ENGINEERING TRIPOS PART IIB

Monday 26 April 2010 9 to 10.30

Module 4A8

ENVIRONMENTAL FLUID MECHANICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SRHB05

1 (a) Consider a momentum–dominated jet being issued into a still atmosphere.

(i) Describe what is meant by an 'entrainment velocity', v_e. Write down an expression that can be used to approximate v_e for a momentum jet. [15%]

(ii) Derive the fundamental equations for the development of a momentum jet from a point source, i.e.:

$$\frac{\mathrm{d}}{\mathrm{d}s}(R^2u) = 2\alpha uR \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}s}(R^2u^2) = 0$$

where s is the stream–wise distance from the source and α is the entrainment coefficient. [25%]

(iii) Assuming that $R \propto s^a$ and $u \propto s^b$, calculate a and b. [20%]

(b) Consider a passive cross–wind line source of pollution in a turbulent atmosphere. K–theory suggests that dispersion of an inert pollutant from such a source is governed by:

$$U\frac{\partial \chi}{\partial x} = \frac{\partial}{\partial z}\left(K_z\frac{\partial \chi}{\partial z}\right)$$

where χ is the pollutant concentration, z is a vertical coordinate, x is a horizontal coordinate and U is the horizontal wind speed (assumed constant with height).

(i) Assuming that $K_z = \kappa u_* z$, a solution is:

$$\chi = \frac{Q}{\kappa u_* x}\exp\left(-\frac{Uz}{\kappa u_* x}\right)$$

where Q is the pollutant emission rate per unit length at $(x,z) = (0,0)$. The upper edge of the plume, z_p, is defined to be the height at which the pollutant concentration is 10% of its value at ground level. Show that $z_p \simeq 2.3\kappa u_* x/U$.

[15%]

(ii) Hence define an effective entrainment velocity for the rate at which the plume entrains clean air.

[25%]

2 (a) Briefly discuss the vertical thermal structure of the atmosphere. Include a sketch of the temperature profile, names of different regions of the atmosphere, and reasons why temperature varies in the way that it does. Also indicate the effect of temperature gradients on the vertical transport of chemical species. [30%]

(b) Consider an adiabatic parcel of air of mass m.

(i) Show that the dry adiabatic lapse rate (DALR) is given by:

$$\mathrm{DALR} = -\frac{\mathrm{d}T}{\mathrm{d}z} = \frac{g}{c_p}$$

where the symbols have their usual meaning. [20%]

(ii) Now assume that the parcel contains a mass of water vapour m_w. The water mixing ratio in the parcel is $w_v = m_w/m$. The specific latent heat of evaporation of water, ΔH_v, can be taken as constant. Neglect the effect of the water vapour on the thermodynamic properties of the parcel. Derive an expression for the moist adiabatic lapse rate (MALR) as a function of $\mathrm{d}w_v/\mathrm{d}z$. [15%]

(iii) Hence show that:

$$\mathrm{MALR} = \frac{g}{c_p + \Delta H_v \frac{\mathrm{d}w_v}{\mathrm{d}T}}$$

Calculate a numerical value for the ratio:

$$\frac{\mathrm{MALR}}{\mathrm{DALR}}$$

You may assume that $\mathrm{d}w_v/\mathrm{d}T = 2.7 \times 10^{-4}\ \mathrm{K}^{-1}$ and $\Delta H_v = 2.5\ \mathrm{MJkg}^{-1}$. [15%]

(iv) Is a cloudy atmosphere inherently less stable than the corresponding dry atmosphere with the same lapse rate? Briefly justify your answer. [20%]

3 (a) Taking the solar constant as $F_S = 1370\ \mathrm{Wm}^{-2}$, the planetary albedo $A = 0.3$, and the Earth's radius $R_E = 6378$ km, show that the mean solar radiation flux absorbed per unit area on the Earth's surface is $F_S(1-A)/4$. [10%]

(b) Assuming the Earth is a black body, estimate the effective temperature of the Earth. Why is this estimate too low? [10%]

(c) Assume the atmosphere is a single isothermal layer that is transparent to solar radiation and absorbs a fraction f of the terrestrial radiation because of greenhouse gases. The temperature of the Earth's surface is T_0 and of the atmospheric layer is T_1. Show that:

$$T_0 = \left[\frac{F_S(1-A)}{4\sigma\left(1-\frac{f}{2}\right)}\right]^{1/4}$$

Assuming $f = 0.77$, calculate T_0 and T_1. Comment on these results. [40%]

(d) Now assume that the outgoing terrestrial flux is reduced by ΔF (i.e. the radiative forcing) due to the addition of mass Δm of a greenhouse gas. This corresponds to a change in the absorption efficiency Δf. For a small perturbation, ΔF, show that:

$$\Delta T_0 = \lambda \Delta F$$

where:

$$\lambda = \frac{1}{4\left(1-\frac{1}{2}f\right)\sigma T_0^3}$$

is the climate sensitivity parameter. [40%]

4 (a) A city administration is considering replacing its existing fleet of buses powered by diesel with a fleet powered by natural gas. These new natural gas buses result in lower CO_2 emissions, higher CH_4 emissions, lower $PM_{2.5}$ emissions and remove SO_2 emissions. Other emissions are expected to remain largely unchanged.

(i) Discuss the effects of such a measure on the air quality immediately adjacent to routes where buses run. [10%]

(ii) Discuss the effects of such a measure on regional air quality. [20%]

(iii) Discuss the effects of such a measure on climate forcing attributable to the bus network. [20%]

(b) A regional authority is experiencing frequent high ozone episodes. The authority responded by mandating a cut in NO_x emissions. However, this resulted in an increase in ozone concentrations. Describe, with an appropriate supporting diagram, why this might happen. Suggest what the authority should do to reduce ozone concentrations. [20%]

(c) The following simplified chemical system governs the steady–state ozone concentration in clean air:

$$\begin{aligned} NO_2 + h\nu &\rightarrow NO + O \\ O + O_2 + M &\rightarrow O_3 + M \\ O_3 + NO &\rightarrow NO_2 + O_2 \end{aligned}$$

which have rate constants j, k_2 and k_3, respectively. Write down an expression for $d[NO_2]/dt$. Hence derive an expression for steady–state ozone concentration. [15%]

(d) Discuss the role of OH relative to anthropogenic pollutants. [15%]

END OF PAPER

4A8 2010 – List of Numerical Answers

1. (a) (iii) $a = 1$, $b = -1$.
   (b) (ii) $v_e = 2.3\ \kappa\ u_*$
2. (b) (iii) MALR/DALR = 0.6.
3. (b) 255 K.
   (c) $T_0 = 288$ K. $T_1 = 241$ K.
4. N/A