ENGINEERING TRIPOS PART IIB

Friday 23 April 2010 9 to 10.30

Module 4A11

TURBOMACHINERY II

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) An axial compressor stator is to be designed for an annulus which has a reduction in radius from inlet to exit, see Fig. 1. The blade must turn the flow from an inlet angle α_1 ($\tan\alpha_1 = V_\theta / V_m$, V_θ and V_m being circumferential and meridional velocity components respectively) to zero swirl at exit. The flow is incompressible. Two designs are being considered: zero sweep, shown in Fig. 1(a) and swept Fig. 1(b).

(i) Taking the pressure distribution shown in Fig. 2 as the idealised case for the unswept blade, show, stating any assumptions made, that the lift factor, *L*, is given by:

$$L = \frac{\text{Actual } F_\theta}{\text{Idealised } F_\theta} = 4(s/c_m)_{\lambda=0} \tan\alpha_1 \cos^2\alpha_1$$

where F_θ is the tangential blade force per unit span and s/c_m is the pitch-to-chord ratio based on meridional chord. The sweep angle λ is defined in Fig 1(b). [30%]

(ii) On a sketch of Fig. 2, show how the idealised pressure distribution would change for the swept blade. [10%]

(iii) Derive a new lift factor for the swept blade. Show that, if the swept and unswept blades achieve the same value of lift factor, then:

$$\frac{(s/c_m)_\lambda}{(s/c_m)_{\lambda=0}} = \left(\cos^2\lambda + \tan^2\alpha_1\right)\cos^2\alpha_1$$ [30%]

(iv) If a sweep angle of $\lambda = 30°$ is chosen and $\alpha_1 = 20°$, estimate the fractional increase in profile loss that might be expected. State your reasoning. [10%]

(b) Within a multi-stage axial compressor, localised sweep is sometimes used at the leading edge close to the endwalls. Explain why this is done. [20%]

(Cont.

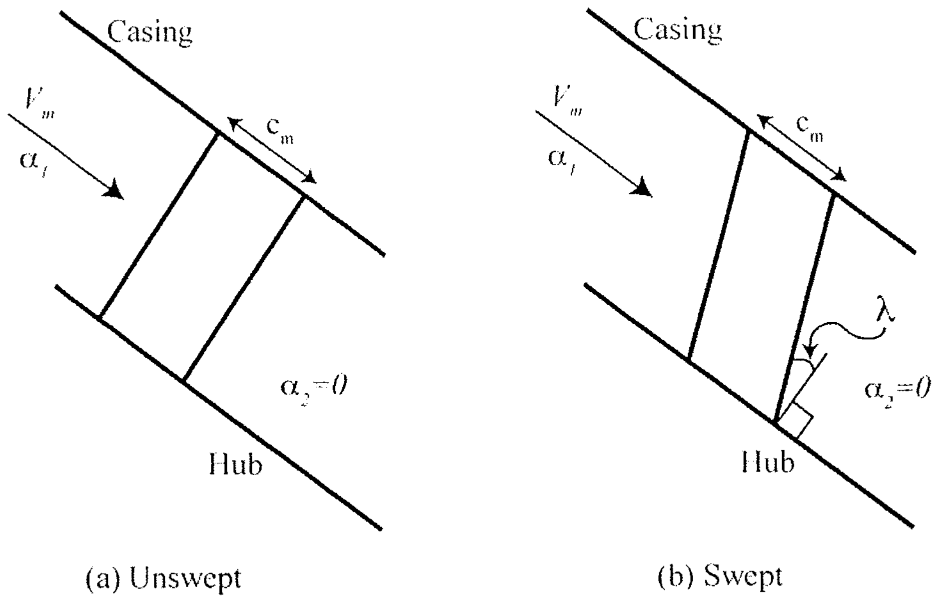
Casing
V_m
α_1
c_m
$\alpha_2=0$
Hub
(a) Unswept
Casing
V_m
α_1
c_m
λ
$\alpha_2=0$
Hub
(b) Swept

