ENGINEERING TRIPOS PART IIB

Wednesday 28 April 2010 2.30 to 4

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

Candidates may bring their notebooks to the examination.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 The equation of motion which governs the sway motion of an unmoored offshore barge has the form

$$M\dot{v} + Cv = F(t)$$

where v is the vessel velocity, M is the mass, C is the damping factor, and $F(t)$ is the random wave force. The wave force has the following *single sided* spectral density:

$$S_{FF}(\omega) = \begin{cases} S_0 & \text{for } \omega_1 \le \omega \le \omega_2 \\ 0 & \text{for } \omega < \omega_1 \text{ and } \omega > \omega_2 \end{cases}$$

(a) Derive an expression for the transfer function between the barge velocity and the wave force, and hence derive an expression for the spectrum of the barge velocity and for the r.m.s. barge velocity. [40%]

(b) Derive an expression for the r.m.s. acceleration of the barge. [30%]

(c) Evaluate the r.m.s. velocity and acceleration of the barge for the case:

$$\begin{aligned} M &= 1.5\times10^{6}\ \text{kg} \\ C &= 2\times10^{4}\ \text{N m}^{-1}\ \text{s} \\ S_0 &= 4.5\times10^{11}\ \text{N}^2\ \text{rad}^{-1}\ \text{s} \\ \omega_1 &= 0.2\ \text{rad s}^{-1} \\ \omega_2 &= 1.0\ \text{rad s}^{-1} \end{aligned}$$

[20%]

(d) Calculate the mean rate at which the barge velocity crosses zero with positive slope. [10%]

(Hint: use may be made of the result $\int (1 + x^2)^{-1} dx = \tan^{-1} x + \text{constant}$)

2 A circuit board is mounted on a satellite structure. During launch, the displacement of the centre of the circuit board can modelled as a stationary random process $x(t)$. The joint probability density function of $x(t)$ and velocity $\dot{x}(t)$ is $p(x,\dot{x})$. The clearance between the centre of the board and sidewall of the satellite is b. The board will be damaged if its centre impacts the sidewall with a velocity greater than V.

(a) Show that the mean rate ν at which the random process $x(t)$ crosses the level b with velocity greater than V is given by

$$\nu = \int_{V}^{\infty} \dot{x} p(b,\dot{x}) \mathrm{d}\dot{x}$$ [30%]

(b) Derive an expression for ν for the case where $x(t)$ is a stationary Gaussian random process with r.m.s. displacement σ_x and r.m.s. velocity $\sigma_{\dot{x}}$. [30%]

(c) The dynamic response of the circuit board is governed by the equation

$$M\ddot{x} + C\dot{x} + Kx = F(t)$$

where $M = 0.01$ kg, $C = 0.3$ N m^{-1} s, $K = 2.25 \times 10^4$ N m^{-1}, and $F(t)$ is white noise with single sided spectral density $S_0 = 5 \times 10^{-3}$ N^2 rad^{-1} s. Calculate the probability of impact damage to the board during launch if $b = 4$ mm, $V = 5$ m s^{-1} and the duration of the excitation is 10 s. Recalculate this probability for the case $b = 3$ mm. Comment on the results of these calculations. [40%]

3 Consider the undamped mass-spring system shown in Fig. 1. The two ends of the coil spring are a distance L apart initially. A mass m is attached to the centre of the spring. The relaxed length of the spring is L_0 and the spring constant is K.

(a) Derive an expression for the potential energy of the system $V(x)$ as a function of displacement x. Hence, derive an expression for the restoring force of the spring as a function of displacement x. Show that the form of the restoring force reduces to the Duffing form for small values of x. [50%]

(b) Starting with the form of the potential function $V(x)$, sketch the behaviour of the system in the phase plane for the cases:

(i) $L = L_0/2$

(ii) $L = 2L_0$

Explain the qualitative difference in behaviour observed in the two cases. [50%]

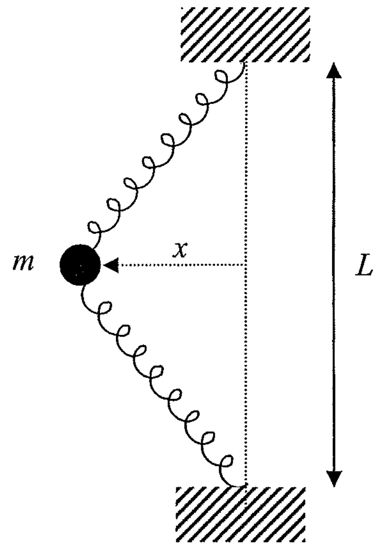

Fig. 1

4 Consider the Van der Pol equation for a single degree-of-freedom resonator:

$$\ddot{x} + \varepsilon(3x^2 - 1)\dot{x} + p^2 x = 0$$

(a) Show that this system is capable of self-oscillation and derive an expression for the steady-state amplitude of oscillation. [40%]

(b) Now consider the case where this nonlinear system is subjected to an external harmonic force at frequency ω:

$$\ddot{x} + \varepsilon(3x^2 - 1)\dot{x} + p^2 x = \varepsilon F \cos \omega t$$

Assume a solution of the following form:

$$x = A \sin pt + B \sin(\omega t + \phi)$$

Use a harmonic balance approach truncated to the frequencies p and ω to obtain the relationship between A, B, ε, F and ω. [50%]

(c) Show that for the case in part (b), there exists a range of values of F and ω for which the limit cycle oscillation is quenched. [10%]

END OF PAPER