ENGINEERING TRIPOS PART IIB

Monday 26 April 2010 9 to 10.30

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

There are no attachments.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
Single-sided script paper	Engineering Data Book CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) Explain briefly what is meant by the Lennard-Jones relation expressing the interaction energy of two atoms. [10%]

(b) Two parallel plane surfaces of the same material are brought into close proximity. The force of attraction per unit area is given by:

$$p(z) = \frac{A}{6\pi z^3}$$

where A is the Hamaker constant and z is their separation.

(i) The work of adhesion w equals the work done per unit area in increasing the separation of two such surfaces from the atomic spacing h_0 to ∞. Confirm that w and A are related by the expression

$$w = \frac{A}{12\pi h_0^2} \quad .$$

[20%]

(ii) Figure 1(a) shows a *rigid* spherical surface of this material held at a minimum distance h_0 above one of the plane surfaces. By considering the forces of attraction operating at radius r, show that the force of adhesion P' acting between the two solids is given by

$$P' = 2\pi R w$$

and sketch the variation of adhesive stress with radius r. [30%]

(c) The DMT (Derjaguin, Muller and Toporov) analysis of an *elastic* adhesive junction assumes an elastic sphere of radius R and contact modulus E^* makes contact with a rigid plane surface, as shown in Fig. 1(b). The contact comprises a central Hertzian zone, of radius a, and an outer annular region over which the profile remains at radius R and the adhesive forces act as in part (b). Express the net compressive load P in terms of a, R, E^* and w. Sketch the variation of contact size a with P, and compare with the Hertz curve corresponding to $w = 0$. Indicate on the graph the conditions for “pull-off”. [30%]

(d) How does the DMT analysis differ from the JKR (Johnson, Kendall and Roberts) treatment of the same problem? [10%]

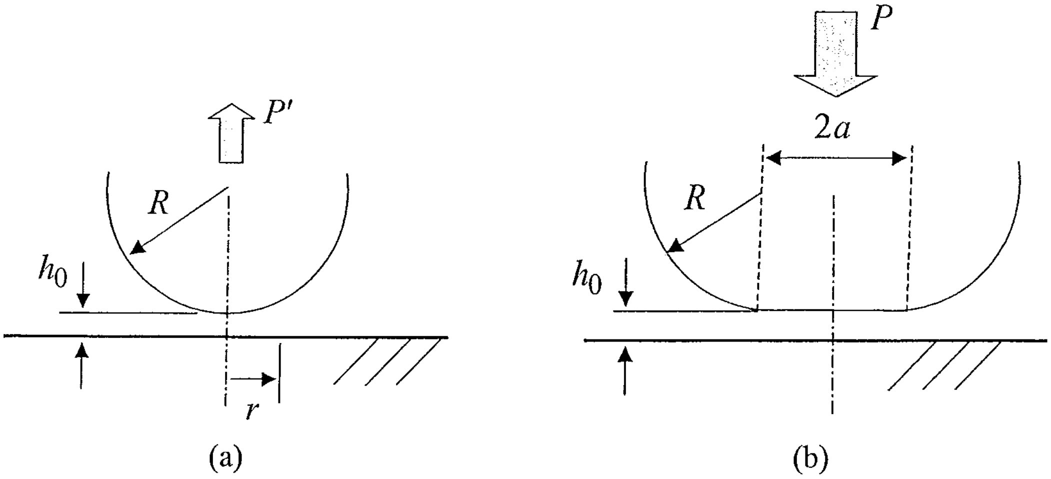

Fig. 1

Hertzian point contact under load P

Reduced radius R given by $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}$

Contact modulus E^* by $\dfrac{1}{E^*}=\dfrac{1-\nu_1^{\,2}}{E_1}+\dfrac{1-\nu_2^{\,2}}{E_2}$

Radius of contact circle $a=\left\{\dfrac{3PR}{4E^*}\right\}^{1/3}$

Maximum contact pressure $p_0=\dfrac{3P}{2\pi a^2}=\left\{\dfrac{6PE^{*2}}{\pi^3R^2}\right\}^{1/3}$

Mean contact pressure $\bar{p}=\dfrac{2}{3}p_0$

Approach of distant points $\delta=\dfrac{a^2}{R}=\left\{\dfrac{9P^2}{16RE^{*2}}\right\}^{1/3}$

Maximum shear stress is of magnitude $0.31p_0$ and at depth $0.48a$.

