ENGINEERING TRIPOS PART IIB

Thursday 29 April 2010 9 to 10.30

Module 4C16

ADVANCED MACHINE DESIGN

*Answer **all** questions.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Attachment:
Module 4C16 data sheet (3 pages).

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 (a) A thin film of incompressible lubricant fills the gap between two solid surfaces one of which is stationary while the other moves with speed U in the Ox direction. Starting from the formula in the data sheet, show that Reynolds' equation for pressure $p(x,y)$ for two-dimensional steady flow can be written as

$$\frac{\partial}{\partial x}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{h^3}{12\eta}\frac{\partial p}{\partial y}\right) = \frac{U}{2}\frac{\partial h}{\partial x}$$

where η is the lubricant viscosity and $h(x)$ is the gap between the surfaces measured normal to the x-y plane. [10%]

(b) If such a bearing is 'short' in the Oy direction, then pressure gradients in the Ox direction can be considered small compared to those acting parallel to the y axis.

(i) Show that Reynolds' equation can be simplified to

$$\frac{\partial^2 p}{\partial y^2} = \frac{6U\eta}{h^3}\frac{\mathrm{d}h}{\mathrm{d}x}.$$ [10%]

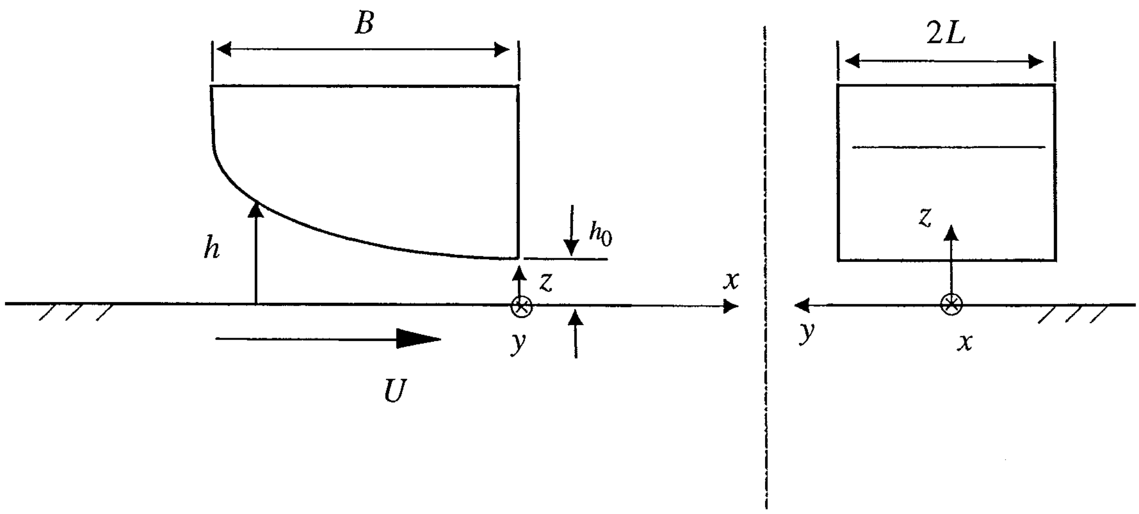
(ii) If the width of the bearing in the Oy direction is $2L$, that is $-L \leq y \leq +L$, obtain an expression that describes the pressure distribution within the fluid in terms of $h, \frac{\mathrm{d}h}{\mathrm{d}x}, L, \eta, U$ and y. [20%]

(c) In a particular case the geometry of the surfaces is as illustrated in Fig. 1 and the gap is described by the equation

$$h = h_0 + \frac{x^2}{2R} \quad \text{for} \quad x \leq 0.$$

What is the physical interpretation of the dimension R? At what location in this bearing will the hydrodynamic pressure be a maximum? It can be assumed that dimension B is large compared to the fluid film thickness. [40%]

(d) By utilizing your answers to parts (b) and (c) obtain an estimate of the load-carrying capacity of such a bearing. [20%]



NOT TO SCALE

Fig. 1

2 Figure 2(a) shows the arrangement of a hybrid drive for a road vehicle. An internal combustion engine is connected to the carrier of a planetary gear. An electrical machine is connected to the sun. A second electrical machine is connected to the annulus, to which is also connected the drive to the road wheels. Electronics control the flow of electrical energy between the two electrical machines and a battery (not shown).

