ENGINEERING TRIPOS PART IIB

Wednesday 5 May 2010 9 to 10.30

Module 4C2

DESIGNING WITH COMPOSITES

Answer not more than **three** *questions.*

All questions carry the same number of marks.

The **approximate** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachments:
Special datasheet (6 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

naf02

1 (a) Consider an epoxy-glass fibre composite lamina with the following elastic constants: $E_1 = 50$ GPa, $E_2 = 5$ GPa, $G_{12} = 10$ GPa, $\nu_{12} = 0.3$.

(i) Calculate the shear strain induced in the lamina when a normal tensile stress of 100 MPa is applied at an angle of 30° to the fibre axis. [15%]

(ii) Estimate the ratio of maximum to minimum Young's moduli of the lamina when loaded at different angles to the fibre axis. [15%]

(b) Explain what is meant by "a *balanced* laminate". Show that a $\pm\phi$ "angle-ply" laminate, with a ply thickness $t/2$, is balanced when loaded at an angle equally inclined to the $+\phi$ and $-\phi$ plies. [35%]

(c) Determine the extensional stiffness matrix for a [-45 / 0 / +45 / 90] laminate, for a ply thickness $t/4$, in terms of the components of the lamina stiffness tensor in principal material axes. How would the Young's modulus of this laminate vary with loading angle? [35%]

2 (a) A GFRP panel in an automotive engine compartment, which is loaded in bending, is found to suffer premature failure when the compartment becomes overheated.

(i) Suggest one possible micromechanism responsible for the failure, explaining why you have made this suggestion. [10%]

(ii) For the micromechanism you have chosen, outline what steps you would take to verify that this was indeed the cause of the failure. [20%]

(iii) How can the performance of the component be improved in hot conditions? [20%]

(b) Describe in detail the resin transfer and injection moulding processes for composite materials. Compare and contrast these processes. Suggest typical applications for each process, explaining why these applications are appropriate. [50%]

3 (a) Describe how interlaminar failure can arise due to:

(i) axial tension of a $\pm 45°$ laminate and

(ii) bending of a thick curved plate. [20%]

(b) An AS/3501 carbon fibre-epoxy $[0°, 90°]_S$ laminate of overall thickness 1 mm (material properties on the datasheet) is subject to tensile stresses σ_1 and σ_2 along the 0° and 90° fibre directions, respectively. The $[Q]$ matrix for a unidirectional lamina of this material and the $[A]^{-1}$ matrix for the laminate are as follows:

$$[Q] = \begin{bmatrix} 139 & 2.7 & 0 \\ 2.7 & 9.0 & 0 \\ 0 & 0 & 6.9 \end{bmatrix} \text{GPa}, \quad [A]^{-1} = \begin{bmatrix} 0.0135 & -0.0005 & 0 \\ -0.0005 & 0.0135 & 0 \\ 0 & 0 & 0.145 \end{bmatrix} \frac{1}{\text{GPa mm}}$$

(i) For an equal biaxial stress loading $\sigma = \sigma_1 = \sigma_2$, calculate the first ply failure stress σ using the Tsai-Hill failure criterion. [50%]

(ii) Sketch the failure surface for the laminate in the (σ_1, σ_2) plane, making further calculations as appropriate. [30%]

4 Consider the design of a GFRP tubular beam for part of a lorry trailer of length $L = 3$ m, radius $R = 50$ mm and wall thickness t. The beam is built-in at one end and has a torsional load $Q = 200$ Nm applied at the free end, as shown in Fig. 1. The beam is to be made from pre-preg material with a ply thickness of 0.5 mm, for which carpet plots are given in Fig. 2. The beam has the design requirements that the rotation at the load point due to the applied torque must be less than 0.03 radians, while the lowest resonant frequency f of the beam in bending must be less than 10 Hz.

(a) Why might GFRP be chosen for such an application? [10%]

(b) Outline a design procedure to include multiple design constraints in choosing an appropriate composite laminate for the beam. [15%]

(c) Identify an appropriate lay-up for the beam to minimise its weight. It can be assumed that laminate failure is not critical. Thin-walled analysis may be used for the design, with $t \ll R$ and second moment of area $I = \pi R^3 t$. Note that the lowest resonant frequency f of a beam of length L, mass per unit length m, second moment of area I, made from an isotropic material with Young's modulus E is given by

$$f = 0.56\sqrt{\frac{EI}{mL^4}}.$$

[75%]