(TURN OVER

2 A single-axis accelerometer (Fig. 2) is fabricated in a polysilicon surface micromachining process comprising a 2 μm thick structural layer. The design is identical to the Analog Devices ADXL76 single-axis accelerometer and consists of a spring supported proof mass embedded by parallel-plate capacitive electrodes for sensing. The spring constant along the x-axis is 10 N m^{-1} and the mass is 1 μg . You may assume that the proof mass is embedded by 30 identical sets of differential electrodes each of overlap length 200 μm and an electrode spacing of 1 μm.

(a) Estimate the displacement of the mass for an acceleration of 1 g (9.8 m s^{-2}) along the x-axis. [10%]

(b) Estimate the scale factor of the accelerometer, as defined by the change in capacitance per unit g of acceleration along the x-axis. [20%]

(c) The proof mass is also sensitive to acceleration changes along the z-axis perpendicular to the fabrication plane. Estimate the change in capacitance per unit g of acceleration along the z-axis assuming that the spring constant for proof mass motion in this direction is 9 N m^{-1} . Comment on the results of your calculation. [20%]

(d) Explain the concept of *self-test* in the context of an accelerometer and give an example of an implementation of self-test for this device. [25%]

(e) Explain how Brownian motion of the proof mass sets a fundamental limit on sensor performance. Estimate the noise floor of this device in units of $g/\sqrt{\mathrm{Hz}}$ at room temperature (300 K) assuming a quality factor of 10 for motion along the x-axis. [25%]

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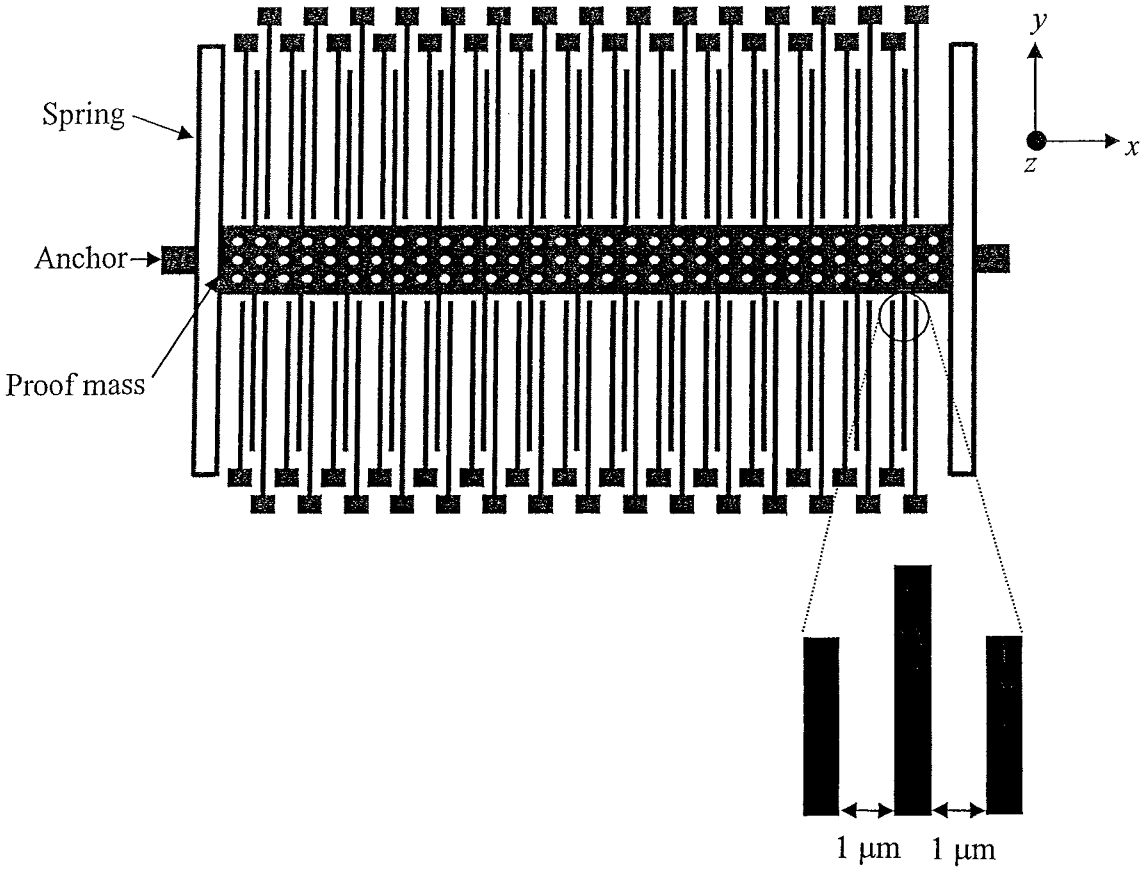