(a) Outline the main objectives of the energy control, and state some of the difficulties that may be encountered in devising a control strategy to achieve these objectives. [15%]

(b) If there is no energy flow to or from the battery the two electrical machines can function as a continuously variable transmission (CVT), as depicted in Fig. 2(b). The shaft speeds and powers are assigned variables and directions as indicated in the figure. The efficiency of the CVT is denoted $\eta = P_V / P_S$ and the speed ratio is $V = \omega_a / \omega_s$. The ratio of annulus to sun diameter is R.

(i) Show that the ratio of annulus to carrier speeds is given by:

$$\frac{\omega_a}{\omega_c} = \frac{1+R}{\frac{1}{V}+R}.$$

[20%]

(ii) By finding the ratio of torques on the sun and annulus, or otherwise, show that the fraction of total power to the wheels provided by the CVT is given by:

$$\frac{P_V}{P_0} = \frac{\eta}{\eta + RV}.$$

[35%]

(iii) Find the range of ratio of annulus to carrier speeds for which there is no recirculation of power. [30%]

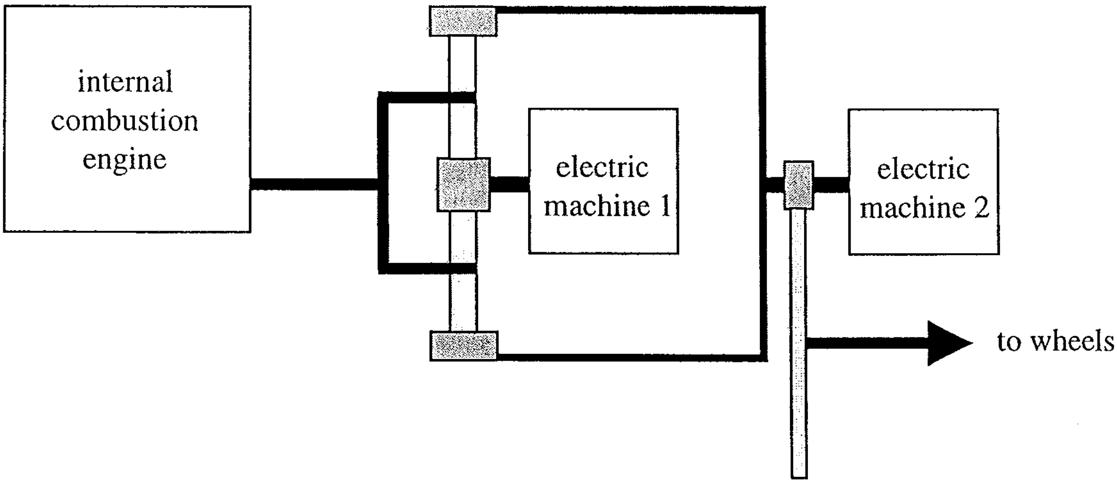

planetary gear
internal combustion engine
electric machine 1
electric machine 2
to wheels

Fig. 2(a)

