ENGINEERING TRIPOS PART IIB

Friday 23 April 2010 2.30 to 4

Module 4C8

APPLICATIONS OF DYNAMICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
4C8 datasheet (4 pages)

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator.

1 (a) A 'coned' railway wheelset with effective conicity ε, average wheel radius r and track gauge $2d$ is moving along a straight track at steady speed u. It has small lateral tracking error y and small yaw angle θ. The coefficients of *both* lateral and longitudinal creep of the wheels are C.

Show that the net lateral force Y and net moment N acting on the wheelset due to the creep forces are given by:

$$Y = 2C\left(\theta + \frac{\dot{y}}{u}\right)$$

$$N = 2dC\left(\frac{\varepsilon\, y}{r} - \frac{d\dot{\theta}}{u}\right)$$

and indicate their directions on a sketch of the wheelset. State your assumptions. [30%]

(b) Figure 1 shows a railway wheelset that is attached to a vehicle body through a suspension system with four springs of stiffness k. The spacing between the longitudinal springs is $2b$ and it can be assumed that the lateral springs do not exert any yaw moment on the axle. The wheelset may be assumed to be massless and the vehicle body may be assumed to track perfectly, with no lateral or yaw motion.

(i) Derive an equation for the lateral motion of the wheelset. [40%]

(ii) Find an expression for the wavelength of the hunting motion. Compare it with the hunting wavelength of a free wheelset. [20%]

(iii) Find an expression for the damping ratio of the hunting motion and sketch a graph of its variation with k. Determine the value of the damping ratio for very large values of k and comment on the variation of damping with k and u. [10%]

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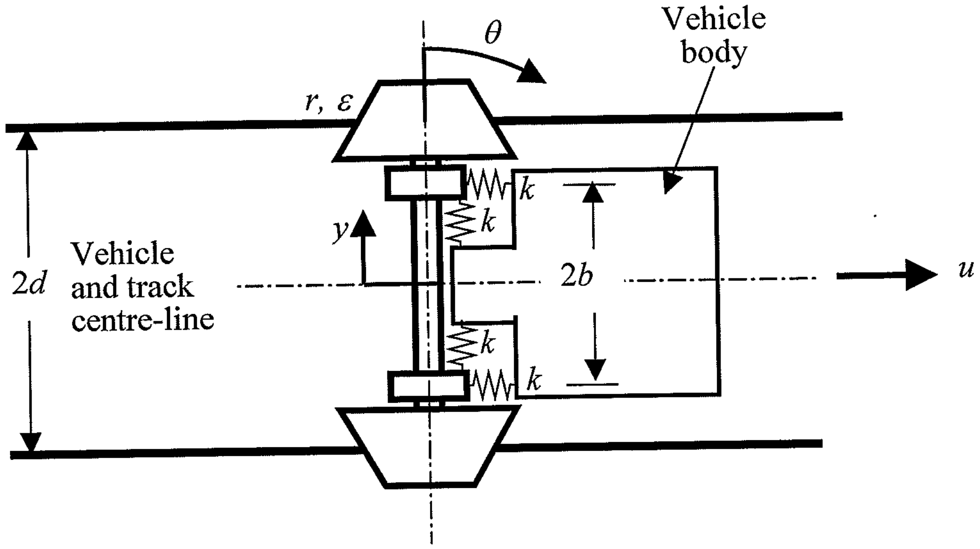

Fig. 1

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2 (a) A 'bicycle' model of a car, with freedom to sideslip with velocity v and yaw at rate Ω, is shown in Fig. 2. The car moves at steady forward speed u. It has mass m, yaw moment of inertia I, and lateral creep coefficients C_f and C_r at the front and rear tyres. The lengths a and b and the steering angle δ are defined in the figure. Show that the equations of motion in a coordinate frame rotating with the vehicle are given by:

$$m(\dot{v}+u\Omega)+\left(C_f+C_r\right)\frac{v}{u}+\left(aC_f-bC_r\right)\frac{\Omega}{u}=C_f\delta$$

$$I\dot{\Omega}+\left(aC_f-bC_r\right)\frac{v}{u}+\left(a^2C_f+b^2C_r\right)\frac{\Omega}{u}=aC_f\delta$$

State your assumptions. [30%]

(b) The steering angle is set to a constant value δ and the vehicle follows a steady circular path of radius R. Use the equations of motion to derive an expression for the steady state curvature response $(1/R)$. Sketch a handling diagram, showing responses for under-steering, neutral steering and over-steering vehicles. Explain the behaviour with increasing speed and state the parameters that determine the type of response obtained. [40%]

[Recall that a handling diagram is a graph of u^2/R vs $(\delta - L/R)$, where $L = a + b$.]