Fig. 1

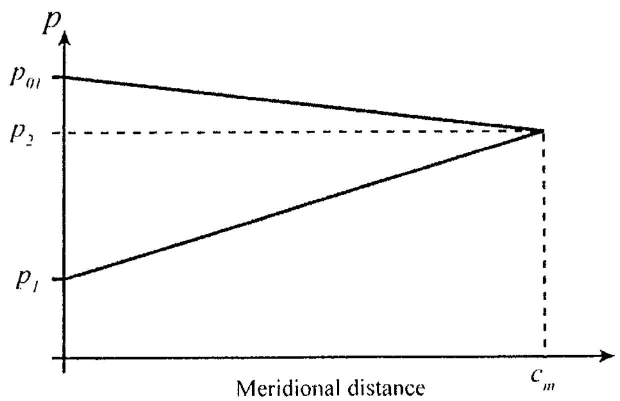
p
p_{01}
p_2
p_1
Meridional distance
c_m

Fig. 2

2 (a) It is observed that for both axial compressor and axial turbine stages the degree of reaction is always lower at the hub than at the casing. Explain why this is the case. Describe any practical problems which can arise from this. [15%]

(b) In the *Meridional Streamline Curvature Method* the flow within the blade rows of an axial flow turbomachine is modelled as an axisymmetric flow. A body force $\boldsymbol{F}$ per unit volume acts on the flow to produce the same momentum changes as would have been produced by the real blades.

(i) Assuming that the radial velocity is small, show, by applying a force analysis on a small control volume, that the simple radial equilibrium equation in stationary coordinates including the effect of the radial body force F_r is:

$$\rho\frac{V_\theta^2}{r} = \frac{dp}{dr} - F_r$$ [15%]

(ii) Derive the equation relating the circumferential body force F_θ to the angular momentum rV_θ, where V_θ is the circumferential velocity. [15%]

(iii) Show that the body force components F_r and F_θ are related by $F_r = F_\theta \tan\delta$, where δ is the angle between the stacking axis and the radial direction. [10%]

(iv) Hence show that the difference between the hub and the casing axial velocities is given by:

$$V_{x,c} - V_{x,h} = \tan\delta \ln\left(\frac{r_h}{r_c}\right)\frac{d(rV_\theta)}{dx}$$

where subscript c denotes the casing and h the hub. Assume that the stagnation pressure and temperature are constant within the blade row and rV_θ is a function of x only. [30%]

(c) Using the results obtained in part (b), explain, using a sketch where necessary, how blade lean can be used to relieve the low reaction problem at the hub. [15%]

3 (a) Discuss shrouded and unshrouded blade tip clearance leakage losses in axial turbine stages. Discuss also why the stage reaction is an important aerodynamic design parameter to consider in selecting the shrouded or unshrouded rotor tip configurations. [25%]

(b) A jet with mass flow rate $\dot{m}_j$ mixes with a mainstream flow of mass flow rate $\dot{m}_m$ at constant pressure. Assuming $\dot{m}_j << \dot{m}_m$ and the two streams have the same stagnation temperature, show that the kinetic energy dissipation per unit mass flow rate of the mainstream is:

$$\Delta h = \frac{\dot{m}_j}{\dot{m}_m}\left(1 - \frac{V_j}{V_m}\cos\alpha\right)V_m^2$$

where V_m and V_j are the mainstream velocity and the jet velocity respectively, and α the angle between the two streams. [20%]

(c) Using the result obtained above, show that an estimate of the mixing loss for a shrouded turbine rotor due to the shroud clearance leakage flow, see Fig. 3, can be expressed with an entropy loss coefficient ς :

$$\varsigma = 2\frac{\dot{m}_l}{\dot{m}_m}\left(1 - \frac{\tan\alpha_1}{\tan\alpha_2}\right)\sin^2\alpha_2$$

where the mass flow rate through the rotor is denoted as $\dot{m}_m$ and that of the leakage flow $\dot{m}_l$. The subscripts 1 and 2 denote the blade inlet and exit respectively, and α the swirl angle measured from the axial direction. State clearly the assumptions made in order to derive this expression. [30%]

(d) It is expected that for a shrouded turbine rotor even with zero shroud clearance (the ideal seal case), there would still be aerodynamic losses associated with the shrouded rotor tip. Discuss the sources of these losses and any practical measures which could be used to reduce these losses. [25%]

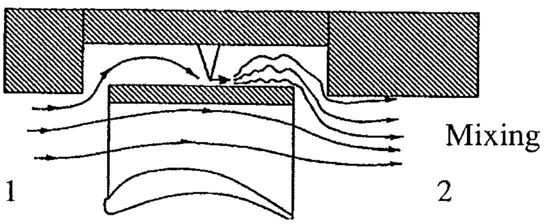

Fig. 3

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