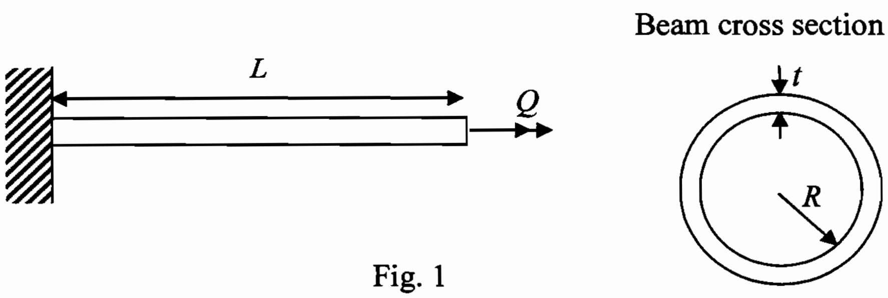


Fig. 1

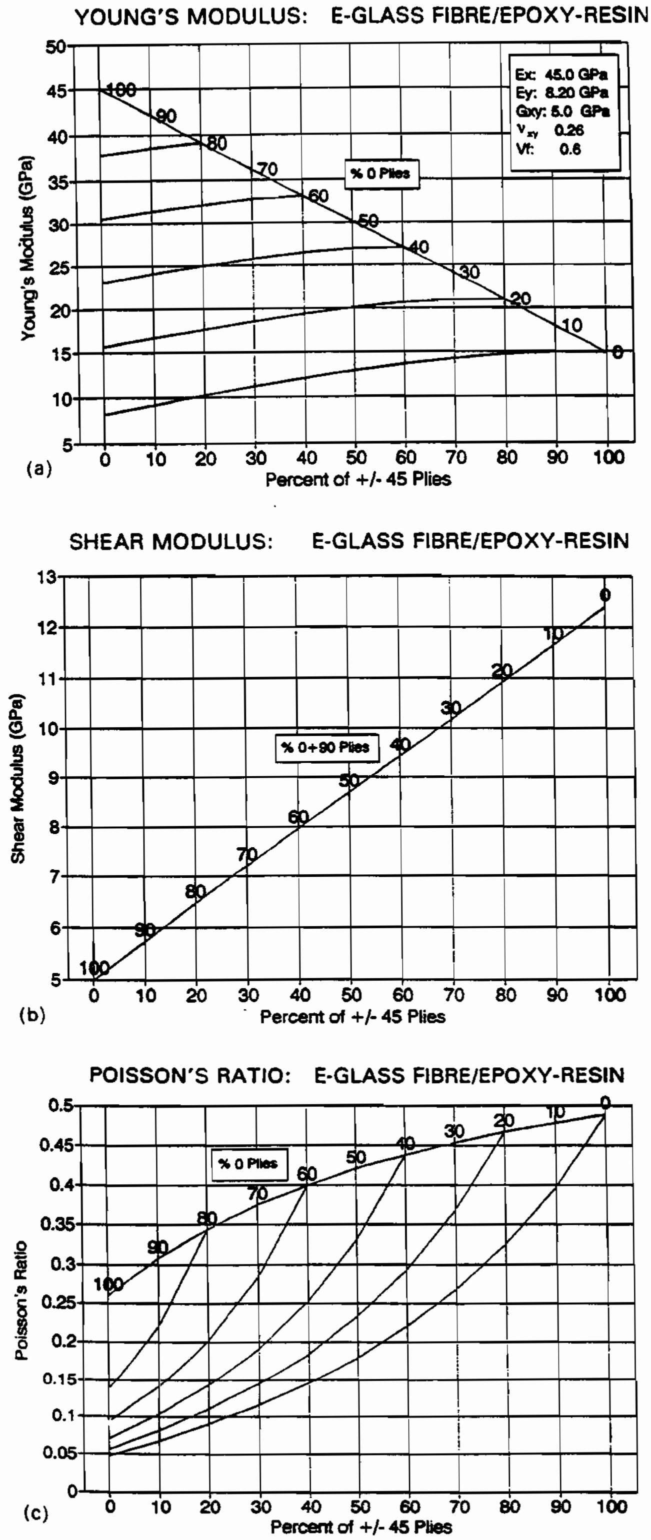


Fig. 2

END OF PAPER

ENGINEERING TRIPOS PART II B

Module 4C2 – Designing with Composites

DATA SHEET

The in-plane compliance matrix [S] for a transversely isotropic lamina is defined by

$$\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{pmatrix}=[S]\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{pmatrix} \quad \text{where } [S]=\begin{bmatrix}1/E_1 & -\nu_{21}/E_2 & 0\\ -\nu_{12}/E_1 & 1/E_2 & 0\\ 0 & 0 & 1/G_{12}\end{bmatrix}$$