Fig. 2

3 A micromachined electro-mechanical switch (Fig. 3) consists of a fixed plate and a parallel moveable plate, of overlap area A and air-gap g. The fixed plate is covered with a thin dielectric of thickness t and permittivity ε_D, while the permittivity of the air-gap is ε_0. The moveable plate is supported by a linear spring of stiffness K and the spring is relaxed when the air-gap equals g_0. You may neglect inertia and assume that $\varepsilon_D >> \varepsilon_0$ and $g_0 >> t$.

(a) The switch is closed by applying a voltage across the fixed and moveable plates. Derive an expression for the minimum voltage across the plates to close the switch, assuming the plates are initially uncharged. This voltage is the pull-in voltage V_{PI}. [30%]

(b) The moveable plate separates from the fixed plate when the voltage is reduced. Snap-off from the pulled-in condition occurs at a pull-out voltage V_{POUT}. Derive an expression for V_{POUT} and an equation for g immediately after pull-out. [40%]

(c) Sketch the gap versus voltage characteristic of the switch, showing its operational cycle. [30%]

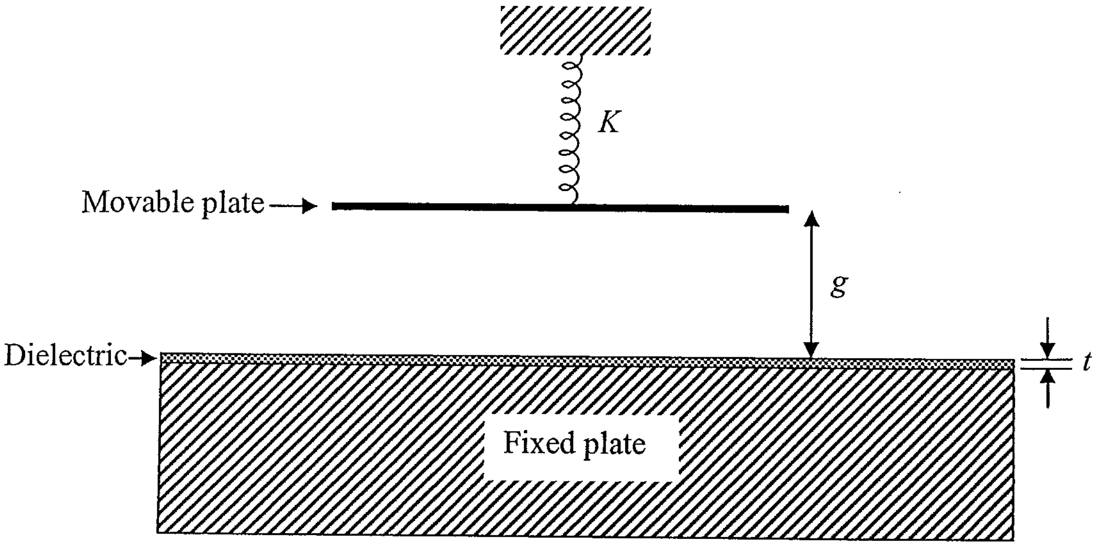

Fig. 3

4 A single degree-of-freedom micro-electro-mechanical resonator is excited by a parallel-plate electrostatic actuator of overlap area A and gap g. A parallel plate capacitor of identical geometry to that of the actuator, is used to sense the motion of the resonator. A DC voltage V_{DC} is applied to the resonator body while the fixed electrodes of the actuator and sensor are initially grounded.

(a) A small signal AC excitation v_{ac} at frequency ω is applied to the actuator electrode. Derive an expression for the electrostatic force at this frequency acting on the resonator. [20%]

(b) Derive an expression for the capacitive current resulting from the motion of the resonator at frequency ω. [30%]

(c) Derive expressions for the motional capacitance, motional inductance and motional resistance of the resonator by drawing an analogy between the current-to-voltage transfer function of the electro-mechanical resonator with that of a series *L-R-C* equivalent circuit representation. [40%]

(d) Describe methods for minimising the motional resistance of micro-electro-mechanical resonators through process and geometry design. [10%]

END OF PAPER