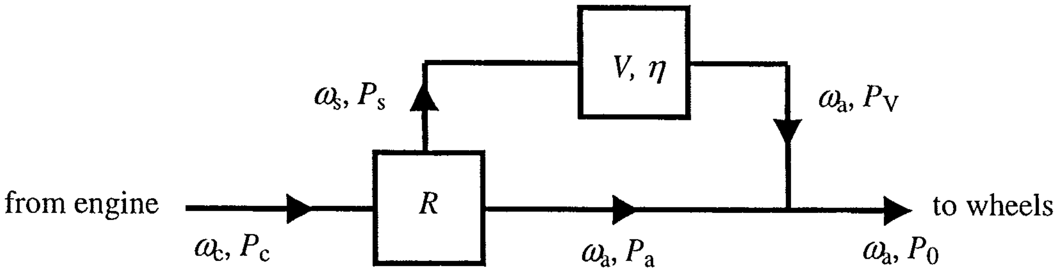

V, η
ωs, Ps
ωa, PV
from engine
R
to wheels
ωc, Pc
ωa, Pa
ωa, P0

Fig. 2(b)

3 A high speed cam is required to lift a mass of 2 kg a distance of 10 mm in 60° of rotation at an angular velocity of 6000 rpm. The lift is to be achieved using a variation of acceleration a with time t of the form

$$a = a_0 \sin(\omega t).$$

(a) Determine the values of the constants a_0 and ω needed to give the specified lift, assuming that the mass is stationary at the start and end of the lift. [30%]

(b) Find the maximum Hertz pressure in the contact between the cam and follower. Assume that the cam and follower have radii of 20 mm and 5 mm, respectively, with a face width of 10 mm. Ignore spring forces and assume that the line of action of the force accelerating the follower is along the common normal to the contact surfaces. The contact modulus E^* is 115 GPa. [20%]

(c) A similar sinusoidal acceleration profile is to be used as detailed above, but the lift is now allowed to start early and finish late, with a permissible lift error of 0.1 mm at the nominal start and finish of the lift. Find the percentage reduction in the maximum Hertz contact pressure, as compared with the profile of part (a). [50%]

END OF PAPER

ENGINEERING TRIPOS Part IIB

Modules 4C16 Data Sheet

HYDRODYNAMIC LUBRICATION

Viscosity: temperature and pressure effects

$\eta = \eta_0$ at $p = 0$ and $T = T_0$

Vogel formula $$\eta = \eta_0 \exp\left\{\frac{b}{T + T_c}\right\}$$

Barus equation $$\eta = \eta_0 \exp\{\alpha p\}$$

Roelands equation $$\eta = \eta_0 \exp\left\{\ln\left(\frac{\eta_0}{\eta_r}\right)\left[\left(1 + \frac{p}{p_r}\right)^{\beta}\left(\frac{T_0 + T_r}{T + T_r}\right) - 1\right]\right\}$$

p_r, T_r and η_r are reference values

Viscous pressure flow

Rate of flow q_x per unit width of fluid of viscosity η down a channel of height h due to pressure gradient $\dfrac{dp}{dx}$

$$q_x = -\frac{h^3}{12\eta}\frac{dp}{dx}$$

Reynolds' Equation for a steady configuration

1–D flow: $$\frac{dp}{dx} = 12\eta\overline{U}\left\{\frac{h - h^*}{h^3}\right\}$$

$\overline{U}$ is the entraining velocity so that $\left|\overline{U}h^*\right|$ is flow per unit width through the contact.

2–D flow: $$\frac{\partial}{\partial x}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial x}\right\} + \frac{\partial}{\partial y}\left\{\frac{h^3}{\eta}\frac{\partial p}{\partial y}\right\} = 12\overline{U}\frac{\partial h}{\partial x}$$

ELASTIC CONTACT STRESS FORMULAE

Suffixes 1, 2 refer to the two bodies in contact.

Effective curvature $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

where R_1, R_2 are the radii of curvature of the two bodies (convex positive).

Contact modulus $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$

where E_1, E_2 and ν_1, ν_2 are Young's moduli and Poisson's ratios.