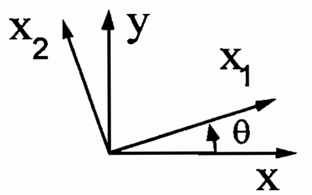
[S] is symmetric, giving $\nu_{12}/E_1 = \nu_{21}/E_2$. The compliance relation can be inverted to give

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{pmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{pmatrix}$$

$$\begin{aligned}\text{where } Q_{11} &= E_1/(1-\nu_{12}\nu_{21})\\ Q_{22} &= E_2/(1-\nu_{12}\nu_{21})\\ Q_{12} &= \nu_{12}E_2/(1-\nu_{12}\nu_{21})\\ Q_{66} &= G_{12}\end{aligned}$$

Rotation of co-ordinates

Assume the principal material directions (x_1, x_2) are rotated anti-clockwise by an angle θ, with respect to the (x, y) axes.



Then,
$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_{12}\end{pmatrix}=[T]\begin{pmatrix}\sigma_x\\ \sigma_y\\ \sigma_{xy}\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \gamma_{12}\end{pmatrix}=[T]^{-T}\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{pmatrix}$$

$$\text{where } [T]=\begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta\\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta-\sin^2\theta\end{bmatrix}$$

$$\text{and } [T]^{-T}=\begin{bmatrix}\cos^2\theta & \sin^2\theta & \sin\theta\cos\theta\\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta\\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & (\cos^2\theta-\sin^2\theta)\end{bmatrix}$$

The stiffness matrix $[Q]$ transforms in a related manner to the matrix $[\bar{Q}]$ when the axes are rotated from (x_1, x_2) to (x, y)

$$[\bar{Q}] = [T]^{-1}[Q][T]^{-T}$$

In component form,

$$[\bar{Q}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \quad \text{where} \quad \begin{aligned} \bar{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}\left(c^4 + s^4\right) \\ \bar{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})sc^3 - (Q_{22} - Q_{12} - 2Q_{66})s^3c \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})s^3c - (Q_{22} - Q_{12} - 2Q_{66})sc^3 \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}\left(s^4 + c^4\right) \\ &\text{with } c = \cos\theta, \quad s = \sin\theta \end{aligned}$$

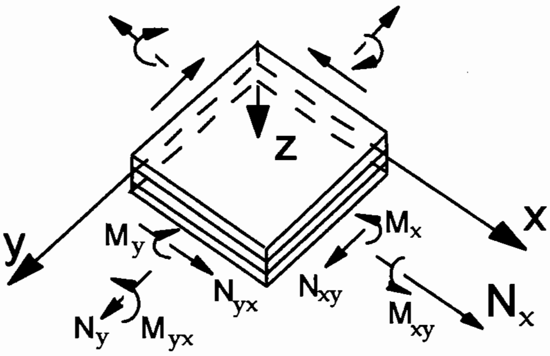
The compliance matrix $[S] \equiv [Q]^{-1}$ transforms to $[\bar{S}] \equiv [\bar{Q}]^{-1}$ under a rotation of co-ordinates by θ from (x_1, x_2) to (x, y), as

$$[\bar{S}] = [T]^T[S][T]$$

and in component form,

$$\begin{aligned} \bar{S}_{11} &= S_{11}c^4 + S_{22}s^4 + (2S_{12} + S_{66})s^2c^2 \\ \bar{S}_{12} &= S_{12}\left(c^4 + s^4\right) + (S_{11} + S_{22} - S_{66})s^2c^2 \\ \bar{S}_{22} &= S_{11}s^4 + S_{22}c^4 + (2S_{12} + S_{66})s^2c^2 \\ \bar{S}_{16} &= (2S_{11} - 2S_{12} - S_{66})sc^3 - (2S_{22} - 2S_{12} - S_{66})s^3c \\ \bar{S}_{26} &= (2S_{11} - 2S_{12} - S_{66})s^3c - (2S_{22} - 2S_{12} - S_{66})sc^3 \\ \bar{S}_{66} &= (4S_{11} + 4S_{22} - 8S_{12} - 2S_{66})s^2c^2 + S_{66}\left(c^4 + s^4\right) \\ &\text{with } c = \cos\theta, \quad s = \sin\theta \end{aligned}$$

