ENGINEERING TRIPOS PART IIB

Wednesday 21 April 2010 2.30 to 4

Module 4A3

TURBOMACHINERY I

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment: Compressible Flow Data Book (38 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

cah05

1 (a) Sketch the velocity triangles for a repeating stage turbine with 50% reaction. Show that the ratio of the exit velocity from the stator V_2 to the rotor blade speed U is given by:

$$\frac{V_2}{U} = \sqrt{\phi^2 + \left(\frac{\psi + 1}{2}\right)^2}$$

where ϕ is the flow coefficient and ψ is the stage loading. [30%]

(b) The total-to-total efficiency for an axial turbine stage is given by the following relationship:

$$\eta_{TT} = 1 - \frac{0.04}{\psi}\left[\left(\frac{V_2}{U}\right)^2 + \left(\frac{W_3}{U}\right)^2\right]$$

where W_3 is the relative velocity at exit from the rotor. Using the result from part (a), show that a repeating stage turbine with 50% reaction and a flow coefficient of 0.5 has maximum efficiency when the stage loading is equal to $\sqrt{2}$. For this design, determine the total-to-total efficiency and the total-to-static efficiency of the stage. Also calculate the flow angles at inlet and exit from the turbine stator. [35%]

(c) The repeating stage design parameters in part (b) are used in a 4-stage air turbine. The turbine is to have a mass flow rate of 25 kg s^{-1} and a power output of 3.5 MW. The rotational speed is 3000 rpm and the density of the air at the inlet is 1.65 kg m^{-3}. Determine the mean radius of the turbine, the flow velocity at inlet and the height of the stator blades in the first stage. [20%]

(d) State the advantages of turbine designs with 50% stage reaction relative to designs with low reaction. [15%]

2 (a) The Lieblein Diffusion Factor for a compressor cascade is defined as:

$$DF = 1 - \frac{V_2}{V_1} + \frac{|V_{\theta 2} - V_{\theta 1}|}{2V_1}\frac{s}{c}$$

where V_1 is the inlet velocity to the cascade, V_2 is the exit velocity, $|V_{\theta 2} - V_{\theta 1}|$ is the change in tangential velocity across the cascade, s is the blade pitch and c is the blade chord. Explain what the Diffusion Factor represents and briefly describe how it is used in the preliminary design of compressors. [20%]

(b) A two-dimensional compressor cascade operates in air. The inlet metal angle of the blades is 55° and the exit metal angle is 37°. When the flow is at zero incidence with an inlet Mach number of 0.65, the exit Mach number is 0.44 and the stagnation pressure loss coefficient is given by:

$$Y_p = \frac{p_{01} - p_{02}}{p_{01} - p_1} = 0.038$$

Determine the exit flow angle and give two reasons why this is greater than the exit metal angle. [25%]

(c) Find the blade pitch-to-chord ratio needed such that $DF = 0.45$ when the cascade is at the operating point described in part (b). [20%]

(d) Assuming that the exit flow angle and loss remain constant, estimate the new value of DF when the incidence of the flow is increased to 5° while maintaining an inlet Mach number of 0.65. Use the same pitch-to-chord ratio as found in part (c). [20%]

(e) If the cascade throat width to pitch ratio o/s is 0.6, determine the incidence of the flow onto the blades at which the cascade will choke with an inlet Mach number of 0.65. It can be assumed that there is no loss upstream of the cascade throat. [15%]

Use $\gamma = 1.4$, $R = 287$ J kg^{-1} K^{-1} and $c_p = 1005$ J kg^{-1} K^{-1} for air throughout the question.

3 (a) Show that the following equations can be used to characterise the working line of a turbojet engine on the compressor performance map:

$$\frac{p_{03}}{p_{02}} = \left(1 + C_0 \frac{T_{04}}{T_{02}}\right)^{C_1}$$

$$\frac{\dot{m}\sqrt{c_p T_{02}}}{p_{02} A_2} = C_2 \sqrt{\frac{T_{02}}{T_{04}}} \times C_3 \left(1 + C_0 \frac{T_{04}}{T_{02}}\right)^{C_1}$$

and determine the parameters C_0, C_1, C_2 and C_3 in terms of η_p, c_p, γ, $\dot{m}$, A_2, A_4, A_N, T_{04} and p_{04}. The subscripts 2, 3 and 4 correspond to compressor inlet, compressor outlet and turbine inlet conditions, respectively. A_N is the propelling nozzle area and all other terms have their usual meanings. Assume that both the turbine and compressor polytropic efficiency are constant and equal to η_p. The values of c_p and γ can also be assumed to be constant throughout the engine. [40%]

(b) Sketch the typical form of a complete multi-stage compressor map and that for a single stage. Identify the zones of unstable operation, choked flow, and maximum efficiency. Draw a typical engine working line on the multi-stage compressor map and explain how the part (a) derivations can be used to determine this. [20%]

(c) Using the results from part (a) show how the working line moves towards the surge line as η_p decreases. Describe how adjusting A_N can be used to move the working line away from the surge line. [15%]

(d) If the engine throttle setting (T_{04}) suddenly increases, sketch the locus of the operating point on the compressor map and explain the physical basis of this trajectory. [15%]

(e) Why is the surge line so difficult to determine mathematically relative to the working line? [10%]

END OF PAPER

IIB Paper 4A3 2010 Turbomachinery I Numerical answers

1.
(b) 0.903, 0.800, 67.5°, -22.5°
(c) 0.501 m, 85.2 m/s, 61 mm

2.
(b) 41.3°
(c) 0.780
(d) 0.60
(e) -7.95°

3.
None