(c) A particular vehicle develops a small additional steer angle at the front wheel due to 'roll-steer'. This additional small steer angle is proportional to the lateral acceleration at the centre of gravity, with constant of proportionality K. Sketch the effect of the roll steer on your handling diagram from part (b). Explain how the sign of K affects the behaviour of under-steering and over-steering vehicles. [30%]

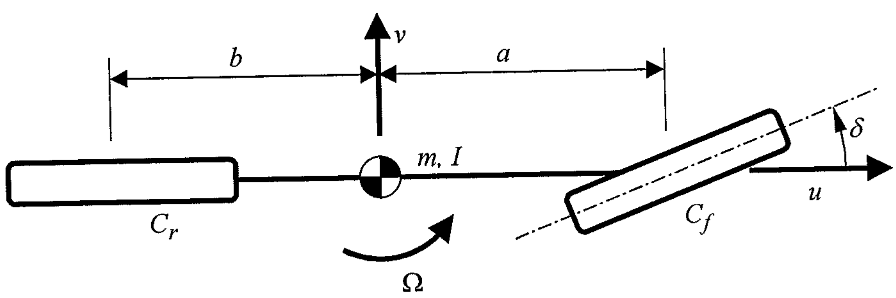

Fig. 2

3 The precession of the circular orbit of a satellite caused by the Earth's equatorial 'bulge' can be calculated from the motion of a rigid ring with the same mass as the satellite, uniformly distributed around its orbital path. The ring rotates in its own plane about the centre of the Earth with the same angular velocity as the satellite ω.

(a) Using virtual work or otherwise, show that the net moment on such a ring would be

$$\frac{3m\mu R^2 J_2 \sin i \cos i}{2a^3}$$

where m and a are the mass and radius of the ring respectively, i is its angle of inclination to the equatorial plane, and the other symbols have their usual meanings. [40%]

(b) Hence show that the rate of precession of the orbital plane is approximately

$$\frac{3\omega R^2 J_2 \cos i}{2a^2}.$$

[40%]

(c) A Galileo GNS satellite has a near-circular orbit of altitude 23,222 km and an inclination of 56°. Find the rate of precession of the satellite's orbit, and hence the number of orbits the satellite will complete before it precesses through one complete revolution. [20%]

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4 Allowing for zonal harmonics only, the Earth's gravitational potential can be written as

$$U(r,\theta)=\frac{\mu}{r}\left[1-\sum_{n=2}^{\infty}(R/r)^n J_n P_n(\cos\theta)\right]$$

(a) Explain why this expression does not include a term for $n=1$, and state the physical basis for the J_2 term. [15%]

(b) Show that a particle of unit mass at a distance r from the centre of the Earth (where $r > R$) in the equatorial plane will experience a force with components

$\frac{\mu}{r^2}\left[1+\frac{3}{2}J_2(R/r)^2-\frac{15}{8}J_4(R/r)^4+\ldots\right]$ towards the Earth's centre, and

$\frac{\mu}{r^2}\left[\frac{3}{2}J_3(R/r)^3+\ldots\right]$ parallel to the polar axis. [35%]

(c) To be in a geostationary orbit, a satellite must stay above the same point on the Earth's surface. Assuming the orbit is a circular one in the equatorial plane, calculate the required mean motion for a geostationary satellite, taking the length of a year to be 365.250 days. Hence show that the radius of the orbit should be approximately 42,165 kilometres, when the effect of the J_2 term alone is included. [30%]

(d) If the mass of the satellite is 100 kg, calculate the magnitude and direction of force it would experience in the direction parallel to the Earth's polar axis, and suggest (without further calculation) how the geostationary orbit assumed above should be modified to accommodate this force. [20%]

END OF PAPER