	Line contact (width $2b$; load W' per unit length)	Circular contact (diameter $2a$; load W)
Semi contact width or contact radius	$b = 2\left\{\frac{W'R}{\pi E^*}\right\}^{1/2}$	$a = \left\{\frac{3WR}{4E^*}\right\}^{1/3}$
Maximum contact pressure ("Hertz stress")	$p_0 = \left\{\frac{W'E^*}{\pi R}\right\}^{1/2}$	$p_0 = \frac{1}{\pi}\left\{\frac{6WE^{*2}}{R^2}\right\}^{1/3}$
Approach of centres	$\delta = \frac{2W'}{\pi}\left[\frac{1-\nu_1^2}{E_1}\left\{\ln\left(\frac{4R_1}{b}\right) - \frac{1}{2}\right\} + \frac{1-\nu_2^2}{E_2}\left\{\ln\left(\frac{4R_2}{b}\right) - \frac{1}{2}\right\}\right]$	$\delta = \frac{a^2}{R} = \frac{1}{2}\left\{\frac{9}{2}\frac{W^2}{E^{*2}R}\right\}^{1/3}$
Mean contact pressure	$\bar{p} = \frac{W'}{2b} = \frac{\pi}{4}p_0$	$\bar{p} = \frac{W}{\pi a^2} = \frac{2}{3}p_0$
Mean shear stress	$\tau_{max} = 0.30p_0$ at $x = 0, z = 0.79b$	$\tau_{max} = 0.31p_0$ at $r = 0, z = 0.48a$ for $\nu = 0.3$
Maximum tensile stress	zero	$\frac{1}{3}(1-2\nu)p_0$ at $r = a, z = 0$

Mildly elliptical contacts

If the gap at zero load is $h = \frac{1}{2}\mathrm{A}x^2 + \frac{1}{2}\mathrm{B}y^2$, and $0.2 < \mathrm{A/B} < 5$

Ratio of semi-axes $b / a \cong (\mathrm{A/B})^{2/3}$

To calculate the contact **area** or Hertz **stress** use the circular contact equations with $R = (\mathrm{AB})^{-1/2}$ or better $R_e = [\mathrm{AB}(\mathrm{A}+\mathrm{B})/2]^{-1/3}$.

For **approach** use circular contact equation with $R = (\mathrm{AB})^{-1/2}$ (**not** R_e)

ELASTOHYDRODYNAMIC LUBRICATION

Formulae for line contact film thickness

$\bar{U}$ is the entraining velocity, R is the effective radius of curvature and E^* is the contact modulus (see elastic contact stress formulae).

Rigid isoviscous (Kapitza)

$$h_c = 4.9 \frac{\bar{U}\eta_0 RL}{W}$$

Ertel-Grubin

$$\frac{\bar{h}}{R} = 1.37 \left(\frac{\eta_0 \alpha (2\bar{U})}{R} \right)^{3/4} \left(\frac{E^* RL}{W} \right)^{1/8}$$

Dowson and Higginson

$$\frac{\bar{h}}{R} = 1.6 \left(2\alpha E^*\right)^{0.54} \left(\frac{\bar{U}\eta_0}{2E^* R} \right)^{0.7} \left(\frac{W/L}{2E^* R} \right)^{-0.13}$$

EPICYCLIC SPEED RULE

$$\omega_s = (1+R)\omega_c - R\omega_a \quad \text{where } R = \frac{A}{S}$$

MPFS, DJC, JAW
January 2009

4C16 2010 Answers

1. b) ii) $p = \frac{3U\eta}{h^3}\left(y^2 - L^2\right)\frac{\mathrm{d}h}{\mathrm{d}x}$

c) $x = -\sqrt{\frac{2h_0R}{5}}$

d) $W = \frac{2U\eta L^3}{h_0^2}$

2. b) iii) $0 < \frac{\omega_a}{\omega_c} < \left(\frac{1}{R} + 1\right)$

3. a) $\omega = 1200\pi \text{ rad/s}$

$a_0 = 7200\pi \text{ m/s}^2$

b) 6.43 GPa

c) 23% reduction