Laminate Plate Theory



Consider a plate subjected to stretching of the mid-plane by $\left(\varepsilon_x^o, \varepsilon_y^o, \varepsilon_{xy}^o\right)^T$ and to a curvature $\left(\kappa_x, \kappa_y, \kappa_{xy}\right)^T$. The stress resultants $\left(N_x, N_y, N_{xy}\right)^T$ and bending moment per unit length $\left(M_x, M_y, M_{xy}\right)^T$ are given by

$$\begin{pmatrix} N \\ \cdots \\ M \end{pmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & . & \cdots \\ B & \vdots & D \end{bmatrix} \begin{pmatrix} \varepsilon^o \\ \cdots \\ \kappa \end{pmatrix}$$

In component form, we have,

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix}$$

where the laminate extensional stiffness, A_{ij}, is given by:

$$A_{ij} = \int_{-t/2}^{t/2} \left(\bar{Q}_{ij}\right)_k dz = \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right)$$

the laminate coupling stiffnesses is given by

$$B_{ij} = \int_{-t/2}^{t/2} \left(\bar{Q}_{ij}\right)_k z dz = \frac{1}{2} \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right)$$

and the laminate bending stiffness are given by:

$$D_{ij} = \int_{-t/2}^{t/2} \left(\bar{Q}_{ij}\right)_k z^2 dz = \frac{1}{3}\sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

with the subscripts $i, j = 1, 2$ or 6.

Here,

$n =$ number of laminae

$t =$ laminate thickness

$z_{k-1} =$ distance from middle surface to the top surface of the *k-th* lamina

$z_k =$ distance from middle surface to the bottom surface of the *k-th* lamina

Quadratic failure criteria.

For plane stress with $\sigma_3 = 0$, failure is predicted when

Tsai-Hill: $\frac{\sigma_1^2}{s_L^2} - \frac{\sigma_1\sigma_2}{s_L^2} + \frac{\sigma_2^2}{s_T^2} + \frac{\tau_{12}^2}{s_{LT}^2} \geq 1$

Tsai-Wu: $F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1$

where $F_{11} = \frac{1}{s_L^+ s_L^-}$, $F_{22} = \frac{1}{s_T^+ s_T^-}$, $F_1 = \frac{1}{s_L^+} - \frac{1}{s_L^-}$, $F_2 = \frac{1}{s_T^+} - \frac{1}{s_T^-}$, $F_{66} = \frac{1}{s_{LT}^2}$

F_{12} should ideally be optimised using appropriate strength data. In the absence of such data, a default value which should be used is

$$F_{12} = -\frac{(F_{11}F_{22})^{1/2}}{2}$$

Fracture mechanics

Consider an orthotropic solid with principal material directions x_1 and x_2. Define two effective elastic moduli E'_A and E'_B as

$$\frac{1}{E'_A} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{22}}{S_{11}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

$$\frac{1}{E'_B} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{11}}{S_{22}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

where S_{11} etc. are the compliances.

Then G and K are related for plane stress conditions by:

crack running in x_1 direction: $G_I E'_A = K_I^2; G_{II} E'_B = K_{II}^2$

crack running in x_2 direction: $G_I E'_B = K_I^2; G_{II} E'_A = K_{II}^2$.

For mixed mode problems, the total strain energy release rate G is given by

$G = G_I + G_{II}$

Approximate design data

	Steel	Aluminium	CFRP	GFRP	Kevlar
Cost C (£/kg)	1	2	100	5	25
E_1 (GPa)	210	70	140	45	80
G (GPa)	80	26	≈35	≈11	≈20
ρ (kg/m^3)	7800	2700	1500	1900	1400
e^+ (%)	0.1-0.8	0.1-0.8	0.4	0.3	0.5
e^- (%)	0.1-0.8	0.1-0.8	0.5	0.7	0.1
e_{LT} (%)	0.15-1	0.15-1	0.5	0.5	0.3

Table 1. Material data for preliminary or conceptual design. Costs are very approximate.

	Aluminium	Carbon/epoxy (AS/3501)	Kevlar/epoxy (Kevlar 49/934)	E-glass/epoxy (Scotchply/1002)
Cost (£/kg)	2	100	25	5
Density (kg/m^3)	2700	1500	1400	1900
E_1 (GPa)	70	138	76	39
E_2 (GPa)	70	9.0	5.5	8.3
ν_{12}	0.33	0.3	0.34	0.26
G_{12} (GPa)	26	6.9	2.3	4.1
s_L^+ (MPa)	300 (yield)	1448	1379	1103
s_L^- (MPa)	300	1172	276	621
s_T^+ (MPa)	300	48.3	27.6	27.6
s_T^- (MPa)	300	248	64.8	138
s_{LT} (MPa)	300	62.1	60.0	82.7

Table 2. Material data for detailed design calculations. Costs are very approximate.

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