a number of sources:
  - Spent fuel cladding
  - Liquid wastes from initial reprocessing stages
  - Waste solvents
  - Spent ion exchange and filter materials
- Liquid medium active wastes are evaporated and the concentrates added to the high level waste streams.
- The solids wastes are encapsulated in cement grout, the aqueous liquid wastes are evaporated, treated chemically then filtered, the filtrate is discharged to sea after monitoring and the filter cake is encapsulated in cement.
- The waste solvents are broken down chemically, concentrated and encapsulated. The spent ion exchange and filter materials are also encapsulated.
- All encapsulated material is placed in 500 litre stainless steel drums for ultimate storage in a waste repository.
- Low level wastes are both liquid and solid, much of this category is in fact non-active but it is easier to treat as low level active waste than to attempt to separate the active from the inactive.
- Much of the low level liquid waste can be discharged directly to sea.
- The low level liquid wastes that contain traces of actinides and other long-lived isotopes are chemically treated in a similar way to the medium active wastes, though without evaporation.



- The low level solid wastes, much of it general domestic waste from the reprocessing areas, are disposed of by landfill using steel containers placed in an engineered disposal facility.

The dissolution process also gives rise to off-gases. These are the fission gases krypton, xenon and iodine together with some  $C_{14}$  as ( $CO_2$ ). By the time the fuel is reprocessed there is in fact very little xenon left as most Xe isotopes have a very short half-life. The main constituent of the off-gases is non-active  $NO_x$ . The gases are passed through a caustic scrubber to remove the  $NO_x$  and the iodine, the scrubber liquid going to the low level waste treatment facility. The  $C_{14}$  is removed by scrubbing with barium salts and precipitated as barium carbonate. No attempt is made at the moment to remove krypton as it is only weakly active and the cost of removing it cannot be justified. [40%]

(c)

Ignoring daughter products

$$A = A_0 \exp(-\lambda t)$$

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

Isotope	Half-life (years)	$\lambda$ (years <sup>-1</sup> )	$A_0$ (Bq)	$A(100)$ (Bq)	$A(1000)$ (Bq)	$A(10000)$ (Bq)
Sr-90	29	0.02390	$2.5 \times 10^6$	229074.2	$1.0430 \times 10^{-4}$	0
Cs-137	30.2	0.02295	$2.5 \times 10^6$	251903.5	$2.6970 \times 10^{-4}$	0
Am-234	7370	$9.405 \times 10^{-5}$	$10^4$	9906.39	9102.37	3904.33
Totals			$5.01 \times 10^6$	490884.1	9102.37	3904.33

[20%]

(d)

The assumption re daughter products is questionable. If the daughter products are radioactive their neglect will result in an underestimate of the activity: if the daughter product half-lives are short the current activity will be underestimated; if long, the long-term activity will be underestimated. However, with the exception of Am-234, their neglect makes very little difference to the final answer. The decay chains for fission products, such as Sr-90 and Cs-137, are relatively short-lived, unlike those for actinides.

It can be seen from the results that, for the first 100 years, most of the activity comes from the fission products, which are strong gamma emitters and hence very good shielding is needed, but by 1000 years, most of the gamma activity has decayed leaving only the long-lived actinides, where the problem is to prevent them entering the water or food chain, hence the need for very good immobilisation and secure storage. [15%]

## Assessor's comment:

Question 4: A fairly popular question on fuel reprocessing and waste management. Again, the descriptive parts were disappointingly answered. Many candidates did not seem to use the mark scheme as a guide for how much to write. The (straightforward) calculation was well done by most, although a surprising number of candidates could not use the isotope half-lives correctly in calculating the reduction in activity with time.